(MARKING SCHEME)

11 March 2011

ECSE 351B Electromagnetic Fields
Assignment 5

Due: 22 March 2011

1/2 1. *Problem 3-18 from the course text.*

1/2 2. Problem 3-19 from the course text.

1 3. Problem 3-24 from the course text.

5 4. Problem 3-26 from the course text.

1 5. Problem 3-28 form the course text.

1 6. A very long (infinite) cylindrical magnet has constant magnetization everywhere inside the magnet equal to **M** = 5000 (A/m) directed along the axis. The diameter of the magnet is 0.04 (m). Calculate the magnetic flux density due to this magnet everywhere in space. Design an equivalent free-space system (involving only conductors) that produces an identical magnetic flux density everywhere in space.

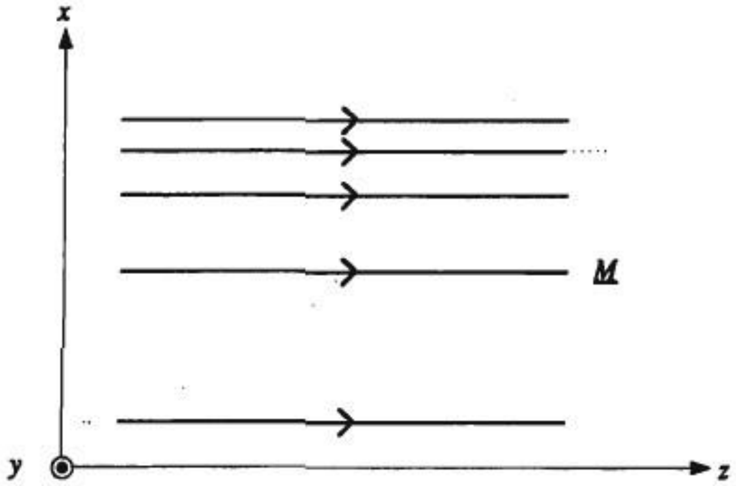
1 7. Consider the flux plot shown below of the static magnetization field $\underline{M}$ in a linear *material region. Assuming that $\underline{M}$ does not vary in the y direction, determine the* direction of the net *bound* volume current density at any given point within the region.

$\frac{}{10}$

(a) Negative *x* direction. (b) Negative *y* direction.

(c) Positive *x* direction. (d) Positive *y* direction.

(e) I don't know …



Problems # 1, 2, 3

3-18.) Given $N = 3.3\times10^{28}$ atoms/m^3, $\bar{H} = 4\bar{a}_z$ A/m, $m = \bar{a}_z\, 1.2\times10^{-24}$ A·m^2.

(a) The volume magnetization density becomes, from (3-55):

$$\bar{M} = N\bar{m} = (3.3\times10^{28})\bar{a}_z 1.2\times10^{-24}$$
$$= 39.6 \text{ kA/m} \quad ...(1)$$

making, from (3-60):

$$\chi_m = \frac{M}{H} = \frac{39600}{4} = 9900 \quad ...(2)$$

so the relative permeability, from (3-62), is

$$\mu_r = 1 + \chi_m = 9901 \quad ...(3)$$

(b) From (3-64c):

$$\bar{B} = \mu\bar{H} = \mu_r\mu_0\bar{H}$$
$$= (9901)(4\pi\cdot10^{-7})4\bar{a}_z$$
$$= 49.8 \text{ mWb/m}^2 \quad ...(4)$$

3.19) Given $\chi_m = 59$,
$\bar{B} = 0.01\,\bar{a}_x$ Wb/m^2.

$$\mu_r = 1 + \chi_m = 60.$$

From $\bar{B} = \mu\bar{H} = \mu_r\mu_0\bar{H}$

$$\Rightarrow \bar{H} = \frac{\bar{B}}{\mu_r\mu_0} = \frac{0.01}{60(4\pi10^{-7})}\bar{a}_x$$
$$= \bar{a}_x 132.63 \text{ A/m}.$$

HE VOLUME MAGNETIZATION DENSITY
N BE FOUND FROM:

