$X \sim B(n,p)$, $P(X=i) = \binom{n}{i} \cdot p^i \cdot q^{n-i}$ $\quad i=0,1,2 \ldots n$

$$\Phi_X(w) = \mathcal{E}\{e^{jwX}\} = \sum_{k=0}^{n} e^{jwk} P(X=k)$$

$$= \sum_{k=0}^{n} e^{jwk} \cdot \binom{n}{k} \cdot p^k \cdot q^{n-k}$$

Binomial Theorem;

Characteristic Func;

$$\boxed{\Phi_X = \sum_{k=0}^{n} \binom{n}{k} \cdot (p \cdot e^{jw})^k \cdot q^{n-k} = (p \cdot e^{jw} + q)^n}$$

Generating Func;

$$P_X(z) = \sum_{i=0}^{\infty} P(X=i) \cdot z^i = \sum_{k=0}^{n} \binom{n}{k} \cdot p^k \cdot q^{n-k} \cdot z^k$$

$$\boxed{P_X(z) = \sum_{k=0}^{n} \binom{n}{k} \cdot (p \cdot z)^k \cdot q^{n-k} = (pz+q)^n}$$

Mean of X;

$$P_X'(1) = \mathcal{E}\{X\} \qquad P_X'(z) = n.p\,(pz+q)^{n-1} = np \cdot \underbrace{(p+q)^{n-1}}_{1}$$

$$\boxed{P_X'(1) = \mathcal{E}\{X\} = np}$$

2) $P_X(z) = \sum_{i=0}^{\infty} P(X=i) \cdot z^i$

a) $a_i = P(X \leq i+2)$

$$P(X=i) = \begin{cases} a_{i-2} - a_{i-3} & i=3,4,5 \\ p_i & i=0,1,2 \end{cases}$$

$$P_X(z) = p_0 + p_1 z + p_2 z^2 + \sum_{i=3}^{\infty} (a_{i-2} - a_{i-3}) \cdot z^i$$

$$= p_0 + p_1 z + p_2 z^2 + z^2 \cdot \sum_{i=3}^{\infty} (a_{i-2} - a_{i-3}) \cdot z^{i-2} = p_0 + p_1 z + p_2 z^2 + z^2 \left(\sum_{i=3}^{\infty} (a_{i-2}) \cdot z^{i-2}\right) - z^3 \left(\sum_{i=3}^{\infty} (a_{i-3}) \cdot z^{i-3}\right)$$

$$= p_0 + p_1 z + p_2 z^2 + z^2 \left[\sum_{i=0}^{\infty} a_i \cdot z^i - a_0\right] - z^3 \left(\sum_{i=0}^{\infty} a_i z^i\right)$$

3)

$$P_X(z) = p_0 + p_1 z + p_2 z^2 + z^2 P_a(z) - z^3 P_a(z) - a_0 \cdot z^2$$

$$P_X(z) = p_0 + p_1 z + p_2 z^2 + P_a(z)(z^2 - z^3) - (p_0 + p_1 + p_2) z^2$$

$$= p_0(1 - z^2) + p_1(z - z^2) + p_2\underbrace{\cancel{(z^2 - z^2)}}_{0} + P_a(z) \cdot (z^2 - z^3)$$

$$= (1 - z)\left(p_0(1 + z) + p_1 z + P_a(z) \cdot z^2\right)$$

$$\boxed{P_a(z) = \frac{P_X(z)}{1 - z} - p_0 - p_1 z - p_0 z}$$

b) for $b_i = P(X > i + 2)$

$$P(X = i) = \begin{cases} p_i & i = 0, 1, 2 \\ b_{i-3} - b_{i-2} & i = 3, 4, 5 \ldots \end{cases}$$

$$P_X(z) = p_0 + p_1 z + p_2 z^2 + \sum_{i=3}^{\infty} (b_{i-3} - b_{i-2}) \cdot z^i$$

$$= p_0 + p_1 z + p_2 z^2 + z^3 \cdot \sum_{i=3}^{\infty} (b_{i-3}) z^{i-3} - z^2 \cdot \sum_{i=3}^{\infty} (b_{i-2}) \cdot z^{i-2}$$

