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CW
R20

THE UNIVERSITY OF CALGARY

FACULTY OF ENGINEERING

Final Examination

PROBABILITY AND STATISTICS FOR ENGINEERS
ENGG 319

December, 2001

Time: 3 hrs.

THE EXAM IS OPEN TEXT
CALCULATORS ARE ALLOWED

There are 40 questions.
Answer all 40 questions by indicating the letter of the correct answer on the scoring sheet.

Each question answered correctly is awarded 1 mark.
Each question answered incorrectly is awarded 0 marks.

Total possible for entire exam is 40 marks.

01 A recent study of response times for ambulances has provided the following data, in minutes.

3.9 7.8 5.2 6.7 4.4 4.4 5.6

What is the mean of this sample?

$\sum x_i = 38.0 \quad n = 7$

$\bar{x} = 5.4$

(a) 4.4
(b) 5.2
(c) 5.4
(d) 5.6

02 The production engineer for Golden Valley Ltd. is concerned that there is too much variability in the net weight of its 50 kg jasmine rice bags. The manufacturing process was designed to yield a true variance of 7 kg^2. If a sample of 5 bags is used to routinely track the variability in net weights, what value of the sample variance will only be exceeded 5% of the time?

$n = 5 \quad \sigma^2 = 7 \text{ kg}^2$

(a) 6.7
(b) 14.9
(c) 16.6
(d) 19.4

$\chi^2_{0.05,4} = 9.488 = \frac{(5-1)s^2}{7}$

$s^2 = 16.6$

03 The City of Dunbar has investigated the number of emergency road crews needed during any given shift. The results of this study include the following probability distribution:

X	0	1	2	3	4	≥ 5
$f(x)$	0.47	0.20	0.17	0.09	0.05	0.02

where X = number of emergencies during a shift needing a road crew.

If the City has only 3 emergency road crews, what is the probability that some emergencies will not be attended to during the shift?

$P(x > 3) = P(x = 4) + P(x \geq 5)$

$= 0.07$

(a) 0.06
(b) 0.07
(c) 0.09
(d) 0.16

04 A new fire retardant coating is expected to provide a mean delay of 5 minutes. A leading testing facility was hired to conduct 6 tests, yielding the following data:

4.5 4.1 5.6 3.7 5.0 3.4

$\bar{x} = 4.38$ $t_{obs} = \frac{4.38-5.0}{0.82/\sqrt{6}}$

$s = 0.82$

Using a one-tailed test with an alternative hypothesis that the mean is less than 5 minutes, and α of 5%, what are the statistics for the test and associated conclusion? (Assume that the delay achieved by the coating is normally distributed.)

$t_{obs} = -1.852$

(a) -2.025 < -1.645, and the test rejects that the true mean is 5.0 minutes. $t_{0.05,5} = -2.015$
(b) -2.025 < -2.015, and the test rejects that the true mean is 5.0 minutes.
(c) -1.852 < -1.645, and the test rejects that the true mean is 5.0 minutes.
(d) -1.852 > -2.015, and the test fails to reject that the true mean is 5.0 minutes.

05 The time-to-failure of a shock absorber used in a rally car follows the normal distribution with a mean of 21 hours and a standard deviation of 13 hours. A typical race team will therefore carry 16 shock absorbers to ensure an ample supply of spare parts. What is the probability that a shock absorber will last at least 30 hours before failing?

$P(x > 30) = 1 - P(x < 30)$

$= 1 - P(z < (30-21)/13)$

$= 1 - P(z < 0.69)$

$= 1 - 0.7549$

$= 0.2451$

(a) 0.0000
(b) 0.0028
(c) 0.2397
(d) 0.2451

06 A city engineer is reviewing her annual snow removal budget of \$ 1.4 million dollars. This budget will allow for 24 snow days. The past 5 years of data indicate that the mean number of snow days is 19.0 days per year, with a standard deviation of 6.4 days per year. What is the 95% confidence interval for the true mean number of snow days per year? (Assume that the number of snow days in any given year follows the normal distribution.)