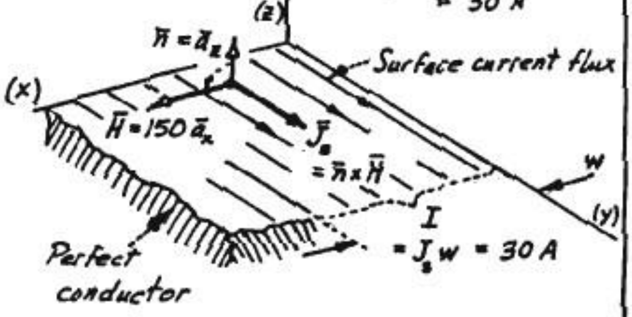
$$\bar{M} = \chi_m\bar{H} = 59(132.63)$$
$$= 7{,}825\,\bar{a}_x \text{ A/m}.$$

$$\bar{H} = \frac{\bar{B}}{\mu_0} - \bar{M} \Rightarrow \bar{M} = \frac{\bar{B}}{\mu_0} - \bar{H} = 7{,}825\bar{a}_x \text{ A/m}$$

3-24.) (a) From (3-72),

$$\bar{J}_s = \bar{n}\times\bar{H} = \bar{a}_z\times\bar{a}_x 150 = 150\bar{a}_y \text{ A/m}$$

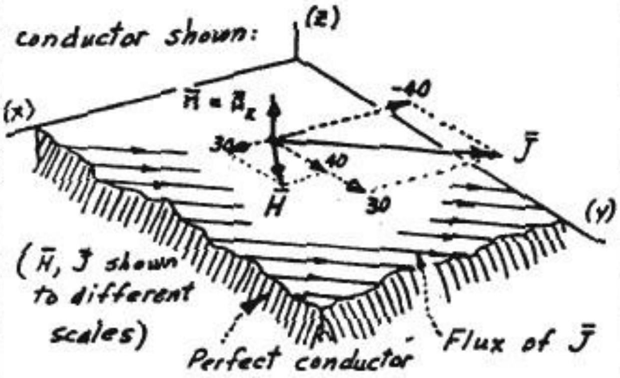
In $w = 20$ cm width, $I = J_s w = 150(0.2)$
$= 30$ A



(b) Given $\bar{H} = \bar{a}_x 30 + \bar{a}_y 40$ A/m:

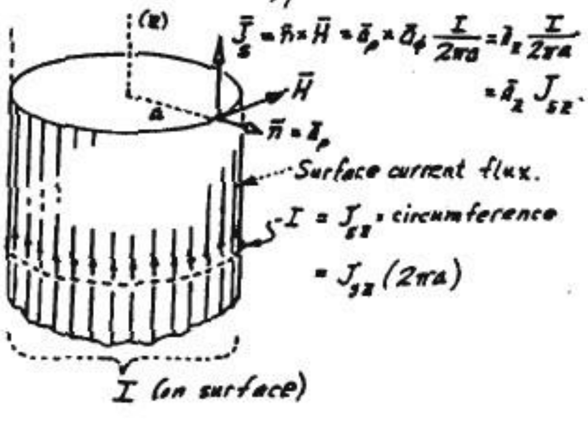
$$\bar{J}_s = \bar{n}\times\bar{H} = \bar{a}_z\times(\bar{a}_x 30 + \bar{a}_y 40)$$
$$= -40\bar{a}_x + 30\bar{a}_y \text{ A/m}$$

on the surface $z=0$ of the perfect conductor shown:



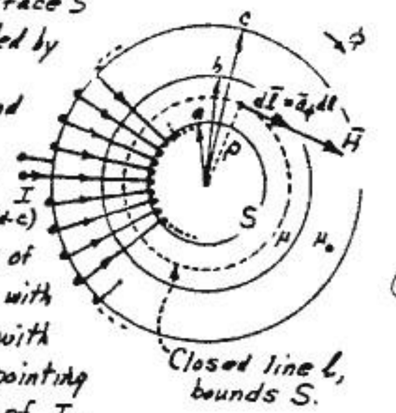
(c) Given is $B_\phi = \frac{\mu_0 I}{2\pi\rho}$, or $\bar{H} = \bar{a}_\phi\frac{I}{2\pi\rho}$

outside a round, perfect conductor:



Problem # 4

3-26.) (The surface S shown is bounded by the symmetric closed path ℓ, and is pierced by the total current $i = nI$.)



