PHYS 3701 - Assignment #1
Due Wednesday January 28, 2015

1. Question #1.11 from Chapter 1 of Scherrer (Compton scattering problem).
   A gamma ray with energy 1 MeV is scattered off of an unknown particle which is at rest. The gamma ray is reflected directly backward with a final energy of 0.98 MeV. What is the $m_o c^2$ for the unknown particle? (Express your answer in MeV).

2. Solve for the eigenvalues and normalized eigenvectors of the operator
   $\hat{O} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. Show that the eigenvectors comprise an orthonormal basis.

   Determine the projections of $|v\rangle = \begin{bmatrix} 1 \\ i \\ -1 \end{bmatrix}$ along the 'direction' of each eigenvector (i.e. write $|v\rangle$ as a linear combination of the eigenvectors).

3 a) Show that $\left(e^{z}\right)^{*} = e^{z^{*}}$ (for z complex),
   b) Show that $e^{z+i\pi} = -e^{z}$ (for z complex),
   c) Rewrite $\cos\theta$ and $\sin\theta$ in terms of complex exponentials
   d) Show $\sin^2 z + \cos^2 z = 1$ (for z complex).

4. Show that the polynomials of degree 4 constitute a vector space. Use the complex numbers as the scalars. What is the simplest set of basis vectors in this case? Show how to write an arbitrary vector as a linear combination of these basis vectors.

5. A particle is described by the wavefunction

$$\psi(x) = \begin{cases} 0 & x < 0 \\ Ax(a-x) & 0 \le x \le a \\ 0 & x > a \end{cases}$$

   for some real constant a. Determine the normalization constant A. Calculate $\langle \hat{x} \rangle$, $\langle \hat{p} \rangle$, $\langle \hat{x}^2 \rangle$, and $\langle \hat{p}^2 \rangle$.