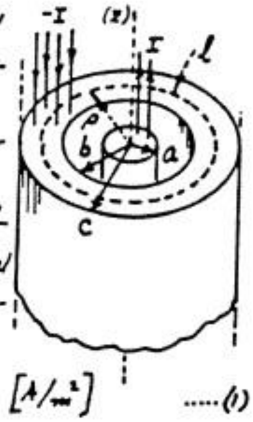
Problem # 1

1-34. (a) The +z and -z directed current densities in the inner and outer conductors respectively, from their total currents divided by their cross-sectional areas, yields current densities:



$$\bar{J}_1 = \bar{a}_z \frac{I}{A_{inner}} = \bar{a}_z \frac{I}{\pi a^2} \quad [A/m^2] \quad \text{.....(1)}$$

and

$$\bar{J}_2 = -\bar{a}_z \frac{I}{A_{outer}} = -\bar{a}_z \frac{I}{\pi(c^2-b^2)} \quad [A/m^2] \quad \text{....(2)}$$

(b) The $\bar{B}$ fields in the inner conductor ($0<\rho<a$) and in the adjacent air region ($a<\rho<b$) follow from Ampère's law (1-63) applied to the symmetric closed line ℓ located in those regions, yielding the same results as (1-64) obtained in Example 1-13.

Placing ℓ now as shown in the diagram here, for $b<\rho<c$ in the outer conductor, Ampère's law becomes

$$\frac{1}{\mu_0}\oint_\ell B_\phi \bar{a}_\phi \cdot \bar{a}_\phi \, d\ell = i \Big]_{\substack{\text{through } S \\ \text{bounded by } \ell}}$$

$$= \underset{\substack{\text{Current} \\ \text{in center conductor.}}}{I} - \underbrace{I \frac{\pi\rho^2 - \pi b^2}{\pi c^2 - \pi b^2}}_{\substack{\text{Current (down)} \\ \text{in the portion of the} \\ \text{outer conductor bounded} \\ \text{by } \ell.}},$$

whence

$$\frac{B_\phi}{\mu_0} 2\pi\rho = I\left[1 - \frac{\rho^2-b^2}{c^2-b^2}\right]$$

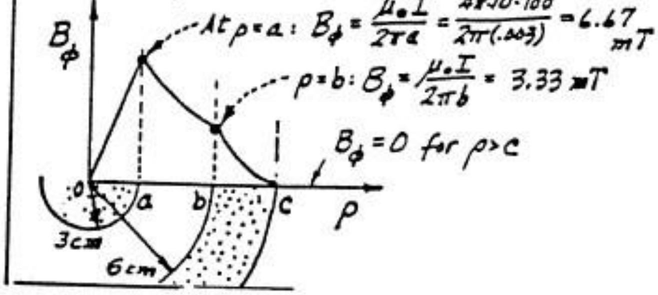
$$= I\left[\frac{c^2-b^2-(\rho^2-b^2)}{c^2-b^2}\right]$$

so that within the outer conductor:

$$B_\phi = \frac{\mu_0 I}{2\pi\rho}\frac{c^2-\rho^2}{c^2-b^2} \quad (b<\rho<c) \quad \text{....(3)}$$

Outside this system (for $\rho>c$), $B_\phi = 0$, since the net current i (through S bounded by the closed line ℓ required upon applying Ampère's law) is $I - I = 0$.

(C) Combining the results of (1-64) and (3) above yields the graph of B_ϕ versus ρ:

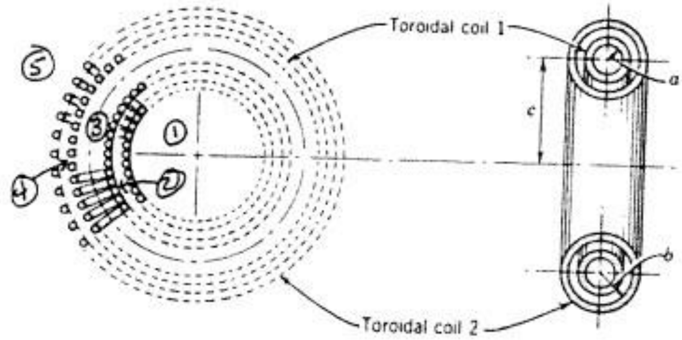


$$\underline{B} = \frac{\mu_0 I \rho}{2\pi a^2} \underline{a}_\phi \; (Wb/m^2), \; 0<\rho<a$$

$$\underline{B} = \frac{\mu_0 I}{2\pi\rho} \underline{a}_\phi \; (Wb/m^2), \; a<\rho<b$$