$$= p_0 + p_1 z + p_2 z^2 + z^3 \cdot P_b(z) - z^2 [P_b(z) - b_0]$$

$b_0 = 1 - p_0 - p_1 - p_2$

$$P_X(z) = p_0 + p_1 z + \cancel{p_2 z^2} + (z^3 - z^2) P_b(z) + z^2 - p_0 z^2 - p_1 z^2 - \cancel{p_2 z^2}$$

$$= p_0 + p_1 z + z^2(1 - p_0 - p_1) + z^2(z - 1) \cdot P_b(z)$$

$$\boxed{P_b(z) = \frac{P_X(z) - p_0 - p_1 z - z^2(1 - p_0 - p_1)}{z^2(z - 1)}}$$

c) $c_i = \begin{cases} P(X = i+1) & , i \text{ odd} \\ 0 & , i \text{ even} \end{cases}$

$$P_c(z) = \sum_{i=0}^{\infty} c_i \cdot z^i = \sum_{i \text{ odd}}^{\infty} p_{i+1} \cdot z^i = z^{-1} \cdot \sum_{i \text{ even}}^{\infty} p_i \cdot z^i$$

$$P_c(z) = z^{-1} \sum p_i \left(z^i + (-z)^i\right)/2 = \frac{1}{2} z^{-1} \left\{ (P_X(z) - p_0) + (P_X(-z) - p_0) \right\}$$

$$\boxed{P_c(z) = \frac{P_X(z) + P_X(-z) - 2p_0}{2z}}$$

3)

$F_{XY}(x,y)$ is the probability of joint event of event $\{X \leq x\}$ and event $\{Y \leq y\}$. The event space of joint event is an algebra in all cases. Therefore the probability of this intersection can always be determined regardless of x and y, and we can guarantee that $F_{XY}(x,y)$ is defined on the entire Cartesian Plane.

4)

$h(x,y) = F_X(x).F_Y(y)$ Let's check if it satisfies the five properties;

i) $F_{XY}(\infty,\infty) = \lim\limits_{\substack{x\to\infty \\ y\to\infty}} F_{XY}(x,y) = 1$

satisfied

$\lim\limits_{\substack{x\to\infty \\ y\to\infty}} h(x,y) = \lim\limits_{x\to\infty} F_X(x) . \lim\limits_{y\to\infty} F_Y(y) = 1.1 = 1$

ii) For any $x,y \in R$;

$F_{XY}(x,-\infty) = \lim\limits_{y\to-\infty} F_{XY}(x,y) = 0$ and $F_{XY}(-\infty,y) = \lim\limits_{x\to-\infty} F_{XY}(x,y) = 0$

$\lim\limits_{y\to-\infty} h(x,y) = F_X(x) . \lim\limits_{y\to-\infty} F_Y(y) = F_X(x).0 = 0$ ✓

satisfied

$\lim\limits_{x\to-\infty} h(x,y) = F_Y(y) . \lim\limits_{x\to-\infty} F_X(x) = F_Y(y).0 = 0$ ✓

iii) $F_{XY}(x,y)$ is a monotonic increasing function;

$F_{XY}(x_1,y_1) \leq F_{XY}(x_2,y_2)$ if $x_1 \leq x_2$, $y_1 \leq y_2$

$h(x_1,y_1) = F_X(x_1) . F_Y(y_1)$

if $x_1 \leq x_2$ then $F_X(x_1) \leq F_X(x_2)$

$h(x_2,y_2) = F_X(x_2) . F_Y(y_2)$

if $y_1 \leq y_2$ then $F_Y(y_1) \leq F_Y(y_2)$

therefore

$h(x_1,y_1) \leq h(x_2,y_2)$ ✓ satisfied

iv) For any real numbers a, b, c, d which $a \leq c$, $b \leq d$,

$$F_{XY}(a,b) + F_{XY}(c,d) - F_{XY}(a,d) - F_{XY}(c,b) \geq 0$$

$$F_X(a) \cdot F_Y(b) + F_X(c) \cdot F_Y(d) - F_X(a) \cdot F_Y(d) - F_X(c) \cdot F_Y(b) \geq 0$$

$$F_X(a)\Big(F_Y(b) - F_Y(d)\Big) + F_X(c)\Big(F_Y(d) - F_Y(b)\Big) \geq 0$$

$F_X(x)$ and $F_Y(y)$ are nonnegative functions;

if $b \leq d$; if $b \leq d$;