$\bar{x} = 19.0$ $s_{\bar{x}} = s_x/\sqrt{n} = 6.4/\sqrt{5} = 2.9$

$t_{0.025,4} = 2.776$

(a) $19.0 \pm 1.960 \times 2.9$
(b) $19.0 \pm 2.776 \times 2.9$
(c) $19.0 \pm 1.960 \times 6.4$
(d) $19.0 \pm 2.776 \times 6.4$

$19.0 \pm 2.776 \times 2.9$

8/83

07 *Thermal Solutions Ltd.* has developed an advanced thermal absorption product that it sells in the form of sheets. The performance of this new product is very sensitive to its carbon content, especially to high levels of carbon where the sheets become brittle. The actual carbon content present in a single sheet, which ideally must be 124 grams, is approximately normally distributed. The company's process engineer wants to use samples of 7 sheets to determine both the mean and standard deviation of the carbon content (in grams), and conduct hypothesis tests using these sample values. How should the null and alternative hypotheses be defined to achieve a strong conclusion that the mean carbon content is too high?

(a) H_0: $\mu < 124$ and H_1: $\mu \geq 124$
(b) H_0: $\mu \leq 124$ and H_1: $\mu > 124$
(c) H_0: $\mu \geq 124$ and H_1: $\mu < 124$
(d) H_0: $\mu > 124$ and H_1: $\mu \leq 124$

Questions 08 and 09
A drilling company wants to develop a greater understanding of the relationship between the number of hours of operation of a drill bit (x, in hours) and the wear of the drill bit (y, in millimeters). To establish this relationship, data has been collected and the following values calculated:

$\sum_i x_i = 401.2$ $\sum_i x_i^2 = 17438.76$ $\bar{x} = 40.12$ note: $n = 10$

$\sum_i y_i = 41.8$ $\sum_i y_i^2 = 189.4$ $\bar{y} = 4.18$

$\sum_i x_i y_i = 1812.88$

$S_{xx} = 1342.616$ $S_{yy} = 14.676$ $S_{xy} = 135.864$

08 *Using the method of least squares, what is the regression line for the relationship $x = g(y)$?*

$$b = \frac{S_{xy}}{S_{yy}} = \frac{135.864}{14.676} = 9.26$$

(a) $x = 1.41 + 9.26\,y$
(b) $x = 9.26 + 1.41\,y$
(c) $x = 0.128 + 0.101\,y$
(d) $x = 0.101 + 0.128\,y$

$a = 40.12 - 9.26 \times 4.18 = 1.41$

09 What is the 95% confidence interval for the wear of a drill bit after 35 hours of operation?

(a) $3.66 \pm 2.262 \times 0.119$
(b) $3.66 \pm 2.262 \times 0.365$
(c) $3.66 \pm 2.306 \times 0.119$
(d) $3.66 \pm 2.306 \times 0.365$

$a = 0.128$
$b = 0.101$
$\hat{y} = 0.128 + 0.101 \times 35 = 3.66$
$s^2 = (14.676 - 0.101 \times 135.864)/8$
$s = 0.345$
$t_{0.025,8} = 2.306$
$s\left(1 + 1/10 + \frac{(35-40.12)^2}{1342.616}\right)^{1/2} = 0.365$

10 A bicycle company is developing a carbon fiber composite material for its new line of road bicycles. Optimal strength characteristics for this material are achieved at a density of 3.4 kg/m^3. Research into the process to be used to manufacture the bicycles indicates that any variability in the material remains constant, and therefore the true standard deviation for the density is constant at 0.3 kg/m^3. A sample of 12 densities obtained for the manufacturing process yielded a mean density of 3.1 kg/m^3. Using a two-tailed test with $\alpha = 10\%$, what is the appropriate statistic and conclusion regarding the ability of the manufacturing process to yield the optimal material density? (Assume that the density follows the normal distribution.)

(a) $-3.46 < -1.645$, therefore reject H_0.
(b) $-3.46 < -1.796$, therefore reject H_0.
(c) $-1.00 < -1.645$, therefore reject H_0.
(d) $-1.00 < -1.796$, therefore reject H_0.