Problem # 2

1-39)



- The field inside an n-turn, closely-wound toroid of circular cross-section is given by:

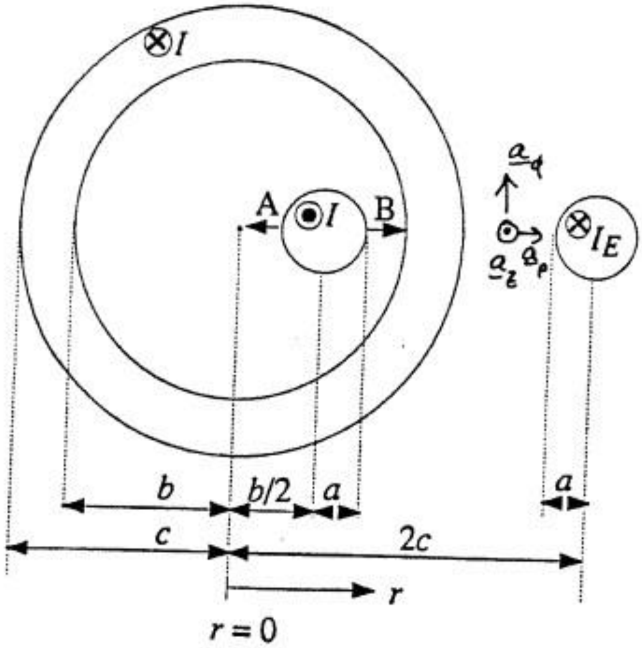
$$\underline{B} = \underline{a}_\phi \frac{\mu_0 n I}{2\pi\rho} \quad \text{(see Example 1-17)}$$

where I is the current in each turn.

- By superposition, for a pair of co-axial toroids each with n-turns and current I in each turn; the $\underline{B}$-field in each of the five regions shown above is given by:

(a) Currents in same direction	(b) Currents in opposite directions
① $\underline{B} = 0$ (Wb/m²)	① $\underline{B} = 0$ (Wb/m²)
② $\underline{B} = \underline{a}_\phi \frac{\mu_0 n I}{2\pi\rho}$ (Wb/m²)	② $\underline{B} = \frac{\mu_0 n I}{2\pi\rho}$ (Wb/m²)
③ $\underline{B} = \underline{a}_\phi \frac{\mu_0 n I}{\pi\rho}$ (Wb/m²)	③ $\underline{B} = 0$ (Wb/m²)
④ $\underline{B} = \underline{a}_\phi \frac{\mu_0 n I}{2\pi\rho}$ (Wb/m²)	④ $\underline{B} = -\underline{a}_\phi \frac{\mu_0 n I}{2\pi\rho}$ (Wb/m²)
⑤ $\underline{B} = 0$ (Wb/m²)	⑤ $\underline{B} = 0$ (Wb/m²)

Problem # 3



- From symmetry (circ. cylindrical coords.), all $\underline{B}$-fields are ϕ-DIRECTED, i.e.,

$$\underline{B}(\rho) = \underline{a}_\phi B_\phi(\rho).$$

- From Ampère's Law (for static fields), we find the following at the location of I_E:

$$|\underline{B}|_{I_\odot} \propto \frac{\mu_0 I}{2\pi(2c-b/2)}$$ (directed upwards, i.e., in the $\underline{a}_\phi$ direction.)

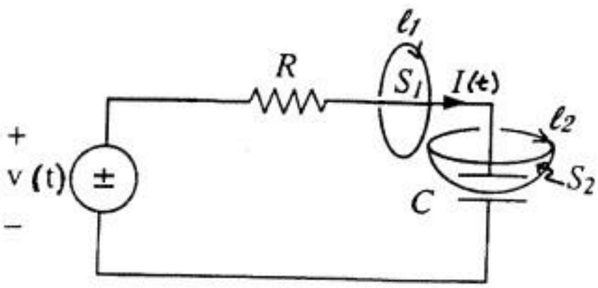
$$|\underline{B}|_{I_\otimes} \propto \frac{\mu_0 I}{2\pi(2c)}$$ (directed downwards, i.e., in the -ve $\underline{a}_\phi$ direction.)

