PHYS 3701 - Assignment #4
Due Wednesday February 25, 2015

1. For the quantum harmonic oscillator, where $\hat{a}_{\pm} = \frac{1}{\sqrt{2\hbar m\omega}}\left(\mp i\hat{p} + m\omega\hat{x}\right)$, show that

   a) $\hat{H} = \hbar\omega\left[\hat{a}_{+}\hat{a}_{-} + 1/2\right] = \hbar\omega\left[\hat{a}_{-}\hat{a}_{+} - 1/2\right]$

   b) $\left[\hat{a}_{+}, \hat{a}_{-}\right] = -1$

   c) and $\hat{H}\left(\hat{a}_{+}|\psi\rangle\right) = (E + \hbar\omega)\left(\hat{a}_{+}|\psi\rangle\right)$ when $\hat{H}|\psi\rangle = E|\psi\rangle$.

2. Find $\langle\hat{x}\rangle$, $\langle\hat{x}^2\rangle$, $\langle\hat{p}\rangle$, and $\langle\hat{p}^2\rangle$ for the n^th^ stationary state of the harmonic oscillator. Check that the uncertainty principle is satisfied. Do this by first by showing $\hat{a}_{+}|n\rangle = \sqrt{n+1}|n+1\rangle$ and $\hat{a}_{-}|n\rangle = \sqrt{n}|n-1\rangle$ and then writing $\hat{x}$ and $\hat{p}$ in terms of these operators. (Hint: look at $\langle\psi_n|\hat{a}_{+}{}^{t}\hat{a}_{+}|\psi_n\rangle$ etc. where t denotes Hermitian conjugation).

3. Suppose a particle is in state $\psi(x,0) = 12\psi_0 + 5\psi_3$ of the quantum mechanical harmonic oscillator.
   a) Normalize this state.
   b) Calculate $\psi(x,t)$ and $|\psi(x,t)|^2$.
   c) What energies might you measure and with what probabilities?
   b) For particles measured with the ground state energy, what is the probability of then finding the particle between x=0 and x=1?