$H_0: \mu = 3.4$ $\sigma = 0.3$
$H_1: \mu \neq 3.4$ $n = 12$

$z_{obs} = \frac{3.1 - 3.4}{0.3/\sqrt{12}} = -3.46$

$\pm z_{0.05} = \pm 1.645$ ∴ reject H_0

11 Plastic sheets produced by a machine are periodically monitored for possible fluctuations in thickness. Uncontrollable heterogeneity in the viscosity of the liquid mold makes some variation in thickness measurements unavoidable. However, if the true standard deviation of thickness exceeds 1.5 millimetres, there is a cause to be concerned about the product quality. Thickness measurements (in millimetres) of 5 specimens produced on a particular shift resulted in the following data:

226 228 226 228 232

$s = 2.45$

What is the variance of this sample?

$s^2 = 6.00$

(a) 2.19
(b) 2.45
(c) 4.80
(d) 6.00

83

12 A drilling company wants to investigate the variability in the life of its drill bits. After analyzing the available data, two regression models were developed to explain the variability in the life of a drill bit. The first model (Model 1) relates the life of the drill bit (hours), y, to the penetration test of the hardest material drilled (millimeters), x_1. The second model (Model 2) defines the relationship between the life of the drill bit, y, and the total depth drilled (m), x_2.

$y = 26.6 + 31.2x_1 \quad r_1 = 0.92$ $\qquad r_1^2 = 0.85$

$y = 17.9 + 0.31x_2 \quad r_2 = 0.97$ $\qquad r_2^2 = 0.94$

Which model should be used, and why?

(a) Model 1 should be used because Model 1 explains 85% of the variability of y, compared to 94% for Model 2.
(b) Model 1 should be used because Model 1 explains 92% of the variability of y, compared to 97% for Model 2.
(c) Model 2 should be used because Model 2 explains 94% of the variability of y, compared to 85% for Model 1.
(d) Model 2 should be used because Model 2 explains 97% of the variability of y, compared to 92% for Model 1.

13 The City of Charleston has measured the levels of suspended solids in water runoff during 54 rainstorms. These data have been collated in the following table:

$z < -1.57 \quad -1.57 < z < -0.14 \quad -0.14 < z < 1.29 \quad z > 1.29$

Suspended Solids (mg/L)	x < 50	50 ≤ x < 100	100 ≤ x < 150	150 ≤ x
Frequency of Storms	9	23	15	7

Combine

$n = 54$

E: $0.0582 \times 54 \quad 0.3861 \times 54 \quad 0.4572 \times 54 \quad 0.0985 \times 54$

$3.1 \quad 20.8 \quad 24.7 \quad 5.3$

At the 5% level of significance, test the hypothesis that X, the suspended solids concentration (in mg/L), follows a normal distribution with a mean of 105 mg/L and a standard deviation 35 mg/L. What are the results of your test?

(a) $\chi^2_{obs.} = 7.10 > \chi^2_{0.05,2} = 5.991$; therefore, reject H_o
(b) $\chi^2_{obs.} = 8.74 > \chi^2_{0.05,2} = 5.991$; therefore, reject H_o
(c) $\chi^2_{obs.} = 10.76 > \chi^2_{0.05,3} = 7.815$; therefore, reject H_o
(d) $\chi^2_{obs.} = 15.82 > \chi^2_{0.05,3} = 7.815$; therefore, reject H_o

$$\chi^2_{obs} = \frac{(32-23.9)^2}{23.9} + \frac{(15-24.7)^2}{24.7} + \frac{(7-5.3)^2}{5.3} = 7.10$$

$$\chi^2_{0.05,2} = 5.991$$

Ø3

14 *AAA Photocopiers* is concerned with the variability in the time-to-service of the Series 5000 Business Photocopier. The following data have been collected on the time-to-service (in hours) of 6 photocopiers:

67 98 75 48 88 82

$s^2 = 305.9$

If the true standard deviation of the time-to-service for the Series 5000 Business Photocopier should be 15 hours, what is the value of the appropriate statistic for the hypothesis test H_0: $\sigma \le 15$ hours? Use $\alpha = 5\%$.

(a) 3.68
(b) 4.29
(c) 5.83
(d) 6.80

$$\chi^2_{obs} = \frac{(6-1) \times 305.9}{225} = 6.80$$

15 The chlorine injector at a water treatment plant should be set to discharge 7 kg per minute into the treated water. Past experience with these injectors suggest that the true standard deviation of the discharge is 0.75 kg per minute. The plant engineer has decided to collect 8 samples to determine the true mean discharge:

8.2 6.7 8.1 7.5 6.9 6.3 7.2 5.8 [kg per minute]

$\bar{x} = 7.1$
$s = 0.84$

What of the 99% confidence interval for the true mean discharge in kg per minute? (Assume the discharge is normally distributed.)

