PHY1124 Test #2 solutions 2012

1. $\frac{1}{2}kx^2 = U = \frac{1}{2}E = \frac{1}{2}\left(\frac{1}{2}kx_m^2\right)$

$\therefore\ x^2 = \frac{1}{2}x_m^2 \Rightarrow |x| = \frac{1}{\sqrt{2}}x_m = 0.707 \times 5\,\text{cm} = 3.5\,\text{cm}$ (d)

2.

$\vec{E} = 0$ here $\Rightarrow q_1, q_2$ of opposite sign (c)

3. $\frac{1}{2}kx_m^2 = E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$

$x_m = \left(x^2 + \frac{m}{k}v^2\right)^{1/2} = \left((0.15\,\text{m})^2 + \frac{0.25\,\text{kg}}{200\,\text{N/m}}(3\,\tfrac{\text{m}}{\text{s}})^2\right)^{1/2} = 0.18\,\text{m}$ (c)

4. $x(0) = x_m\cos\phi > 0 \Rightarrow \cos\phi > 0 \Rightarrow -\pi/2 < \phi < \pi/2$

$v(0) = -\omega x_m \sin\phi > 0 \Rightarrow \sin\phi < 0 \Rightarrow -\pi < \phi < 0$

$\left.\right\}\ -\pi/2 < \phi < 0$ or $3\pi/2 < \phi < 2\pi$ (d)

5. $E_P = mgh_1 = E_Q = mgh_2 + \frac{1}{2}mv^2$

$\therefore\ v^2 = 2g(h_1 - h_2) \Rightarrow v = \sqrt{2g(h_1 - h_2)}$ (d)

6. $F_i = \frac{kq_1q_2}{r^2}$ $\quad F_f = \frac{kq_1q_2}{(1.6r)^2} = F_i\left(\frac{1}{1.6}\right)^2 = \frac{15\,\text{N}}{2.56} = 5.9\,\text{N}$ (b)

7. $\frac{1}{2}mv_1^2 + U(x_1) = \frac{1}{2}mv_2^2 + U(x_2)$ where $x_1 = 1\,\text{m}$, $x_2 = 0$

$\frac{1}{2}m(v_2^2 - v_1^2) = U(x_1) - U(x_2)$

$v_2^2 = v_1^2 + \frac{2}{m}\left(8x_1^2 + 2x_1^4 - (8x_2^2 + 2x_2^4)\right)$ (e)

$= (5)^2 + \frac{2}{0.2}\left(8(1)^2 + 2(1)^4 - 0\right) = 125 \quad \therefore\ v_2 = 11.2\,\tfrac{\text{m}}{\text{s}}$

8. $F_x = -\frac{dU}{dx} = -8(2x) - 2(4x^3) = -16 - 8 = -24\,\text{N}$ at $x = 1\,\text{m}$.

$a_x = \frac{F_x}{m} = \frac{-24\,\text{N}}{0.2\,\text{kg}} = -120\,\text{m/s}^2$ (a)

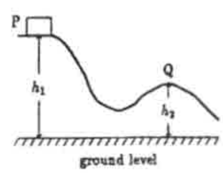
1. A mass on a spring oscillates with an amplitude 5.0 cm. How far is the mass from its equilibrium position when the kinetic and potential energies are equal?
   a. There is not enough information given to answer this question.
   b. 1.2 cm c. 2.5 cm d. 3.5 cm e. 3.8 cm

2. Two point charges of unknown magnitude and sign are located at $x = 0$ and $x = d$ on the x-axis. If the electric field strength is zero at some point $x > d$ on the x-axis, you can conclude that
   a. the charges are equal in magnitude but opposite in sign.
   b. the charges are equal in magnitude and have the same in sign.
   c. the charges are not necessarily equal in magnitude but have opposite in sign.
   d. the charges are not necessarily equal in magnitude but have the same sign.
   e. there is not enough information to say anything specific about the charges.

3. An 0.25 kg block oscillates on the end of a spring with a spring constant of 200 N/m. The oscillation is started at time $t = 0$ by elongating the spring to $x = 0.15$ m and giving the block a speed of 3.0 m/s in the direction away from its equilibrium position $x = 0$. The amplitude of the oscillation is
   a. 0.13 m b. 3.7 m c. 0.18 m d. 0.52 m e. 1.3 m

4. Let the displacement of the mass on the spring in the above question be $x(t) = x_m \cos(\omega t + \phi)$. The value of the constant ϕ (in radians) is between
   a. 0 and $\pi/2$ b. $\pi/2$ and π c. π and $3\pi/2$ d. $3\pi/2$ and 2π
   e. None of the above; ϕ is exactly 0, $\pi/2$, π, or $3\pi/2$.

