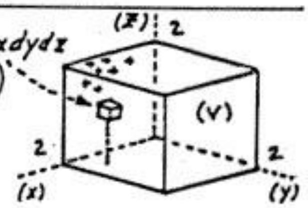
Problem # 1

1-21.)

(a) $\rho_v = xy^2e^{-2z}$ (mC/m^3) in the cube:

$dv = dx\,dy\,dz$



$$q = \int_V \rho_v \, dv$$

$$= \iiint xy^2e^{-2z} \; dx\,dy\,dz$$ (and use product-separability)

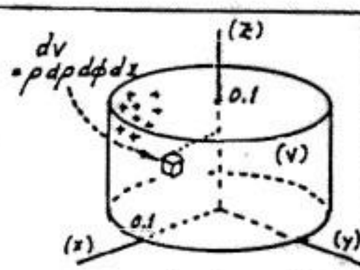
$$= \int_0^2 x\,dx \int_0^2 y^2 dy \int_0^2 e^{-2z} dz = \left[\frac{x^2}{2}\right]_0^2 \left[\frac{y^3}{3}\right]_0^2 \left[-\frac{e^{-2z}}{2}\right]_0^2$$

$$= \frac{-1}{12}(2)^2(2)^3(e^{-4}-1) = \underline{2.62(mC)}$$

(b)

$= 10\rho z$ in the cylinder:

$dv = \rho\,d\rho\,d\phi\,dz$



$$q = \int_V \rho_v \, dv$$

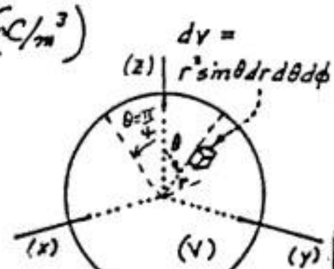
$$\iiint (10\rho z)\rho\,d\rho\,d\phi\,dz$$ (and use the product separability)

$$10\int_0^{0.1} \rho^2 d\rho \int_0^{2\pi} d\phi \int_0^{0.1} z\,dz = 10\left[\frac{\rho^3}{3}\right]_0^{0.1} 2\pi \left[\frac{z^2}{2}\right]_0^{0.1}$$

$$= \frac{10}{6}(0.1)^3 2\pi (0.1)^2 = \underline{104.7\,(\mu C)}$$

(c) $\rho_v = 20r^2\cos^2\theta$ (mC/m^3) in the cone:

$dv = r^2 \sin\theta\,dr\,d\theta\,d\phi$



$$\int_V \rho_v \, dv$$

$$= \iiint (20r^2\cos^2\theta) r^2 \sin\theta\,dr\,d\theta\,d\phi$$

$$= 20\int_0^2 r^4 dr \int_0^{\pi/4} \cos^2\theta \sin\theta\,d\theta \int_0^{2\pi} d\phi$$

form: $\int u^2 du = \frac{u^3}{3} = -\frac{\cos^3\theta}{3}\Big]_0^{\frac{\pi}{4}}$

$$= 20\left.\frac{r^5}{5}\right]_0^2 (0.215)\, 2\pi = 0.173\,(C)$$

Problem # 2

(a)

$\mathbf{F} = Q\,(\mathbf{E} + \mathbf{u}\times\mathbf{B})$, where $\mathbf{E} = x\,\hat{\mathbf{a}}_x + xy\,\hat{\mathbf{a}}_z$ [V/m] , $\mathbf{B} = y\,\hat{\mathbf{a}}_x + x\,\hat{\mathbf{a}}_y$ [T] ,