$z_{0.005} = 2.576$

(a) $7.1 \pm 2.576 \times 0.27$
(b) $7.1 \pm 2.576 \times 0.75$
(c) $7.1 \pm 3.499 \times 0.30$
(d) $7.1 \pm 3.499 \times 0.84$

$7.1 \pm 2.576 \times 0.75/\sqrt{8}$

$7.1 \pm 2.576 \times 0.27$

16 A new high-pressure process is being investigated that modifies the density of polymer sheets. A study investigating this process has generated the following data:

Density (kg/m³) Before X_1	Density (kg/m³) After X_2
2.3	2.8
2.5	2.6
2.4	2.7
2.6	2.9

What is the most appropriate test to conduct if we are interested in determining if there is a significant difference between the densities before and after the high-pressure process? (Assume that both densities follow the normal distribution, and that the true variances of these densities are equal.)

(a) $Z = \dfrac{\overline{D} - \mu_D}{\sigma_D/\sqrt{n}}$, $D = X_1 - X_2$

Paired observations

σ_D unknown

(b) $T = \dfrac{\overline{D} - \mu_D}{s_D/\sqrt{n}}$, $D = X_1 - X_2$

(c) $Z = \dfrac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$

(d) $T = \dfrac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{S_p\sqrt{1/n_1 + 1/n_2}}$, $\upsilon = n_1 + n_2 - 2$

17 A manufacturer of submersible pumps claims that at most 30% of the pumps require repairs within the first 5 years of operation. A random sample of 120 of these pumps includes 47 which required repairs within the first 5 years. Using a one-tailed test ($p > 0.30$), does the data support the manufacturer's claim at the 5% level of significance?

(a) Yes, since $z_{obs.} = 0.017 < 1.645$
(b) Yes, since $z_{obs.} = 0.017 < 1.960$
(c) No, since $z_{obs.} = 2.064 > 1.645$
(d) No, since $z_{obs.} = 2.064 > 1.960$

$\hat{p} = 47/120 = 0.392$

$z_{obs} = \dfrac{0.392 - 0.30}{(0.30 \times 0.70/120)^{1/2}} = 2.20$

$z_{0.05} = 1.645$

D3

18 If the probability is 0.10 that a certain wide flange column will fail under a given axial load, what is the probability that among 16 such columns two will fail?

(a) 0.2108
(b) 0.2745
(c) 0.4853
(d) 0.7892

$x \sim b(0.1, 16)$

$P(x=2) = P(x \le 2) - P(x \le 1)$

$= 0.7892 - 0.5147$

$= 0.2745$

19 The *Kilgaroon Diamond Mine* in South Africa has yielded diamonds ranging in size from $1/16^{th}$ of a carat to 3 carats. A geological engineer has established that the sizes of these diamonds are actually normally distributed with a mean of ¾ carat and a standard deviation of ¼ carat. In the diamond industry, diamonds of 1 carat or more are sold at auction rather than through a diamond wholesaler, and the proportion of the diamonds that are sent to auction is a measure of a mine's quality. A new seam currently being worked at the Kilgaroon Diamond Mine has yielded 100 auction quality diamonds out of the 500 diamonds extracted to date. Using a 5% significance level, what is the acceptance region for the hypothesis that the new seam is of higher quality than the rest of the Kilgaroon Mine? *(Note: use new seam minus Kilgaroon Mine.)*

(a) $(-1.645, \infty)$
(b) $(-\infty, +1.645)$
(c) $(+1.645, \infty)$
(d) $(-\infty, -1.645)$

$H_0: p_1 - p_2 \le 0$

$H_1: p_1 - p_2 > 0$

$\therefore (-\infty, +1.645)$

20 A manufacturing engineer is investigating the number of returns of HighTork camshafts. A study of the manufacturing process has revealed that 9% of all HighTork camshafts sold are defective, and represent a serious safety hazard if installed. Each camshaft returned to the company is therefore examined to establish if it is one of the defective units. Of the camshafts returned, 23% are defective. If 10% of all camshafts sold are returned, what is the proportion of camshafts in use (not returned) defective?