HENCE, THE DIRECTION OF THE NET FORCE (per unit length) ACTING ON I_E WILL BE GIVEN BY:

$$\underline{F} \propto I_E(-\underline{a}_z) \times \left[\frac{\mu_0 I}{2\pi(2c-b/2)} - \frac{\mu_0 I}{4\pi c}\right](+\underline{a}_\phi)$$

$\Rightarrow$ $+\underline{a}_\rho$ DIRECTION, i.e., "B" DIRECTION.

Problem # 4



- FOR l_1 & S_1:

$$\frac{1}{\mu_0}\int_{l_1} \underline{B}\cdot d\underline{l} \neq 0 \quad \text{\&} \quad \int_{S_1} \underline{J}\cdot d\underline{s} = I(t) \neq 0$$

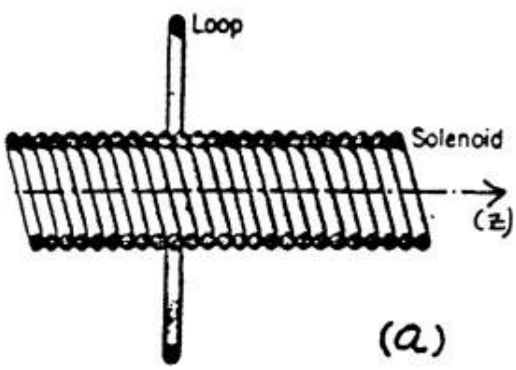
BUT, FOR l_2 & S_2:

$$\frac{1}{\mu_0}\int_{l_2} \underline{B}\cdot d\underline{l} \neq 0$$ (SAME $\underline{B}$ AS FOR l_1!)

& $\int_{S_2} \underline{J}\cdot d\underline{s} = 0 \Rightarrow$ A CONTRADICTION!

UNLESS, WE TAKE INTO ACCOUNT THE DISPLACEMENT CURRENT TERM: $\frac{1}{\mu_0}\oint_{l_2} \underline{B}\cdot d\underline{l} = \int_{S_2} \underline{J}\cdot d\underline{s} + \underbrace{\frac{d}{dt}\int_{S_2} (\varepsilon_0 \underline{E})\cdot d\underline{s}}_{\neq 0}$.

Problem #5



(a)

SOLENOID:

radius = 0.15(m)

winding density:

$\frac{n}{d} = 25\times 10^3$ turns/m

$I(t) = (10t - 50)$

(a) • For an infinitely long, closely-wound solenoid having n-turns in every length d, the $\underline{B}$-field will be z-directed and completely contained within the solenoidal coil, i.e.,

$$(1)\ \underline{B} = \underline{a}_z\, \mu_0 \frac{n}{d} I \quad \text{(See Example 1-17)}$$

• Assuming the magnetoquasistatic approximation is valid:

$$(2)\ \underline{B}(t) = \underline{a}_z \mu_0 \frac{n}{d}(10t - 50)$$

• Applying Faraday's Law around the loop (ring) shown

$$(3)\ V(t) = -\frac{d}{dt}\int_S \underline{B}\cdot d\underline{s} = -\frac{d}{dt}\left[\mu_0 \frac{n}{d}(10t-50)\right]\int_S ds$$

CIRCULAR CROSS-SECTIONAL AREA OF SOLENOID $= \pi(0.15)^2$

$$= -10\mu_0 (2500)(225\cdot 10^{-4})\pi = -562.5\mu_0\pi \text{ (VOLTS)}$$

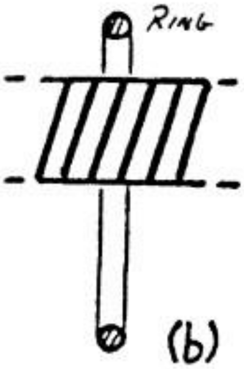
⇒ The induced current in the ring is given by

$$(4)\ |I_{ind}| = |V(t)|/R = \frac{562.5}{25}\mu_0\pi = 22.5\mu_0\pi \text{ (A)} \ //$$