(a) Grasping any of the wires shown with the right hand, with the thumb thus pointing in the direction of I, the fingers within the interior of the toroid then point in the direction of $\bar{H}$ produced by I. $\bar{H}$ is thus ϕ-directed as shown (with the z axis for this circular cylindrical system into the paper). (1/4)

(b) Ampère's integral law in static form, from (3-66), is applied to the symmetric closed path ℓ shown; so $\oint_\ell \bar{H}\cdot d\bar{\ell} = \int_S \bar{J}\cdot d\bar{s} = i$ becomes

$$\oint_\ell \bar{H}\cdot d\bar{\ell} = \oint_\ell (\bar{a}_\phi H_\phi)\cdot\bar{a}_\phi d\ell = i\Big]_{\text{thru } S} = nI$$

and with H_ϕ constant on the symmetric ℓ, solving for it obtains

$$H_\phi = \frac{nI}{\oint_\ell d\ell} = \frac{nI}{2\pi\rho} \qquad \ldots(1) \quad (1/2)$$

a result correct for any radius ρ within the toroidal winding whether in the air or the iron region, since no more current is intercepted by ℓ upon expanding its radius into the air region ($b<\rho<c$).

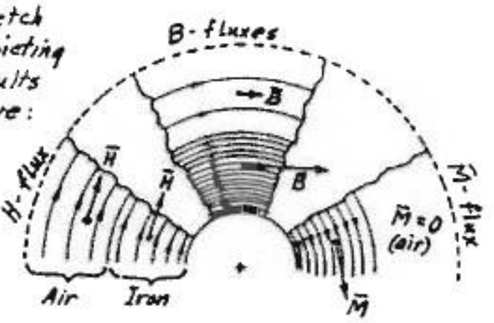
$\bar{B}$ is next found from (3-64c), $\bar{B} = \mu\bar{H}$. Thus,

$$\bar{B} = \begin{cases} \mu\bar{H} = \bar{a}_\phi \dfrac{\mu n I}{2\pi\rho} & (\text{in Fe: } a<\rho<b) \quad \ldots(2) \quad (1/2) \\ \mu_0\bar{H} = \bar{a}_\phi \dfrac{\mu_0 n I}{2\pi\rho} & (\text{in air}: b<\rho<c) \quad \ldots(3) \quad (1/2) \end{cases}$$

(c) The volume magnetization intensity $\bar{M}$ is found from (3-60) and (3-62):

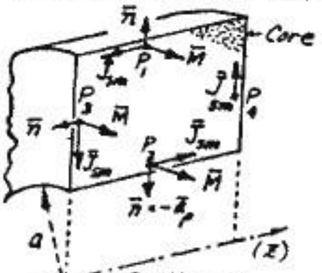
$$\bar{M} = \begin{cases} \chi_m\bar{H} = \bar{a}_\phi(\mu_r - 1)\dfrac{nI}{2\pi\rho} & (a<\rho<b) \quad \ldots(4) \quad (1/4) \\ (\chi_m = 0): = 0 & (\text{air}: b<\rho<c) \quad \ldots(5) \quad (1/4) \end{cases}$$

Sketch depicting results above:



(1/2)

(d) With $\bar{M}$ in the iron core $\frac{1}{\rho}$-dependent, from (4), using (3-56), the volume density of (bound) magnetization current $\bar{J}_m$ $(= \nabla\times\bar{M})$ is zero.



Section of the Iron Core

From (3-74), the (bound) surface magnetization current densities are given by

$$\bar{J}_{sm} = -\bar{n}\times\bar{M}$$

Thus, at P_2 in the diagram (at $\rho = a$):

$$\bar{J}_{sm} = -\bar{n}\times\bar{M} = \bar{a}_\rho\times\bar{a}_\phi M_\phi = \bar{a}_z(\mu_r - 1)\frac{nI}{2\pi a} \quad [A/m] \quad \ldots(6) \quad (1/4)$$

Similarly, at P_1, P_3, and P_4 shown:

$$P_1:\ \bar{J}_{sm} = -\bar{a}_z(\mu_r - 1)\frac{nI}{2\pi b} \qquad \ldots(7) \quad (1/4)$$

$$P_3:\ \bar{J}_{sm} = -\bar{a}_\rho(\mu_r - 1)\frac{nI}{2\pi\rho} \quad (a<\rho<b) \qquad \ldots(8) \quad (1/4)$$

$$P_4:\ \bar{J}_{sm} = \bar{a}_\rho(\mu_r - 1)\frac{nI}{2\pi\rho} \quad (a<\rho<b) \qquad \ldots(9) \quad (1/4)$$