(a) 0.032
(b) 0.069
(c) 0.074
(d) 0.081

$P(D) = 0.090 \quad P(D|R) = 0.23 \quad P(R) = 0.10$

$P(D'|R) = 0.77 \quad P(R') = 0.90$

$P(D \cap R) = P(D|R)P(R) = 0.23 \times 0.10 = 0.023$

$P(D \cap R') = P(D) - P(D \cap R) = 0.090 - 0.023$
$= 0.067$

$$P(D|R') = \frac{P(D \cap R')}{P(R')} = \frac{0.067}{0.90} = 0.074$$

D3

21. If the distribution of the weights of all men travelling by air between Dallas and El Paso has a mean of 163 pounds and has a standard deviation of 18 pounds, what is the probability that the combined gross weight of 36 men travelling on a plane between these two cities is more than 6,000 pounds? Assume the individual weights are normally distributed.

$\mu = 163 \quad \sigma = 18 \quad n = 36 \quad \bar{x} = 166.67$

(a) 0.89
(b) 0.41
(c) 0.11
(d) 0.48

$$P(\bar{x} > 166.67) = P(z > (166.67 - 163)/(18/\sqrt{36}))$$
$$= P(z > 1.223)$$
$$= 1 - P(z < 1.223)$$
$$= 1 - 0.89 = 0.11$$

22. The diameter of rotor shafts in a lot have a mean of 0.249 inches and a standard deviation of 0.003 inches. The inner diameter of bearings in another lot have a mean of 0.255 inches and a standard deviation of 0.002 inches. If a shaft and a bearing are selected at random, what is the probability that the shaft will not fit inside the bearing? (Assume both dimensions are normally distributed).

$\mu_{RS} = 0.249 \quad \sigma_{RS} = 0.003$
$\mu_B = 0.255 \quad \sigma_B = 0.002$

(a) 0.9520
(b) 0.4681
(c) 0.5319
(d) 0.0480

$$P(x_{RS} - x_B > 0) = P\left(z > \frac{0 - (0.249 - 0.255)}{(0.003^2 + 0.002^2)^{1/2}}\right)$$
$$= P(z > 1.664) = 1 - 0.952 = 0.0480$$

23. The following sulfur emission data are ordered from smallest to greatest value:

6.2	7.7	8.3	9.0	9.4	9.8	10.5	10.7	11.0	11.2	11.8
12.3	12.8	13.2	13.3	13.5	13.9	14.4	14.5	14.7	15.2	15.5
15.8	15.9	16.2	16.7	16.9	17.0	17.3	17.5	17.6	17.9	18.0
18.0	18.1	18.1	18.4	18.5	18.7	19.0	19.1	19.2	19.3	19.4
19.4	20.0	20.1	20.1	20.4	20.5	20.8	20.9	21.4	21.6	21.9
22.3	22.5	22.7	22.7	22.9	23.0	23.5	23.7	23.9	24.1	24.3
24.6	24.6	24.8	25.7	25.9	26.1	26.4	26.6	26.8	27.5	28.5
28.6	29.6	31.8								

$n = 80$

Compute the 50th percentile.

$$\tilde{x} = \frac{x_{40} + x_{41}}{2} = \frac{19.0 + 19.1}{2} = 19.05$$

(a) 19.10
(b) 19.01
(c) 19.05
(d) 20.91

24. J.J. Thomson (1856-1940) discovered the electron by isolating negatively charged particles for which he could measure the mass-charge ratio. This ratio appeared to be constant over a wide range of environmental conditions, and consequently, could be a characteristic of a new particle. His observations, from two different cathode-ray tubes that used air as the gas, are:

Tube 1	0.57	0.34	0.43	0.32	0.48	0.40	0.40
Tube 2	0.53	0.47	0.47	0.51	0.63	0.61	0.48

Find the 90% confidence interval for the difference between the means of the mass-charge ratio if you know both population variances are equal to 0.01 and the observations are normally distributed.

