PHYS 3701 - Assignment #6
Due Wednesday March 25, 2015

1. a) Show that $\left[\hat{L}_z, \hat{L}_y\right] = -i\hbar\hat{L}_x$.

   b) Show that $\left[\hat{L}_+, \hat{L}_-\right] = 2\hbar\hat{L}_z$

   c) Using the relationship between Cartesian and Spherical coordinates, show that $\hat{L}_z = -i\hbar\frac{\partial}{\partial\phi}$.

   d) Show that $\hat{L}_x$ and $\hat{L}_y$ are Hermitian.

2. a) Show for a particle in an eigenstate of $\hat{L}^2$ and $\hat{L}_z$ that $\left\langle \hat{L}_x^2 \right\rangle = \left\langle \hat{L}_y^2 \right\rangle$ and $\frac{\hbar^2 l}{2} \le \left\langle \hat{L}_x^2 \right\rangle \le \frac{\hbar^2 l(l+1)}{2}$.

   b) Show that $\hbar$ has units of angular momentum.

3. Suppose you have a spin-1/2 particle in state $\frac{1}{\sqrt{2}}\left[\left|\frac{1}{2}\ \frac{1}{2}\right\rangle - \left|\frac{1}{2}\ -\frac{1}{2}\right\rangle\right]$.

   a) If you measured z-component of spin, what values could you measure and with what probabilities?
   b) What is the expectation value for the z-component of spin for this state?
   c) Show that the uncertainty principle for $\hat{S}_x$ and $\hat{S}_y$ holds.