Problem # 1

3-1.) (a) Free-electron mobility μ_e is found from (3-9), $\sigma = ne\mu_e$, or

$$\mu_e = \frac{\sigma}{ne} = \frac{5.8\times10^{7}}{10^{29}\times1.6\times10^{-19}} = 3.6\times10^{-3}\ m^2/V\cdot s$$

(b) $\rho_v = -ne = -10^{29}\cdot1.6\times10^{-19} = -1.6\times10^{10}\ C/m^3$

$= -1.6\times10^{4}\ C/mm^3$ cm^3

(c) From (3-4):

$$\bar{v}_d = -\mu_e\bar{E} = -3.6\times10^{-3}\times1\bar{a}_x = -0.0036\bar{a}_x\ m/s$$

$$= -3.6\bar{a}_x\ mm/s$$

Corresponding current density:

$$\bar{J} = \rho_v\bar{v}_d = -ne\bar{v}_d = -1.6\cdot10^{10}(-0.0036\bar{a}_x)$$

$$= 57.6\ MA/m^2$$

$$= 5.76\ kA/cm^2$$

(Use (3-7) to check: $\bar{J} = \sigma\bar{E}$ means

$\sigma = \frac{J_x}{E_x} = \frac{57.6\ M}{1} = 5.7\times10^{7}\ ℧/m$, which checks the given conductivity σ.)

Problem # 2

3-7.) (a) Given: $\bar{E} = \bar{a}_x 1000 x^2 \sin\omega t$ V/m.

$\epsilon_o \bar{E} = \bar{a}_x 8.84 \times 10^{-9} x^2 \sin\omega t$ C/m²

$\bar{P} = \chi_e \epsilon_o \bar{E}$ (with $\chi_e = \epsilon_r - 1 = 1.26$)

$= 1.26\,(8.84\times10^{-9} x^2 \sin\omega t)\bar{a}_x$

$= \bar{a}_x\, 11.1\, x^2 \sin\omega t$ nC/m² $(n = 10^{-9})$

$\bar{D} = \epsilon_r \epsilon_o \bar{E} = 2.26\,(\bar{a}_x 8.84\times10^{-9} x^2 \sin\omega t)$

$= \bar{a}_x\, 20.0\, x^2 \sin\omega t$ nC/m²

$\rho_p = -\nabla\cdot\bar{P} = -\dfrac{\partial P_x}{\partial x} = -\dfrac{\partial}{\partial x}(11.1\, x^2 \sin\omega t)$

$= -22.3\, x \sin\omega t$ nC/m³

$\bar{J}_p = \dfrac{\partial \bar{P}}{\partial t} = \bar{a}_x\, 11.1\, x^2 \dfrac{\partial}{\partial t}(\sin\omega t)$

$= \bar{a}_x\, 11.1\, x^2 \omega\cos\omega t$ nA/m²

(b) Given: $\bar{E}(\rho, t) = \bar{a}_\rho 10^3 \rho \sin\omega t$ V/m.

$\epsilon_o \bar{E} = \bar{a}_\rho 8.84\times10^{-9} \rho \sin\omega t$ C/m²

$\bar{P} = \chi_e \epsilon_o \bar{E} = 1.26\,\epsilon_o \bar{E}$

$= \bar{a}_\rho\, 11.1\,\rho \sin\omega t$ nC/m²

$\bar{D} = \epsilon_r \epsilon_o \bar{E} = 2.26\,\epsilon_o \bar{E}$

$= \bar{a}_\rho\, 20.0\,\rho \sin\omega t$ nC/m²

$\rho_p = -\nabla\cdot\bar{P} = -\dfrac{1}{\rho}\dfrac{\partial}{\partial\rho}(\rho P_\rho) = \dfrac{1}{\rho}\dfrac{\partial}{\partial\rho}(11.1\rho^2 \sin\omega t)$

$= -22.2 \sin\omega t$ nC/m³

$\bar{J}_p = \dfrac{\partial\bar{P}}{\partial t} = \bar{a}_\rho\, 11.1\,\rho\,\omega\cos\omega t$ nA/m²

(c) Given: $\bar{E}(r, t) = \bar{a}_r (10^3/r^2)\sin\omega t$ V/m.