(e) At $\rho = a+ = 1$ cm (in Fe core):

$$\bar{H} = \bar{a}_\phi\frac{nI}{2\pi a} = \bar{a}_\phi\frac{100(0.1)}{2\pi(0.01)} = \bar{a}_\phi\ 159.2\ A/m \quad (1/4)$$

$$\bar{B} = \mu\bar{H} = \bar{a}_\phi(10^3)4\pi\cdot10^{-7}\times159.2 = \bar{a}_\phi\ 0.2\ Wb/m^2 \quad (1/4)$$

At $\rho = b- = 1.5$ cm (in Fe core):

$$\bar{H} = \bar{a}_\phi\frac{10}{2\pi(0.015)} = 106.1\,\bar{a}_\phi\ A/m, \quad \bar{B} = 0.133\ Wb/m^2 \quad (1/4)$$

At $\rho = b+ = 1.5$ cm (in air core):

$$\bar{H} = 106.1\,\bar{a}_\phi\ A/m, \quad \bar{B} = \mu_0\bar{H} = 4\pi\cdot10^{-7}(106.1) = 0.133\ mWb/m^2 \quad (1/4)$$

At $\rho = c = 2$cm (in air core):

$$\bar{H} = \bar{a}_\phi\frac{10}{2\pi(0.02)} = 79.6\,\bar{a}_\phi\ A/m, \quad \bar{B} = 0.1\ mWb/m^2 \quad (1/4)$$

/(5)

Problem # 5

3-28) (a) FROM AMPÈRE'S LAW AND SYMMETRY:

$$\oint_{\ell} \bar{H}\cdot d\bar{\ell} = \int_{s} \bar{J}\cdot d\bar{s} = i, \quad \bar{H} = \bar{a}_\phi H_\phi$$

FOR $0<\rho<a$: $\bar{J} = \bar{a}_z J_z$ WITH $J_z = \frac{I}{\pi a^2}$ [A/m²]

$$\therefore,\ 2\pi\rho H_\phi = \frac{I}{\pi a^2}\int_s ds = \frac{I}{\pi a^2}\pi\rho^2 = \frac{I\rho^2}{a^2}$$

$$\Rightarrow H_\phi = \frac{I\rho}{2\pi a^2} \text{ A/m}$$

FOR $a<\rho<b$: $i = I \Rightarrow H_\phi = \frac{I}{2\pi\rho}$

FOR $\rho>b$: $i = I \Rightarrow H_\phi = \frac{I}{2\pi\rho}$

SUMMARY:

$$\begin{cases} 0<\rho<a: \bar{H} = \bar{a}_\phi \frac{I\rho}{2\pi a^2} \text{ A/m}, & \bar{B} = \mu_0\bar{H} = \bar{a}_\phi \frac{\mu_0 I\rho}{2\pi a^2} \text{ wb/m}^2 \\ a<\rho<b: \bar{H} = \bar{a}_\phi \frac{I}{2\pi\rho} \text{ A/m}, & \bar{B} = \mu\bar{H} = \bar{a}_\phi \frac{\mu I}{2\pi\rho} \text{ wb/m}^2 \\ \rho>b: \bar{H} = \bar{a}_\phi \frac{I}{2\pi\rho} \text{ A/m}, & \bar{B} = \mu_0\bar{H} = \bar{a}_\phi \frac{\mu_0 I}{2\pi\rho} \text{ wb/m}^2 \end{cases}$$

(b) $\mu_r = 1 + \chi_m \Rightarrow \chi_m = \mu_r - 1$

$$\bar{M} = \chi_m \bar{H} = (\mu_r - 1)\bar{H} = \underbrace{\bar{a}_\phi (\mu_r - 1)\frac{I}{2\pi\rho}}_{\text{IN THE MAGNETIC SLEEVE}} \text{ [A/m]}$$

H_ϕ IS CONTINUOUS AT THE INTERFACES

B_ϕ & M_ϕ ARE DIS-CONTINUOUS AT THE INTERFACES

(N.B. SEE p.4 FOR SKETCH OF H_ϕ, B_ϕ, & M_ϕ v.s. ρ).