$\mathbf{u} = 3\hat{\mathbf{a}}_x - 2.5\hat{\mathbf{a}}_y + 1.5\hat{\mathbf{a}}_z$ [m/s] , and $Q = 1$ [μC] . At $P(1,-2,1)$, $\mathbf{B} = \hat{\mathbf{a}}_x - 2\,\hat{\mathbf{a}}_z$, $\mathrm{E} = -2\,\hat{\mathbf{a}}_x + \hat{\mathbf{a}}_y$

$$\mathbf{u}\times\mathbf{B} = \begin{vmatrix} \hat{\mathbf{a}}_x & \hat{\mathbf{a}}_y & \hat{\mathbf{a}}_z \\ 3 & -2.5 & 1.5 \\ -2 & 1 & 0 \end{vmatrix} = -1.5\,\hat{\mathbf{a}}_x - 3\,\hat{\mathbf{a}}_y - 2\,\hat{\mathbf{a}}_z$$

Substituting: $\mathbf{F} = -0.5\,\hat{\mathbf{a}}_x - 3\,\hat{\mathbf{a}}_y - 4\,\hat{\mathbf{a}}_z$ [μN] .

(b)

Given: $\mathbf{F} = -320\,\hat{\mathbf{a}}_x + 150\hat{\mathbf{a}}_y$ [μN] when $Q = +3$ [μC], $\mathbf{u} = 2.5\,\hat{\mathbf{a}}_x - 3.2\hat{\mathbf{a}}_y + 4.0\,\hat{\mathbf{a}}_z$ [m/s] ,

and $\mathbf{B} = 200\,\hat{\mathbf{a}}_y$ [T] , so $\mathbf{u}\times\mathbf{B} = \begin{vmatrix} \hat{\mathbf{a}}_x & \hat{\mathbf{a}}_y & \hat{\mathbf{a}}_z \\ 2.5 & -3.2 & 4 \\ 0 & 20 & 0 \end{vmatrix} = -800\,\hat{\mathbf{a}}_x + 500\,\hat{\mathbf{a}}_z$ [V/m]

Solving $\mathbf{F} = Q\,(\mathbf{E} + \mathbf{u}\times\mathbf{B})$ for $\mathbf{E}$ and substituting we find:

$$\mathbf{E} = \frac{\mathbf{F}}{Q} - \mathbf{u}\times\mathbf{B} = 693.3\,\hat{\mathbf{a}}_x + 50\,\hat{\mathbf{a}}_y - 500\,\hat{\mathbf{a}}_z \text{ [V/m]}$$

Problem # 3

(a) Total charge q in sphere of radius r_o:

$q = \int_v \rho_v \, dv$, with $\rho_v = \rho_o \frac{r}{r_o}$

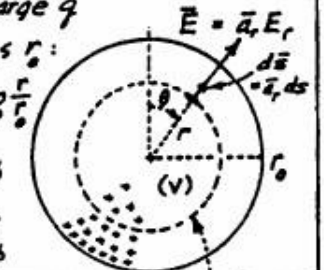
$= \int_v \rho_o \frac{r}{r_o} r^2 \sin\theta \, dr \, d\theta \, d\phi$

$= \frac{\rho_o}{r_o} \int_0^{r_o} r^3 dr \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi$

(product-separable)

$= \frac{\rho_o}{r_o} \left[\frac{r^4}{4}\right]_0^{r_o} (2)(2\pi)$

$= \pi \rho_o r_o^3$ [coulomb]. $= 9.8696 (\mu C)$



(b) Find $\bar{E}$ ($= \bar{a}_r E_r$ only, from symmetry) first inside the charged sphere ($r < r_o$):
Construct a symmetric Gaussian surface S (sphere) of radius r, as shown, on which E_r = constant, from the symmetry. Use Gauss's law (1-53)

$\oint_S \epsilon_o \bar{E} \cdot d\bar{s} = \int_v \rho_v \, dv$ (S, v having the meanings on the diagram)

in which $\bar{E} = \bar{a}_r E_r$, $d\bar{s} = \bar{a}_r ds$ on S, and so with left integral yielding (E_r const. on S):

$\epsilon_o \oint_S E_r \bar{a}_r \cdot \bar{a}_r \, ds = \epsilon_o E_r \oint_S ds = \epsilon_o E_r 4\pi r^2$ ...(1)