$F_Y(b) - F_Y(d)$ is negative α, $F_Y(d) - F_Y(b)$ is positive α

if $a \leq c$; $|F_Y(b) - F_Y(d)| = |F_Y(d) - F_Y(b)|$

$F_X(c) \geq F_X(a)$ therefore

$$F_X(c) \cdot (\alpha) + F_X(a) \cdot (-\alpha) \geq 0$$

$$F_X(c) \geq F_X(a) \quad \checkmark$$

<u>satisfied</u>

v) $\lim_{\substack{\varepsilon_1 \to 0^+ \\ \varepsilon_2 \to 0^+}} F_{XY}(x+\varepsilon_1, y+\varepsilon_2) = F_{XY}(x,y)$ for all $x, y \in R$

$$\lim_{\substack{\varepsilon_1 \to 0^+ \\ \varepsilon_2 \to 0^+}} h(x+\varepsilon_1, y+\varepsilon_2) = \lim_{\varepsilon_1 \to 0^+} F_X(x+0^+) \cdot \lim_{\varepsilon_2 \to 0^+} F_Y(y+0^+) \overset{?}{=} h(x,y)$$

$$= F_X(x) \cdot F_Y(y) \overset{\checkmark}{=} h(x,y)$$

<u>satisfied</u>

$*$ All properties are satisfied, $h(x,y)$ is a joint distribution function.

5) If X and Y are two independent random var.; ($f_{XY} = f_X(x) \cdot f_Y(y)$)

Then $f_Z(z) = f_X(z) * f_y(z)$; → convolution

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(x) \cdot f_y(z-x) \cdot dx$$

$$f_Z(z) = \int_{-\infty}^{\infty} c \cdot e^{-cx} \cdot u(x) \cdot c^2 \cdot (z-x) \cdot e^{-c(z-x)} \cdot u(x) \cdot dx$$

$$f_Z(z) = \int_{0}^{z} c^3 \cdot \cancel{e^{-cx}} \cdot (z-x) \cdot e^{-cz} \cdot \cancel{e^{cx}} \cdot dx$$

$$f_Z(z) = c^3 \cdot e^{-cz} \cdot \int_0^z (z-x) \cdot dx = c^3 \cdot e^{-cz} \cdot \left(zx - \frac{x^2}{2}\right)\Big|_0^z$$

$$f_Z(z) = c^3 \cdot e^{-cz} \cdot \frac{z^2}{2} \cdot \text{ for } z>0$$

$$\boxed{f_Z(z) = \frac{c^3 \cdot e^{-cz}}{2} \cdot z^2 \cdot u(z)}$$

6)

$X \sim N(0,1)$, $Y \sim N(0,1)$ They are independent;

$$f_X(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-x^2/2} \quad , \quad f_Y(y) = \frac{1}{\sqrt{2\pi}} \cdot e^{-y^2/2}$$

Let's use an auxiliary random variable $W = X$

$g(x,y) = \frac{y}{x}$, $h(x,y) = x$, $z = \frac{y}{x}$, $w = x$

if $x = w$ then $y = zw$ $(w \neq 0)$

The jacobian determinant is:

$$J(x,y) = \begin{vmatrix} -\frac{y}{x^2} & \frac{1}{x} \\ 1 & 0 \end{vmatrix} = -\frac{1}{x} = -\frac{1}{w}$$

jacobian exists thus we have;