(c) WITHIN THE MAGNETIC SLEEVE:

$$\bar{M} = \bar{a}_\phi (\mu_r - 1)\frac{I}{2\pi\rho}, \quad \therefore,\ \bar{J}_m = \bar{\nabla}\times\bar{M} = 0.$$

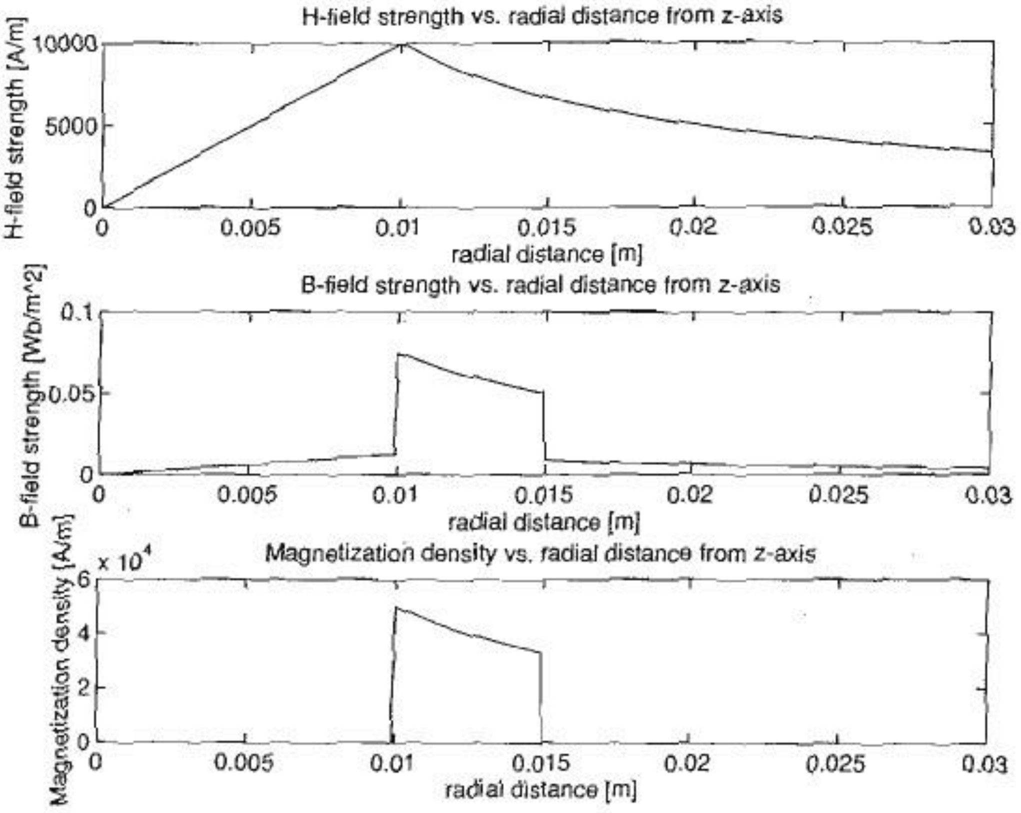
ON THE MAGNETIC SLEEVE SURFACE,

AT $\rho = a$: $\bar{J}_{sm} = -n\times\bar{M} = -(-\bar{a}_\rho)\times\bar{a}_\phi M_\phi = \bar{a}_z(\mu_r - 1)\frac{I}{2\pi\rho}$ [A/m]

AT $\rho = b$: $\bar{J}_{sm} = -n\times\bar{M} = -\bar{a}_\rho\times\bar{a}_\phi M_\phi = -\bar{a}_z(\mu_r - 1)\frac{I}{2\pi\rho}$ [A/m]

—x—

Problem # 5 (continued)



#7) $\underline{J}_m = \nabla \times \underline{M} = -\underline{a}_y C$

i.e. NEGATIVE "y" DIRECTION.

351B ASSIGNMENT #5 – SOLUTIONS OUTLINE

#6).