$\epsilon_o \bar{E} = \bar{a}_r (8.84/r^2)\sin\omega t$ nC/m²

$\bar{P} = \chi_e \epsilon_o \bar{E} = 1.26\,\epsilon_o \bar{E}$

$= \bar{a}_r (11.1/r^2)\sin\omega t$ nC/m²

$\bar{D} = \epsilon_r \epsilon_o \bar{E} = 2.26\,\epsilon_o \bar{E}$

$= \bar{a}_r (20.0/r^2)\sin\omega t$ nC/m²

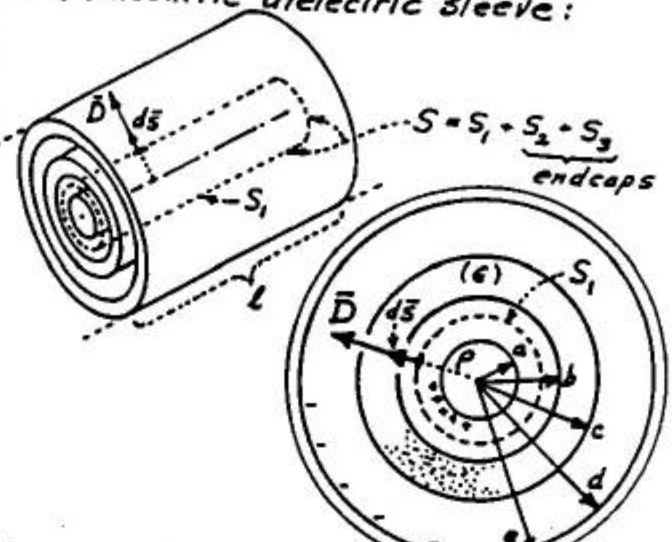
$\rho_p = -\nabla\cdot\bar{P} = \dfrac{-1}{r^2}\dfrac{\partial}{\partial r}(r^2 P_r)$

$= 0$ (since r^2 cancels out).

$\bar{J}_p = \dfrac{\partial\bar{P}}{\partial t} = \bar{a}_r \dfrac{11.1}{r^2}\,\omega\cos\omega t$ nA/m²

Problem # 3

3-11.) Given the coaxial conductor pair with concentric dielectric sleeve:



(a) To find $\bar{D}$ any where between the conductors, construct a symmetric, closed (Gaussian) cylindrical surface S (with endcaps) as shown. Symmetry dictates that $\bar{D} = \bar{a}_\rho D_\rho$ or everywhere radially outward between the conductors. With no flux of $\bar{D}$ emerging from the endcaps, Gauss's law (3-37) applied only to peripheral surface S_1 in the figure yields, with $d\bar{s} = \bar{a}_\rho\, ds$ on S_1:

$$\oint_S \bar{D}\cdot d\bar{s} = \int_{S_1} (\bar{a}_\rho D_\rho)\cdot\bar{a}_\rho\, ds = Q \qquad \ldots(1)$$

with Q the charge enclosed by S in its length ℓ. D_ρ being constant on S_1 from the symmetry, solving (1) for D_ρ obtains

$$D_\rho = \frac{Q}{\int_{S_1} ds} = \frac{Q}{2\pi\rho\ell} \qquad \ldots(2)$$

or, with $\bar{D} = \bar{a}_\rho D_\rho$, the vector for of (2) is

$\bar{D} = \bar{a}_\rho \dfrac{Q}{2\pi\rho\ell}$, making $\bar{E}$ become: 1/2

In the dielectric sleeve ($b<\rho<c$):

$$\bar{E} = \frac{\bar{D}}{\epsilon} = \bar{a}_\rho \frac{Q}{2\pi\epsilon\rho\ell} \qquad (3)$$

In the air regions ($a<\rho<b$ and $c<\rho<d$):

$$\bar{E} = \frac{\bar{D}}{\epsilon_0} = \bar{a}_\rho \frac{Q}{2\pi\epsilon_0\rho\ell} \qquad (4)$$

(b) The volume electric polarization density:

$$\bar{P} = \chi_e\epsilon_0\bar{E} \quad \text{(and making use of (3))}$$

$$= (\epsilon_r - 1)\epsilon_0\bar{E} = (\epsilon_r-1)\epsilon_0 \frac{Q}{2\pi\epsilon_r\epsilon_0\rho\ell}\bar{a}_\rho$$

$$= \frac{\epsilon_r - 1}{\epsilon_r}\frac{Q}{2\pi\rho\ell}\bar{a}_\rho \quad (b<\rho<c) \qquad \ldots(5)$$

From (3-21), the (bound) volume polarization density:

$$\rho_p = -\nabla\cdot\bar{P} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho P_\rho) = 0 \quad \left(\text{since } P_\rho \propto \frac{1}{\rho}\right) \qquad \ldots(6)$$

(c) The surface charge density ρ_s on each conductor is given by (3-45), $\rho_s = \bar{n}\cdot\bar{D}$, with $\bar{n}$ being the unit vector directed normally away from the conductor. So on the inner conductor:

$$\rho_s = \bar{n}\cdot\bar{D} = \bar{a}_\rho\cdot\bar{a}_\rho\frac{Q}{2\pi a\ell} = \frac{Q}{2\pi a\ell} \quad (\rho = a) \ldots(7)$$