$$f_{ZW}(z,w) = |w| \cdot f_{XY}(w, zw)$$

$$f_{XY}(w, zw) = \frac{1}{2\pi} \cdot e^{-(w^2 + z^2w^2)/2}$$

$$f_{ZW}(z,w) = |w| \cdot \frac{1}{2\pi} \cdot e^{-(w^2 + z^2w^2)/2}$$

$$f_Z(z) = \int_{-\infty}^{\infty} |w| \frac{1}{2\pi} \cdot e^{-w^2(1+z^2)/2} \cdot dw$$

$$f_Z(z) = \frac{-1}{\pi(1+z^2)} \cdot e^{-w^2(1+z^2)/2} \Big|_{w=0}^{w=\infty} = \frac{1}{\pi(1+z^2)}$$

$$F_Z(z) = \int_{-\infty}^{z} f_Z(\lambda) \cdot d\lambda = \frac{1}{2} + \frac{1}{\pi} \cdot \arctan(z)$$

) X and Y are two independent rand. var.;

$$f_X(x) = a.e^{-ax}.u(x)$$

$$f_Y(y) = b.e^{-by}.u(y)$$

i) $f_Z(z) = ?$ if $Z = 2X + 3Y$

$$f_Z(z) = \frac{1}{2}\cdot f_X\left(\frac{z}{2}\right) * \frac{1}{3} f_Y\left(\frac{z}{3}\right) = \frac{1}{6}\int_0^\infty ab.e^{-a\lambda/2}.e^{-b(z-\lambda)/3}\cdot u(z-\lambda)d\lambda$$

$$= \frac{ab}{6}\cdot\int_0^z e^{-bz/3}.e^{-\lambda(a/2 - b/3)}\cdot u(z-\lambda).d\lambda$$

$$= -\frac{ab}{6}.e^{-\frac{bz}{3}}\cdot\frac{1}{(a/2 - b/3)}\cdot\left(1 - \underbrace{e^{-\lambda(a/2 - b/3)}\Big|^{z}}\right)$$

for $3a \neq 2b$;
& for $z > 0$;

$$f_Z(z) = \frac{ab}{6}\cdot\frac{6}{3a-2b}\left(e^{-\frac{bz}{3}} - e^{-\frac{az}{2}}\right).u(z)$$

$$\boxed{f_Z(z) = \frac{ab}{3a-2b}\cdot\left(e^{-\frac{bz}{3}} - e^{-\frac{az}{2}}\right).u(z)}$$

ii) $Z = 2Y - X$

$$f_Z(z) = \frac{1}{2}\cdot f_Y\left(\frac{z}{2}\right) * f_X(-x)$$

$$= \frac{1}{2}\int_0^\infty ab.e^{-\frac{b\lambda}{2}}.e^{-a(-(z-\lambda))}.u(-[z-\lambda]).d\lambda$$

$$= \frac{ab}{2}.e^{az}.\int_0^\infty e^{-\lambda(b/2 + a)}.u(\lambda - z).d\lambda$$

$$= \frac{ab}{2a+b}.e^{az}.e^{-\lambda(b/2 + a)}\Big|_{(0,z)}^{\infty}$$

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$$f_Z(z) = \begin{cases} \frac{ab}{2a+b} e^{az}, & z<0 \\ \frac{ab}{2a+b} \cdot e^{-bz/2}, & z>0 \end{cases} \qquad 2a+b>0$$

iii)

$$f_Z(z) = \int_{-\infty}^{\infty} |y| \cdot F_{XY}(zy, y) \cdot dy$$

$$= \int_0^{\infty} y \cdot a \cdot b \cdot e^{-azy} \cdot e^{-by} \cdot u(z)\, dy$$

$$= ab \cdot \int_0^{\infty} y \cdot e^{-(az+b)\cdot y} \cdot u(z) \cdot dy$$

$$\boxed{f_Z(z) = \frac{ab}{(az+b)^2} \cdot u(z)}$$

iv) for $Z = max(X,Y)$

$$F_Z(z) = P(max\{X,Y\} \le z) = P(\{X \le z\} \text{ and } \{Y \le z\})$$

$$F_Z(z) = F_{XY}(z,z) = F_X(z) \cdot F_Y(z)$$

$$f_Z(z) = \frac{\partial}{\partial z} F_Z(z) = f_X(z) \cdot F_Y(z) + f_Y(z) \cdot F_X(z)$$