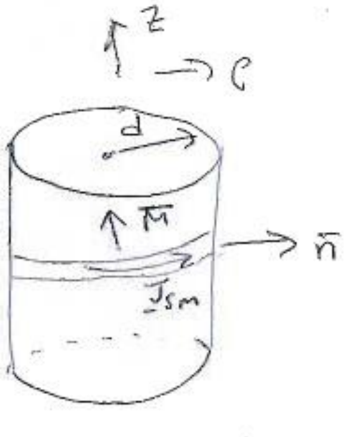


FIG. 1: DIRECTION OF $\bar{J}_{sm}$ FOR UNIFORM MAGNETIZATION

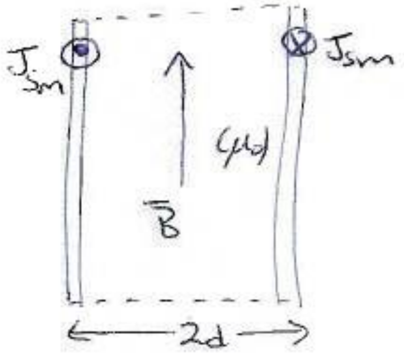


FIG. 2: EQUIVALENT SURFACE CURRENT MODEL

SINCE $\bar{M}$ IS CONSTANT ∴ $\nabla \times \bar{M} = \bar{J}_m = 0$, BUT THERE IS A BOUND SURFACE MAGNETIZATION CURRENT DENSITY:

$$-\bar{n} \times \bar{M} = \bar{J}_{sm} \quad (A/m)$$

$$\Rightarrow \bar{J}_{sm} = -\bar{a}_\rho \times (5000)\,\bar{a}_z = \bar{a}_\phi\,(5000) \quad (A/m)$$

∴ USING THE EQUIVALENT CURRENT-SHEET MODEL (FIG. 2) & AMPÈRE'S LAW: (see p. 45, CHAP. 1. LECTURE NOTES)

$$\boxed{\begin{aligned} \bar{B} &= 2 \times \frac{\mu_0 J_{sm}}{2}\,\bar{a}_z \quad \left(\begin{array}{l}\text{INSIDE THE CYLINDER.}\\ = 0 \ \text{OUTSIDE!}\end{array}\right) \\ \therefore\ \bar{B} &= 4\pi \times 10^{-7} \times 5000 = 0.00628\,\bar{a}_z \ (\text{T or Wb/m}^2) \end{aligned}}$$

CONTINUED . . .

6 (... CONTINUED)

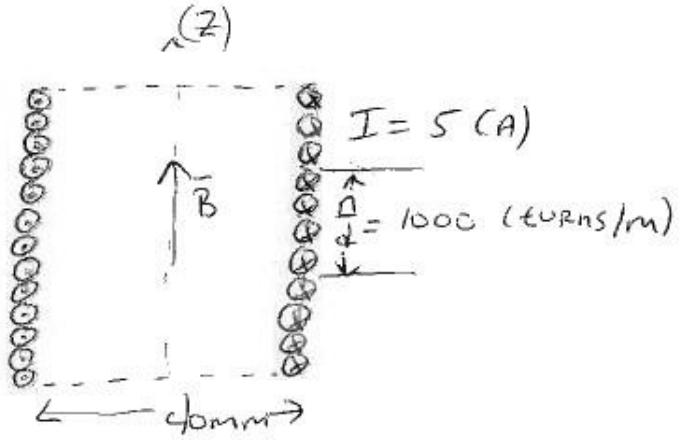


FIG. 3: AN EQUIVALENT SOLENOID IMPLEMENTATION OF THE MODEL IN FIG. (2).

- THE TOTAL CURRENT / METER LENGTH MUST BE 5000 (A/m), OR 5000 A · turns / meter.

∴ ANY CHOICE OF NUMBER OF TURNS IS CORRECT (IN THEORY) IF CHOOSE THE CURRENT APPROPRIATELY.

e.g. CHOOSE 1000 turns/meter

$\Rightarrow I = 5A$

(N.B. 1000 turns/meter ⇒ 1mm diameter WIRE FOR SOLENOID MADE OF A SINGLE LAYER OF WIRES WOUND TIGHTLY. IS 5 (A) TOO HIGH FOR 1(mm) diameter WIRE? $5(A) \Rightarrow 6.4 \times 10^6 \ (A/m^2)$ CURRENT DENSITY, WHICH IS ACCEPTABLE FOR COPPER WIRE!)