The right side (volume integral) of (1-53) yields

$\int_v \rho_v \, dv = \frac{\rho_o}{r_o} \int\int\int r \cdot r^2 \sin\theta \, dr \, d\theta \, d\phi$

$= \frac{\rho_o}{r_o} \int_0^{r} r^3 dr \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi$

$= \frac{\rho_o}{r_o} \frac{r^4}{4} (2) 2\pi = \pi \frac{\rho_o}{r_o} r^4$ [coulomb] ..... (2)

Equating (1), (2) and solving for E_r:

$E_r = \frac{\rho_o r^2}{4 \epsilon_o r_o}$ $(r < r_o)$ ..... (3)

$= 887 r^2$ (MV/m) $(r < r_o)$

If the Gaussian surface S is brought outside the charged spherical cloud ($r > r_o$) then the r-limit in (2) stops at $r = r_o$, yielding $\pi \frac{\rho_o}{r_o} r_o^4 = \pi \rho_o r_o^3$. Equating the latter to (1) then yields $\epsilon_o E_r 4\pi r^2 = \pi \rho_o r_o^3$, or *

$E_r = \frac{\rho_o r_o^3}{4 \epsilon_o r^2}$ $(r > r_o)$ ....(4)

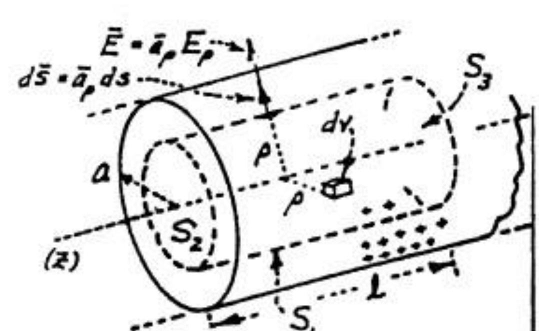
If from part (a), $q = \pi \rho_o r_o^3$ is inserted into (4), it becomes just *

$E_r = \frac{q}{4\pi\epsilon_o r^2}$ $(r > r_o)$: the same form as for a point charge (as though the total charge were concentrated at the origin.

(c) With $\rho_o = 3.14 \times 10^{-3} C/m^3$, $r_o = 0.1$ m into q of part (a): $q = \pi \rho_o r_o^3 = \pi^2 (10^{-3})(0.1)^3 =$ 9.87 (μC), while from (3), E_r at the cloud surface becomes $E_r = 8.87$ MV/m.

* $E_r = \frac{88.7}{r^2}$ (kV/m) $r > r_o$

Problem # 4



Given:
$\rho_v = \rho_0\left(\frac{\rho}{a}\right)$ in long, cylindrical cloud. (Note the axial symmetry.)

(Closed Gaussian surface $S = S_1 + S_2 + S_3$)

(a) Total charge q in length ℓ of cloud:

$$q = \int_V \rho_v\,dv \quad (V \text{ is of radius } a,\ \text{length } \ell)$$

$$= \iiint \rho_0\left(\frac{\rho}{a}\right)\rho\,d\rho\,d\phi\,dz \quad (\text{\& note product-separability})$$

$$= \frac{\rho_0}{a}\int_0^a \rho^2 d\rho \int_0^{2\pi} d\phi \int_0^\ell dz = \frac{\rho_0}{a}\cdot\frac{a^3}{3}\cdot 2\pi\ell$$

$$= \frac{2}{3}\rho_0 \pi a^2 \ell = 9.87\times10^{-5}\,(C) \quad \ldots (1)$$

(b) Make use of symmetry to find $\bar{E}\,(=\bar{a}_\rho E_\rho$ only) through Gauss's law (1-53). Construct the symmetric cylindrical closed Gaussian surface ($S = S_1 + S_2 + S_3$) shown. Gauss's law, first applied within the cloud (at radius $\rho < a$), becomes