On the outer conductor:

$$\rho_s = -\bar{a}_\rho\cdot\bar{a}_\rho\frac{Q}{2\pi d\ell} = -\frac{Q}{2\pi d\ell} \quad (\rho = d) \quad \ldots(8)$$

From (3-47), the bound surface-polarization electric charge densities (with $\bar{P}_1 = 0$ in the adjacent air regions) and using (5) above:

At $\rho = c$: $\rho_{sp} = \bar{n}\cdot\bar{P}_2 = \bar{a}_\rho\cdot\bar{a}_\rho\dfrac{\epsilon_r-1}{\epsilon_r}\dfrac{Q}{2\pi c\ell}$

$$= \frac{\epsilon_r - 1}{\epsilon_r}\frac{Q}{2\pi c\ell} \qquad \ldots(9)$$

At $\rho = b$: $\rho_{sp} = -\bar{a}_\rho\cdot\bar{a}_\rho\dfrac{\epsilon_r-1}{\epsilon_r}\dfrac{Q}{2\pi b\ell}$

$$= \frac{1-\epsilon_r}{\epsilon_r}\frac{Q}{2\pi b\ell} \qquad \ldots(10)$$

(d) With $a = 2$, $b = 4$, $c = 8$ (mm), $d = 1$ cm, $Q/\ell = 10^{-2}\,\mu C/m$, $\epsilon_r = 2.26$:

At $\rho = a$: $\bar{E} = \bar{a}_\rho \dfrac{10^{-8}(10^{12})}{2\pi(8.84)2\cdot10^{-3}} = \bar{a}_\rho\, 90$ kV/m

At $\rho = d$: $\bar{E} = \bar{a}_\rho\, 90\dfrac{a}{d} = \bar{a}_\rho\, 18$ kV/m

At $\rho = a$: $\rho_s = \dfrac{Q}{2\pi a\ell} = \dfrac{10^{-8}}{2\pi(2\cdot10^{-3})} = 0.796\ \mu C/m^2$

At $\rho = d$: $\rho_s = \dfrac{Q}{2\pi d\ell} = 0.796\dfrac{a}{d} = 0.159\ \mu C/m^2$

At $\rho = b\pm$: $\bar{D} = \bar{a}_\rho\dfrac{10^{-8}}{2\pi(4\cdot10^{-3})} = \bar{a}_\rho\, 0.398\ \mu C/m^2$

At $\rho = b+$: $\bar{E} = \dfrac{\bar{D}}{\epsilon} = \bar{a}_\rho\dfrac{0.398}{2.26(8.84)10^{-12}} = \bar{a}_\rho\, 20$ kV

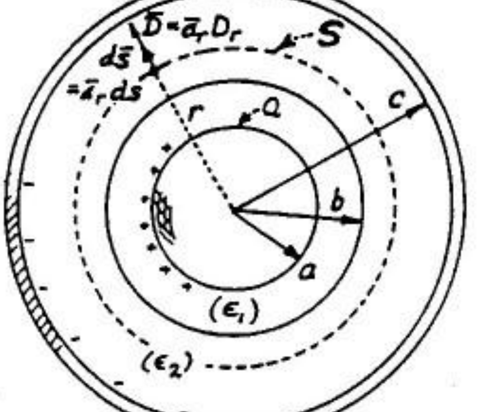
$\rho = b-$: $\bar{E} = \dfrac{\bar{D}}{\epsilon_0} = a_\rho\, 4.5$ (kV/m).

$\rho = b-$: $\underline{P} = 0$

$\rho = b+$: $\underline{P} = \underline{0.222\,\mu C/m^2}\ \underline{a}_\rho$

Problem # 4 (6 MARKS)

3-13.) Insert a spherical, closed Gaussian surface S, of radius r, in either region as shown:



(a)

Symmetry dictates that $\bar{D} = \bar{a}_r D_r$, everywhere radially outward in the region between the conductors. Gauss's law applied to the surface S yields, with charge Q inside S,

$$\oint_s \bar{D} \cdot d\bar{s} = \oint_s (\bar{a}_r D_r) \cdot \bar{a}_r ds = Q$$

and D_r being constant on S, solving obtains

$$D_r = \frac{Q}{\oint_s ds} = \frac{Q}{4\pi r^2} \quad \text{....(1)} \quad 1/2$$

yielding in region 1:

$$\bar{E} = \frac{Q}{4\pi\epsilon_1 r^2}\bar{a}_r \quad (a<r<b) \quad \text{...(2)}$$

and in region 2:

$$\bar{E} = \frac{Q}{4\pi\epsilon_2 r^2}\bar{a}_r \quad (b<r<c) \quad \text{...(3)}$$