$$\boxed{f_Z(z) = \left[a \cdot e^{-az}(1-e^{-bz}) + b \cdot e^{-bz} \cdot (1-e^{-az}) \right] u(z)}$$

v) for $Z = min(X,Y)$

$$F_Z(z) = P(X \le z) + P(Y \le z) - P(X \le z, Y \le z)$$

$$F_Z(z) = F_X(z) + F_Y(z) - F_{XY}(z,z)$$

$$F_Z(z) = F_X(z) + F_Y(z) - F_X(z) \cdot F_Y(z)$$

$$f_Z(z) = f_X(z) + f_Y(z) - f_X(z) \cdot F_Y(z) - f_Y(z) \cdot F_X(z)$$

$$= f_X(z)[1 - F_Y(z)] + f_Y(z)[1 - F_X(z)]$$

$$\boxed{f_Z(z) = a \cdot e^{-az}[1-(1-e^{-bz})] + b \cdot e^{-bz}[1-(1-e^{-az})] = (a+b)\, e^{-(a+b) \cdot z} \cdot u(z)}$$

8) a)

$X \sim N(\mu_x, \sigma_x)$

$Y \sim N(\mu_y, \sigma_y)$ If X and Y are independent rand. var.

$Z = X + Y$

$$f_Z(z) = f_X(z) * f_Y(z) = \frac{1}{2\pi\sigma_x\sigma_y} \cdot e^{-(z-\mu_x)^2/2\sigma_x^2} * e^{-(z-\mu_y)^2/2\sigma_y^2}$$

$$= \frac{1}{2\pi\sigma_x\sigma_y} \cdot \int_{-\infty}^{\infty} e^{-(x-\mu_x)^2/2\sigma_x^2} \cdot e^{-((z-x-\mu_y))^2/2\sigma_y^2} \cdot dx$$

Let $\tau = x - \mu_x$, $x = \tau + \mu_x$; $dx = d\tau$

$$f_Z(z) = \frac{1}{2\pi\sigma_x\cdot\sigma_y} \cdot \int_{-\infty}^{\infty} e^{-\frac{\tau^2}{2\sigma_x^2} - \frac{(z-\tau-\mu_x-\mu_y)^2}{2\sigma_y^2}} \cdot d\tau$$

Let $\frac{1}{2\sigma_x^2} = \alpha$, $\beta = \frac{1}{2\sigma_y^2}$, $\gamma = z - \mu_x - \mu_y$

$$f_Z(z) = \frac{1}{2\pi\sigma_x\cdot\sigma_y} \cdot \int_{-\infty}^{\infty} e^{-[\alpha\tau^2 + \beta(\gamma-\tau)^2]} \cdot d\tau$$

$$\alpha\tau^2 + \beta(\gamma-\tau)^2 = (\alpha+\beta)\left(\tau - \frac{\beta\gamma}{\alpha+\beta}\right)^2 + \frac{\alpha\beta}{\alpha+\beta}\cdot\gamma^2$$

$$f_Z(z) = \frac{1}{2\pi\sigma_x\sigma_y} \cdot e^{-\frac{\alpha\beta}{\alpha+\beta}\cdot\gamma^2} \cdot \underbrace{\int_{-\infty}^{\infty} e^{-(\alpha+\beta)\cdot(\tau-c)^2} \cdot d\tau}_{\int_{-\infty}^{\infty} e^{-(\alpha+\beta)\cdot x^2} \cdot dx = \sqrt{\frac{\pi}{\alpha+\beta}}} \qquad c = \frac{\beta\gamma}{\alpha+\beta}$$

$$f_Z(z) = \frac{1}{\sqrt{4\pi\cdot\sigma_x^2\sigma_y^2\cdot(\alpha+\beta)}} \cdot e^{-(z-\mu_x-\mu_y)^2 / (\alpha+\beta)/\alpha\beta}$$