$$\oint_S \epsilon_0 \bar{E}\cdot d\bar{s} = \int_V \rho_v\,dv \quad (\text{with } \underline{no} \text{ flux of } \epsilon_0\bar{E} \text{ out of endcaps } S_1, S_3)$$

$$\int_{S_1} \epsilon_0 E_\rho \bar{a}_\rho\cdot\bar{a}_\rho\,ds = \int_V \rho_0\left(\frac{\rho}{a}\right)\rho\,d\rho\,d\phi\,dz$$

$$\epsilon_0 E_\rho \int_{S_1} ds = \frac{\rho_0}{a}\int_0^\rho \rho^2 d\rho \int_0^{2\pi} d\phi \int_0^\ell dz$$

$$\epsilon_0 E_\rho 2\pi\rho\ell = \frac{\rho_0}{a}\frac{\rho^3}{3}2\pi\ell, \quad \text{whence}$$

$$E_\rho = \frac{\rho_0\rho^2}{3\epsilon_0 a}\ (N/C)\ (\rho < a) \quad (2)$$

To find E_ρ for $\rho > a$, expand the Gaussian surface S so that it is outside the cloud ($\rho > a$). Then (1-53) is written

$$\int_{S_1} \epsilon_0 E_\rho \bar{a}_\rho\cdot\bar{a}_\rho\,ds = \frac{\rho_0}{a}\int_0^a \rho^2 d\rho \int_0^{2\pi} d\phi \int_0^\ell dz$$

$$\epsilon_0 E_\rho 2\pi\rho\ell = \frac{\rho_0}{a}\cdot\frac{a^3}{3}\cdot 2\pi\ell, \quad \text{whence}$$

$$E_\rho = \frac{\rho_0 a^2}{3\epsilon_0\rho} \quad (\rho > a) \quad \ldots (3)$$

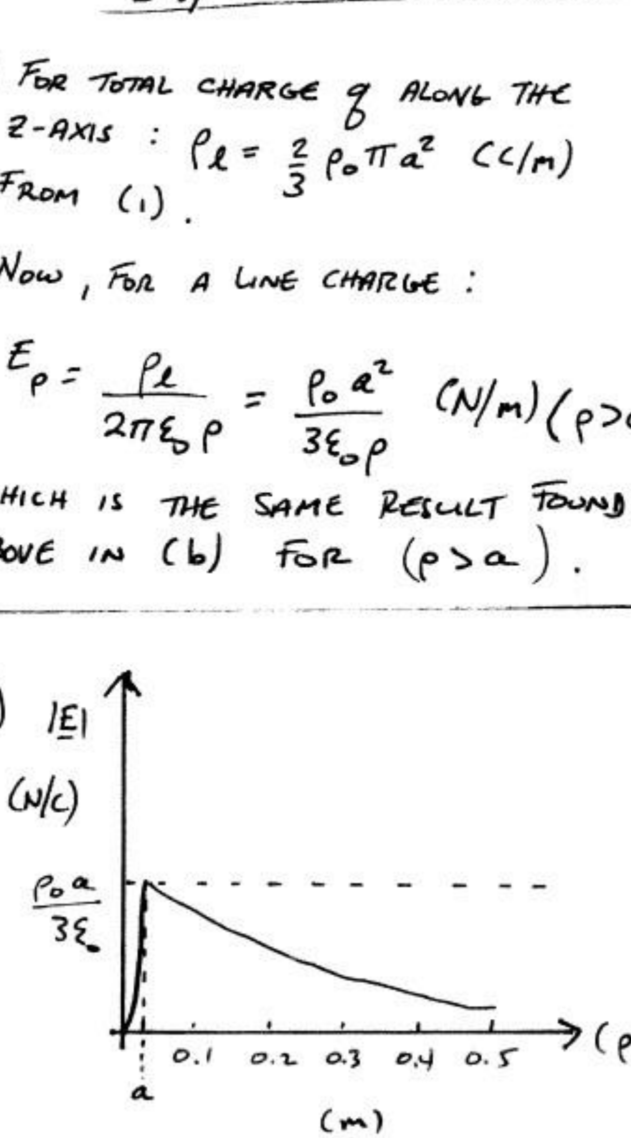
(C) FOR TOTAL CHARGE q ALONG THE Z-AXIS : $\rho_\ell = \frac{2}{3}\rho_0\pi a^2$ (C/m) FROM (1).