1/2

(b) $$\bar{P} = \chi_e \epsilon_0 \bar{E} = (\epsilon_{r1}-1)\epsilon_0 \frac{Q}{4\pi\epsilon_{r1}\epsilon_0 r^2}\bar{a}_r$$

$$= \frac{\epsilon_{r1}-1}{\epsilon_{r1}} \frac{Q}{4\pi r^2}\bar{a}_r (a<r<b) \quad \text{..(4)}$$

$$\bar{P} = \frac{\epsilon_{r2}-1}{\epsilon_{r2}} \frac{Q}{4\pi r^2}\bar{a}_r (b<r<c) \quad \text{...(5)} \quad 1/2$$

From (3-21),

$$-\rho_0 = \nabla \cdot \bar{P} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 P_r) = 0 \quad (\text{since } P_r \propto \tfrac{1}{r^2}) \quad 1/2$$

...(6)

(c) Use boundary condition (3-45) to find ρ_s:

$$\rho_s = \bar{n} \cdot \bar{D} = \bar{a}_r \cdot \bar{a}_r \frac{Q}{4\pi r^2}\Big]_{r=a} = \frac{Q}{4\pi a^2} \quad (r=a) \quad \text{....(6)}$$

$$\rho_s = -\bar{a}_r \cdot \bar{a}_r \frac{Q}{4\pi r^2}\Big]_{r=c} = \frac{-Q}{4\pi c^2} \quad (r=c) \quad \text{...(7)}$$

1

At $r=b$, use (3-46), with $n = -\bar{a}_r$:

$$\rho_{sp} = \bar{n} \cdot (\bar{P}_2 - \bar{P}_1) = -\bar{a}_r \cdot \bar{a}_r \left(\frac{\epsilon_{r2}-1}{\epsilon_{r2}} - \frac{\epsilon_{r1}-1}{\epsilon_{r1}}\right)\frac{Q}{4\pi b^2}$$

$$= -\frac{Q}{4\pi b^2}\left(\frac{1}{\epsilon_{r1}} - \frac{1}{\epsilon_{r2}}\right) \quad \text{... (8)}$$

1

(d) Let $a=1$, $b=1.02$, $c=1.05$ m, $\epsilon_{r1}=2.26$, $\epsilon_{r2}=1$, $Q=0.1\,\mu C$:

At $r=a$: $$\bar{E} = \bar{a}_r \frac{Q}{4\pi\epsilon_1 a^2} = \bar{a}_r \frac{10^{-7}(10^{12})}{4\pi(2.26)8.84(1)^2}$$

$$= \bar{a}_r \, 398 \text{ V/m} \quad \text{...(9)}$$

At $r=b-$: $$\bar{E} = \bar{a}_r \frac{Q}{4\pi\epsilon_1 b^2} = \bar{a}_r \frac{10^{-7}(10^{12})}{4\pi(2.26)8.84(1.02)^2}$$

$$= \bar{a}_r \, 383 \text{ V/m} \quad \text{...(10)}$$

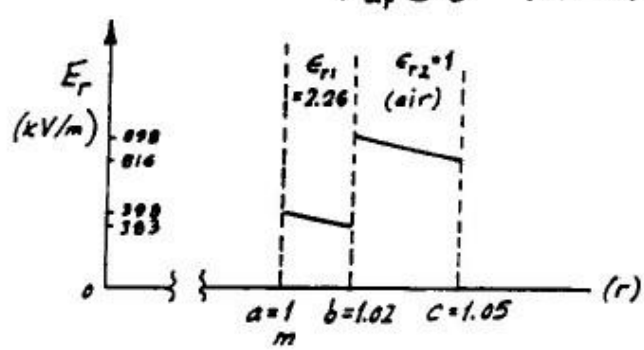
At $r=b+$: (in air) $$\bar{E} = \bar{a}_r \frac{Q}{4\pi\epsilon_2 b^2} = \bar{a}_r \frac{10^{-7}(10^{12})}{4\pi(1)8.84(1.02)^2}$$

$$= \bar{a}_r \, 865 \text{ V/m} \quad \text{..(11)}$$

2

At $r=c$: (in air) $$\bar{E} = \bar{a}_r \frac{Q}{4\pi\epsilon_2 c^2} = \bar{a}_r \frac{10^{-7}(10^{12})}{4\pi(1)8.84(1.05)^2}$$

$$= \bar{a}_r \, 816 \text{ V/m} \quad \text{...(12)}$$



TOTAL: 6

PROBLEM # 5

"A" — UPWARDS

(1 MARK)