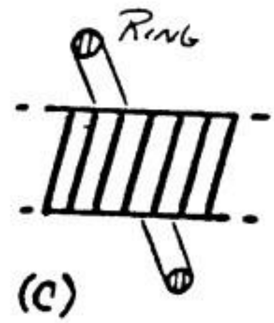
$$= 88.8\ (\mu A)\ //$$

Problem # 5 (CONTINUED)

(b)



NO CHANGE IN THE INDUCED CURRENT; SAME AMOUNT OF B-FIELD FLUX "THREADS" THE LOOP AS IN THE ORIGINAL CASE!



- IF THE RING HAD A RADIUS OF 30 (cm) INSTEAD OF 25 (cm), THE INDUCED CURRENT WOULD NOT CHANGE! (ASSUMING THE SAME RESISTANCE R FOR THE RING.)

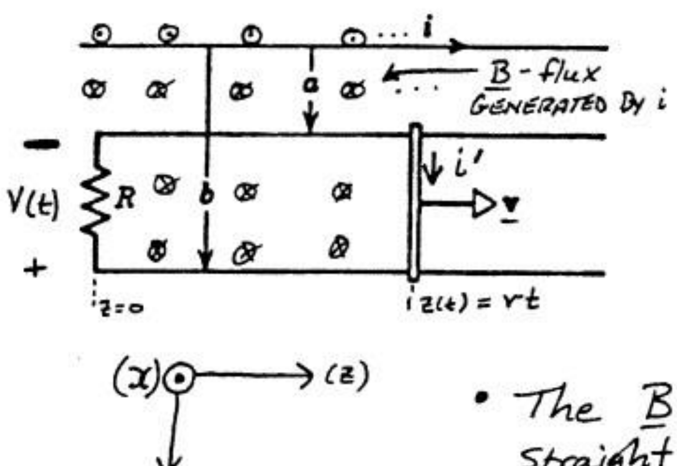
N.B. THE INDUCED emf IS INDEPENDENT OF THE RING RADIUS

IF THE RESISTANCE IS ASSUMED TO INCREASE WITH THE RADIUS, THEN

$$|I| = \frac{|emf|}{R}$$

WILL DECREASE, SINCE emf IS NOT AFFECTED BY THE RADIUS.

Problem # 6



$\underline{v} = 25$ (m/s)

$i = 150$ (A)

$R = 5$ (Ω)

$a = 0.01$ (m)

$b = 0.20$ (m)

- The $\underline{B}$-field produced by a long, infinite, straight wire carrying a current i is given by:

(1) $\underline{B} = \underline{a}_\phi \dfrac{\mu_0 i}{2\pi\rho}$, WHERE ρ is the radial distance from the wire.

- Applying Faraday's Law to the closed loop formed by the conducting rails and the conducting rod:

$$(2)\quad V(t) = -\frac{d}{dt}\int_S \underline{B}\cdot d\underline{s} = -\frac{d}{dt}\iint_S \frac{\mu_0 i}{2\pi y}\,dy\,dz$$

$$= -\frac{d}{dt}\int_{z=0}^{z=vt} dz \int_{y=a}^{b} \frac{\mu_0 i}{2\pi y}\,dy = -\frac{d}{dt}\left[vt\,\frac{\mu_0 i}{2\pi}\ln(b/a)\right]$$

$$\Rightarrow V(t) = -\frac{v\mu_0 i}{2\pi}\ln(b/a) = -\frac{4\pi\cdot 10^{-7}}{2\pi}(150)\ln\left(\frac{0.2}{0.01}\right)(25) \cong -2.25\,(\text{mV})$$

$\Rightarrow$ INDUCED CURRENT $i' = V(t)/R = -0.45$ (mA) //

* N.B. THE NEGATIVE SIGN IN THE ANSWER INDICATES THAT $|V(t)|$ AND $|i'|$ HAVE POLARITIES OPPOSITE TO THOSE SHOWN IN THE DIAGRAM ABOVE.