NOW, FOR A LINE CHARGE :

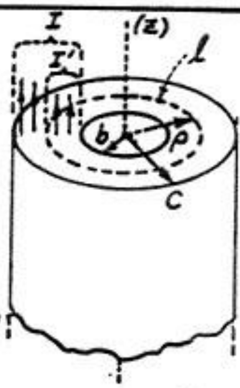
$$E_\rho = \frac{\rho_\ell}{2\pi\epsilon_0\rho} = \frac{\rho_0 a^2}{3\epsilon_0\rho}\ (N/m)\ (\rho > 0)$$

WHICH IS THE SAME RESULT FOUND ABOVE IN (b) FOR ($\rho > a$).

(d)



Problem # 5



(a) In the hollow conductor shown, the constant current density $\bar{J}$ in the cross-section is given by $\bar{J} = \bar{a}_z I/A$, where A is the cross-sectional area and I is the total current. Thus,

$$\bar{J} = \bar{a}_z \frac{I}{\pi c^2 - \pi b^2} = \bar{a}_z \frac{I}{\pi(c^2-b^2)} \left[A/m^2\right] \quad \text{....(1)}$$

(b) Use Ampère's law (1-63) first to find the magnetic $\bar{B}$-field <u>outside</u> the wire. Assume a symmetric, closed Amperian path ℓ as shown, of fixed radius ρ, except locate it <u>outside</u> the wire ($\rho > c$). On ℓ, $d\bar{\ell} = \bar{a}_\phi d\ell$, and from the symmetry, $\bar{B} = \bar{a}_\phi B_\phi$, with B_ϕ constant on the fixed-radius ℓ, of radius ρ. So Ampère's law is written

$$\oint_\ell \frac{\bar{B}}{\mu_0} \cdot d\bar{\ell} = i\Big]_{\substack{\text{thru } S,\\ \text{bounded by } \ell}} \left(\equiv \int_S \bar{J} \cdot d\bar{s} \right) \quad \text{....(1)}$$

or,

$$\oint_\ell \bar{a}_\phi \frac{B_\phi}{\mu_0} \cdot \bar{a}_\phi d\ell = I \quad \text{(since the }\underline{\text{total}}\text{ current } I \text{ passes through } S \text{ bounded by } \ell)$$

from which, with $\bar{a}_\phi \cdot \bar{a}_\phi = 1$, you get

$$\frac{B_\phi}{\mu_0} \oint_\ell d\ell = I, \quad \text{or} \quad \frac{B_\phi}{\mu_0}(2\pi\rho) = I, \text{ so}$$

$$B_\phi = \frac{\mu_0 I}{2\pi\rho} \qquad (\rho > c) \qquad \text{.... (2)}$$

(the same as for the solid wire of Fig. 1-19).

Inside the conductor region (for $b < \rho < c$), Ampère's law is to be applied to the closed line ℓ shown here in the diagram. In this case, the current i of (1) is a <u>fraction</u> of the total current I, found from the ratio A'/A, in which A' denotes the conductor cross-sectional area bounded by ℓ in the diagram. Thus, put this time

