

MAT 1348 – The Most Important Concepts and Questions

NOTE: You must be able to give precise definitions of important concepts, as well as state important theorems. The following is just a partial list of topics covered.

• Logic and Proofs

Concepts

1. propositions, logical connectives, equivalence
2. tautology, contradiction, contingency
3. disjunctive normal forms
4. rules of inference, arguments; validity of an argument
5. counter-examples (i.e. finding valuations which falsify).
6. types of proof: direct, by contraposition, and by contradiction.

Methods

1. truth tables
2. truth trees
3. ordinary mathematical proofs.

Questions

1. Translate English sentences into propositional calculus.
2. Is a given proposition a tautology/contradiction?
3. Are given propositions equivalent?
4. Find a disjunctive normal form of a given proposition.
5. Is a given argument valid?
6. Prove a given theorem using a specified method of proof.

• Sets

Concepts

1. Set, cardinality, subset, equality of sets.
2. Know particular sets studied in this course: $\emptyset, \mathbb{Z}_n, \mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}, \mathbb{R}^n$, etc.
3. set operations: union, intersection, complement, difference, symmetric difference, power set $\mathcal{P}(-)$, cartesian product of sets.

Questions

1. Find the cardinality of a power set, cartesian product etc.

2. Prove a set identity (using a rigorous proof using elements, or by known set identities (Boolean algebra identities)).

- **Functions**

Concepts

1. what is a function?
2. injective (one-to-one) and surjective (onto) functions; bijections
3. invertible functions, inverse
4. composition of functions.

Questions

1. Determine whether a given function is injective (one-to-one), surjective (onto), invertible, or a bijection.
2. Find the inverse of an invertible function.

- **Relations**

Concepts

1. reflexive, symmetric, antisymmetric, transitive relations
2. equivalence relations, equivalence classes, partitions
3. congruence and integers $\text{mod } n$.

Questions

1. Determine whether a given relation is reflexive, symmetric, antisymmetric, or transitive.
2. Prove that a given relation is an equivalence relation.
3. Find the equivalence classes of a given equivalence relation (or the corresponding partition of the set).

- **Counting**

Concepts

1. Product Rule, Sum Rule, Principle of Inclusion – Exclusion
2. Pigeonhole Principle
3. combinations, permutations
4. binomial coefficients, Binomial Theorem

Questions

1. How many...?

2. What is the smallest number of ... to guarantee...? or Show that in any set of... we can find... such that....
3. Find the coefficient of ... in the expansion of...

- **Mathematical Induction**

Concepts

1. Mathematical Induction
2. Strong Induction (not included in the exam)

Questions

1. Prove... using Mathematical Induction.

- **Graphs**

Concepts

1. graph, simple graph, directed graph
2. complete graphs, cycles, paths, complete bipartite graphs (Compare terminology in Rosen)
3. bipartite graphs
4. subgraphs, isomorphism
5. walks, trails, paths, cycles
6. connected graphs
7. Euler tours and Euler trails

Theorems

1. Handshaking Theorem
2. equivalent conditions for a graph to be bipartite
3. necessary and sufficient conditions for a graph to have an Euler tour or an Euler trail

Questions

1. How many edges in a graph with a given degree sequence?
2. Is a given graph bipartite? (If so, give a 2-colouring. If not, find an odd cycle.)
3. Are given graphs isomorphic? (If so, give an isomorphism. If not, find an invariant on which they disagree.)
4. Does a graph have an Euler tour or an Euler trail? Explain why.