5. A block is released from rest at point P and slides along a frictionless track shown. At point Q, its speed is:
   a. $2g\sqrt{h_1 - h_2}$ b. $2g(h_1 - h_2)$
   c. $(h_1 - h_2)/2g$ d. $\sqrt{2g(h_1 - h_2)}$
   e. $(h_1 - h_2)^2/2g$



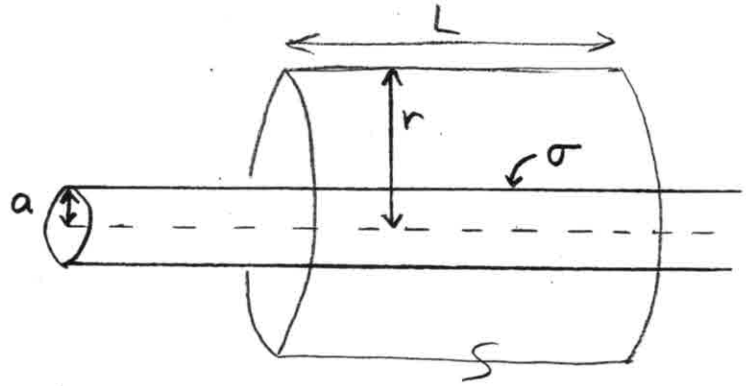
6. Charges q_1 and q_2 exert repulsive forces of 15 N on each other. What is the repulsive force when their separation is increased so that their final separation is 160 % of their initial separation?
   a. 1.3 N b. 5.9 N c. 9.8 N d. 8.7 N e. 6.4 N

7. A 0.20 kg particle moves along the x-axis under the influence of a stationary object. The potential energy is given by $U(x) = 8x^2 + 2x^4$, where U is in Joules and x is in meters. If the particle has a speed of 5.0 m/s when it is at $x = 1.0$ m, its speed when it is at the origin is (in m/s):
   a. 0 b. 2.5 c. 5.7 d. 7.9 e. 11

8. In the question above, the acceleration of the particle when its position is $x = 1.0$ m is (in m/s^2):
   a. -120 b. -80 c. -40 d. 80 e. 24

PART B: ANSWER IN THE SPACE PROVIDED, SHOWING ALL YOUR WORK

9. An infinitely long, straight, cylinder of radius a carries a uniform, surface charge density $\sigma > 0$ as shown. (a) Find the electric charge on a length L of the the cylinder, in terms of the variables given. (b) Use Gauss' law to find the electric field $\vec{E}$ at a distance r from the central axis of the cylinder for $r > \cancel{R}\,a$, showing all your work and stating any assumptions you make based on symmetry. Clearly describe the Gaussian surface you use.



a) $q = \sigma A$

$= \sigma(2\pi a L)$

$= 2\pi\sigma a L$

b) $\oint_S \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0} q_{enc}$ is Gauss' law

S = cylinder of radius r, length L, centred on axis of charged cylinder

Assume, by symmetry, that $\vec{E}$ points radially (outwards) from axis of cylinder, with magnitude $E(r)$.

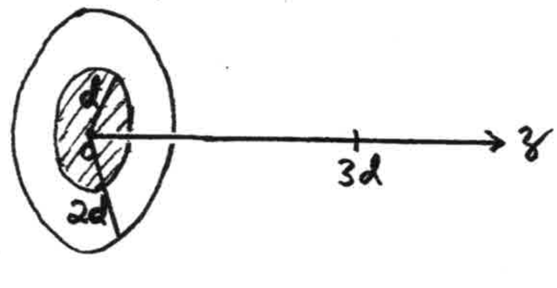
Gauss' law gives:

$$\underbrace{2\pi r L}_{\text{area of } S}\, E(r) = \frac{1}{\varepsilon_0} q = \frac{1}{\varepsilon_0} 2\pi\sigma a L$$

$$\therefore\ E(r) = \frac{\sigma a}{\varepsilon_0 r}$$

and $\vec{E}$ points radially outwards, $\perp$ axis of cylinder

10. A circular ring of radius $2d$ lies in the $z = 0$ plane, centred on the origin as shown. The ring has an electric charge q distributed uniformly on it. In addition, a thin, flat disc of radius d lies in the $z = 0$ plane, also centred on the origin. The disc has a uniform surface charge density distributed on it. At a position $z = 3d$ on the z-axis, the net electric field due to the ring and disc together is equal to half of that due to the ring alone. Find the charge Q on the disc.



$$E_{z,\text{ring}} = \left.\frac{qz}{4\pi\varepsilon_0 (R^2+z^2)^{3/2}}\right|_{z=3d,\,R=2d}$$

$$= \frac{q(3d)}{4\pi\varepsilon_0 ((2d)^2+(3d)^2)^{3/2}}$$

$$= \frac{q}{4\pi\varepsilon_0 d^2}\left(\frac{3}{13^{3/2}}\right)$$

$$E_{z,\text{disc}} = \left.\frac{\sigma}{2\varepsilon_0}\left(\frac{z}{|z|} - \frac{z}{\sqrt{R^2+z^2}}\right)\right|_{z=3d,\,R=d}$$

$$= \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{3d}{\sqrt{d^2+(3d)^2}}\right) = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{3}{\sqrt{10}}\right)$$

Let $E_{z,\text{ring}} + E_{z,\text{disc}} = \frac{1}{2} E_{z,\text{ring}}$

$\therefore\ E_{z,\text{disc}} = -\frac{1}{2} E_{z,\text{ring}}$

$\therefore\ \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{3}{\sqrt{10}}\right) = -\frac{1}{2}\,\frac{q}{4\pi\varepsilon_0 d^2}\left(\frac{3}{13^{3/2}}\right)$

$\therefore\ \sigma = -\frac{q}{4\pi d^2}\,\frac{3}{13^{3/2}(1-3/\sqrt{10})}$

$\therefore\ Q = \pi d^2 \sigma = -\frac{3}{4\cdot 13^{3/2}(1-3/\sqrt{10})}\,q = -0.31\,q$