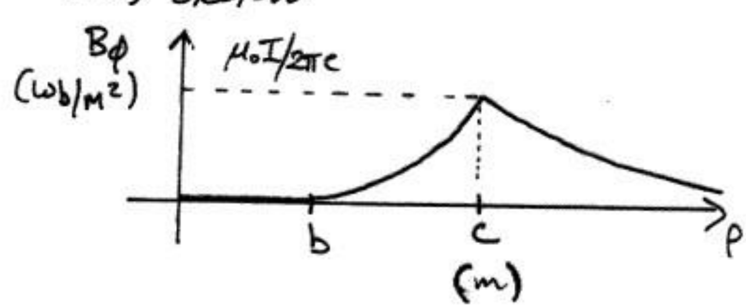
$$\underbrace{\oint_\ell \bar{a}_\phi \frac{B_\phi}{\mu_0} \cdot \bar{a}_\phi d\ell}_{\frac{B_\phi}{\mu_0}(2\pi\rho) \text{ again}} = i\Big]_{\substack{\text{thru } S\\ \text{b'ded. by } \ell}} = I\frac{A'}{A} = I \frac{\pi\rho^2 - \pi b^2}{\pi c^2 - \pi b^2}$$

whence, solving for B_ϕ:

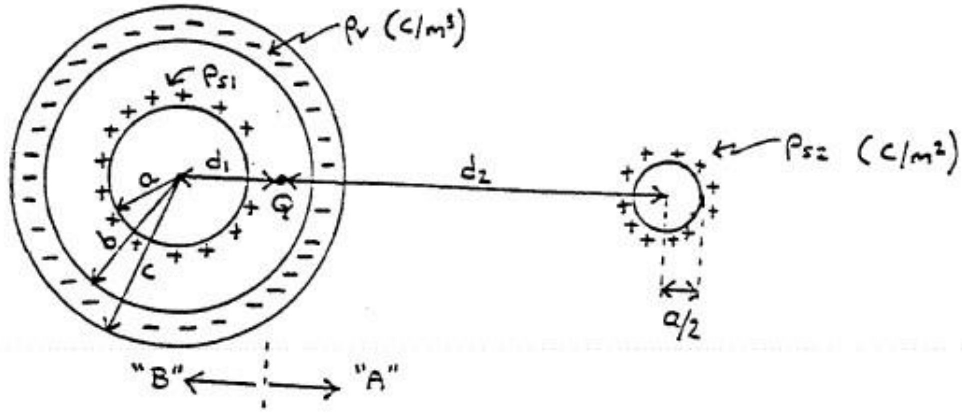
$$B_\phi = \frac{\mu_0 I(\rho^2 - b^2)}{2\pi\rho(c^2-b^2)} \qquad (b < \rho < c) \quad \text{.... (3)}$$

Finally, the field <u>inside</u> the hollow conductor is <u>zero</u> by inspection, since the right side of Ampère's law (1-63) becomes $i\big]_{\text{thru } S} = 0$ upon constructing a closed line within the region $\rho < b$.

(c) Sketch:



Problem # 6



From symmetry (spherical coords.) all E fields are <u>radial</u>, i.e., $\underline{E} = \underline{a}_r E_r(r)$.

- From Gauss's Law, we find the following at the location of Q:

$$|\underline{E}|_{\rho_v} = 0; \quad |\underline{E}|_{\rho_{s1}} \propto \frac{q_{\rho_{s1}}}{d_1^2}; \text{ and } |\underline{E}|_{\rho_{s2}} \propto \frac{q_{\rho_{s2}}}{d_2^2},$$

where q_{s1} & q_{s2} are the total charge due to ρ_{s1} & ρ_{s2}, respectively.

- Hence, $$|\underline{E}|_{\rho_{s1}} \propto \frac{\rho_{s1}(a^2)}{d_1^2}; \quad |\underline{E}|_{\rho_{s2}} \propto \frac{16\rho_{s1}\left(\frac{a}{2}\right)^2}{d_2^2} = \frac{4\rho_{s1}a^2}{16 d_1^2} = \frac{\rho_{s1}a^2}{4d_1^2}$$

- Thus, $|\underline{E}|_{\rho_{s1}} = 4|\underline{E}|_{\rho_{s2}}$ AT Q.

$\Rightarrow$ <u>Net force on Q will be in the direction "B"</u>, SINCE Q IS "-ve".