$$\alpha+\beta = \frac{1}{2}\left(\frac{1}{\sigma_x^2} + \frac{1}{\sigma_y^2}\right) \Rightarrow 2\sigma_x^2\sigma_y^2(\alpha+\beta) = \sigma_x^2 + \sigma_y^2$$

$$f_Z(z) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-(z-\mu)^2/2\sigma^2}$$

$$\sigma^2 = \sigma_x^2 + \sigma_y^2 \qquad \mu = \mu_x + \mu_y$$

$$Z \sim N(\mu, \sigma)$$

b) If X and Y are independent Cauchy rand. variable;

$$f_X(x) = \frac{\alpha/\pi}{\alpha^2 + x^2} \quad , \quad f_Y(y) = \frac{\beta/\pi}{\beta^2 + x^2}$$

for $Z = X + Y$;

$$f_Z(z) = f_X(z) * f_Y(z) = \frac{\alpha\beta}{\pi^2} \cdot \int_{-\infty}^{\infty} \frac{1}{(\alpha^2+x^2)} \cdot \frac{1}{(\beta^2+(z-x)^2)} \cdot dx$$

* The fourier transform of convolution gives multiplication;

$$F\{f_Z(z)\} = \cdot e^{-|w|\alpha} \cdot e^{-|w| \cdot \beta} = e^{-|w|(\alpha+\beta)} \qquad w = 2\pi f$$

$$\boxed{F^{-1}\{F\{f_Z(z)\}\} = f_Z(z) = \frac{\alpha+\beta}{\pi} \cdot \frac{1}{(\alpha+\beta)^2 + z^2}}$$

As can be seen, Z is also a cauchy rand. variable.

Independent random variables.

$*\quad X \sim N(\mu_x, \sigma_x)\ ,\ Y \sim N(\mu_y, \sigma_y)\ ;$

$$P(XY>0) = P([X>0, Y>0] + [X<0, Y<0])$$

$$P(XY>0) = P(X>0).P(Y>0) + P(X<0).P(Y<0)$$

$$= P\left(\frac{(X-\mu_x)}{\sigma_x} > -\frac{\mu_x}{\sigma_x}\right).P\left(\frac{(y-\mu_y)}{\sigma_y} > -\frac{\mu_y}{\sigma_y}\right) + P\left(\frac{(x-\mu_x)}{\sigma_x} < -\frac{\mu_x}{\sigma_x}\right).P\left(\frac{y-\mu_y}{\sigma_y} < -\frac{\mu_y}{\sigma_y}\right)$$

$$= Q\left(-\frac{\mu_x}{\sigma_x}\right).Q\left(\frac{-\mu_y}{\sigma_y}\right) + \left(1-Q\left(-\frac{\mu_x}{\sigma_x}\right)\right).\left(1-Q\left(-\frac{\mu_y}{\sigma_y}\right)\right)$$

$$Q(x) = 1 - Q(-x)$$

$$P(XY>0) = Q(-\mu_x/\sigma_x) \cdot Q(-\mu_y/\sigma_y) + Q\left(\frac{\mu_x}{\sigma_x}\right) \cdot Q\left(\frac{\mu_y}{\sigma_y}\right)$$

$$= \left(1-Q\left(\frac{\mu_x}{\sigma_x}\right)\right)\left(1-Q\left(\frac{\mu_y}{\sigma_y}\right)\right) + Q\left(\frac{\mu_x}{\sigma_x}\right).Q\left(\frac{\mu_y}{\sigma_y}\right)$$

$$\boxed{P(XY>0) = 1 - Q\left(\frac{\mu_x}{\sigma_x}\right) - Q\left(\frac{\mu_y}{\sigma_y}\right) + 2Q\left(\frac{\mu_x}{\sigma_x}\right).Q\left(\frac{\mu_y}{\sigma_y}\right)}$$

10) $$f_X(x) = \frac{x^{m/2-1} \cdot e^{-x/2}}{2^{m/2} \cdot \Gamma(m/2)} \quad , \quad x>0$$

$$f_Y(y) = \frac{y^{n/2-1} \cdot e^{-y/2}}{2^{n/2} \cdot \Gamma(n/2)} \quad , \quad y>0$$

$$Z = \frac{X}{m} \cdot \frac{n}{Y} = \frac{n}{m} \cdot \frac{X}{Y}$$

Let $W = Y$;

$$\boxed{x = \frac{mwz}{n} \quad , \quad y = w ;}$$

Jacobian determinant ; $\det \begin{vmatrix} n/my & -\frac{nx}{y^2} \\ 0 & 1 \end{vmatrix} = \frac{n}{mw}$

$$f_{ZW}(z,w) = \frac{f_{XY}(mwz/n \, , w)}{|n/mw|}$$

$$= \frac{mw}{n} \cdot \frac{(mwz/n)^{m/2-1} \cdot w^{n/2-1} \cdot e^{-\left(\frac{mwz}{n}+w\right)/2}}{2^{(m+n)/2} \cdot \Gamma(m/2) \cdot \Gamma(n/2)} \quad , \quad \begin{matrix} w>0 \\ z>0 \end{matrix}$$

$$\left(\frac{m}{n}\right)^{m/2} \cdot \frac{z^{m/2-1}}{2^{(m+n)/2} \cdot \Gamma(m/2) \cdot \Gamma(n/2)} \cdot w^{\frac{m+n}{2}-1} \cdot e^{-\left(\frac{m}{n}z+1\right)\frac{w}{2}}$$

$$f_Z(z) = \int_0^\infty f_{ZW}(z,w) \, dw$$

$$= \left(\frac{m}{n}\right)^{m/2} \cdot \frac{z^{m/2-1} \cdot u(z)}{2^{(m+n)/2} \, \Gamma(m/2) \, \Gamma(n/2)} \int_0^\infty w^{\left(\frac{m+n}{2}-1\right)} \cdot e^{-\left(\frac{m}{n}z+1\right)w/2} \cdot dw$$

Let $\left(\frac{m}{n}z+1\right)\frac{w}{2} = \lambda$

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$$: w^{\frac{m+n}{2}-1} \cdot e^{-\left(\frac{m}{n}z+1\right)\frac{w}{2}} \cdot dw = \left(\frac{2}{\frac{mz}{n}+1}\right)^{\frac{m+n}{2}} \underbrace{\int_0^{\infty} \lambda^{\frac{m+n}{2}-1} \cdot e^{-\lambda} \cdot d\lambda}_{\Gamma\left(\frac{m+n}{2}\right)}$$

$$f_Z(z) = \underbrace{\left(\frac{m}{n}\right)^{m/2} \cdot \frac{\Gamma\left(\frac{m+n}{2}\right)}{\Gamma(m/2)\cdot\Gamma(n/2)}}_{c} \cdot \frac{z^{m/2-1}}{\sqrt{\left(1+\frac{mz}{n}\right)^{m+n}}} \cdot u(z)$$

11) X and Y are independent zero mean σ^2 variance gaussian r. v.

$\boxed{Z = |Y-X|}$, $\boxed{Y-X = A}$

$$\boxed{f_A(x) = \frac{1}{\sqrt{4\pi\sigma^2}} \cdot e^{-x^2/4\sigma^2}}$$ $\boxed{|A| = Z}$

since $Z^2 = |Y-x|^2 = A^2$, $\boxed{\mathcal{E}\{Z^2\} = 2\sigma^2}$

$$\mathcal{E}\{Z\} = \mathcal{E}\{|A|\} = \int_{-\infty}^{\infty} |A| \cdot f_A(x) \cdot dx = 2 \cdot \int_0^{\infty} \frac{x}{\sqrt{4\pi\sigma^2}} \cdot e^{-x^2/4\sigma^2} \cdot dx$$

$$\boxed{\mathcal{E}\{Z\} \cong \frac{2\sigma}{\sqrt{\pi}}}$$

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