$\bar{x}_1 = 0.42$ $\bar{x}_2 = 0.529$ $\sigma_1^2 = \sigma_2^2 = 0.01$

$s_1 = 0.085$ $s_2 = 0.122$

$n_1 = 7$ $n_2 = 7$

(a) –0.0282, -0.1898
(b) 0.0001, -0.2181
(c) -0.0211, -0.1969
(d) -0.0096, -0.2084

$(\bar{x}_1 - \bar{x}_2) \pm z_{0.05}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

$-0.109 \pm 1.645(0.01/7 + 0.01/7)^{1/2}$.

25. The vessels that contain the reactions at some nuclear power plants consists of two hemispherical components that are welded together. Copper in the welds could cause them to become brittle after years of service. Samples of welding material from one production run that were used in one plant had the copper contents 0.27, 0.35, 0.37. Samples from the next run had values of 0.23, 0.15, 0.25, 0.24, 0.3, 0.33, 0.26. Find a 90% confidence interval for the difference between the two means. Assume the variances are unequal.

$n_1 = 3$ $n_2 = 7$

$\bar{x}_1 = 0.33$ $\bar{x}_2 = 0.251$ $\nu = 4.163$

$s_1 = 0.05292$ $s_2 = 0.05699$ say 4

$t_{0.05,4} = 2.132$

(a) 0.1486, 0.0094
(b) 0.0036, 0.1544
(c) 0.0175, 0.1405
(d) -0.0074, 0.1587

$(0.33 - 0.251) \pm 2.132\left(\frac{0.05292^2}{3} + \frac{0.05699^2}{7}\right)^{1/2}$

26. If only 36 of 100 persons interviewed are familiar with the tax incentives for installing certain energy-saving devices, construct a 95% confidence interval for the corresponding true proportion.

$\hat{p} = 36/100 = 0.36$ $\hat{q} = 0.64$

(a) 0.439, 0.281
(b) 0.454, 0.266
(c) 0.365, 0.356
(d) 0.734, 0.546

$\hat{p} \pm z_{0.025}(\hat{p}\hat{q}/n)^{1/2}$

$0.36 \pm 1.960(0.36 \times 0.64/100)^{1/2}$

D3

27. Suppose that the refractive indices of 20 pieces of glass (randomly selected from a large shipment purchased by an optical firm) has a variance of 1.2 x 10^{-4}. Assuming that the refractive indices are normally distributed, construct a 95% confidence interval for the variance of the population.

$n = 20 \quad s^2 = 1.2 \times 10^{-4}$

(a) 0.000069, 0.000256
(b) 0.000093, 0.000277
(c) 0.000082, 0.000292
(d) 0.000089, 0.000258

$\chi^2_{0.025,19} = 32.852$

$\chi^2_{0.975,19} = 8.907$

$s_L^2 = \frac{19 \times 1.2 \times 10^{-4}}{32.852} \quad s_u^2 = \frac{19 \times 1.2 \times 10^{-4}}{8.907}$

28. Suppose that we want to estimate the true proportion of defectives in a very large shipment of adobe bricks, and that we want to be at least 95% confident that the error is at most 0.04. How large a sample would we need if we had no idea what the true proportion would be?

(a) 601
(b) 423
(c) 422
(d) 600

$n = \frac{z_{\alpha/2}^2}{4e^2} \quad z_{0.025} = 1.960 \quad e = 0.04$

$n = \frac{1.960^2}{4 \times 0.04^2} = 600.25$

29. In a study designed to determine the number of turns required for an artillery-shell to arm, 75 shells averaged 38.7 turns with a standard deviation of 4.3 turns. Establish tolerance limits for which one can assert with 99% confidence that at least 95% of the fuses will arm within these limits.

$n = 75 \quad s = 4.3$

$\bar{x} = 38.7 \quad \gamma = 0.01$

$1 - \alpha = 0.95$

(a) 51.61, 25.79
(b) 47.48, 29.92
(c) 49.16, 28.24
(d) 52.45, 24.95

$k = 2.433$

$38.7 \pm 2.433 \times 4.3$

30. What is the maximum error one can expect to make with probability 0.90 when using the mean of a random sample of size n = 64 to estimate the mean of a population with a true ~~variance~~ of 2.56?
standard deviation

$n = 64 \quad \alpha = 0.10$

$\sigma = 2.56$

$z_{\alpha/2} = z_{0.05} = 1.645$

(a) 0.4099
(b) 0.5264
(c) 0.0658
(d) 0.0512

$z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 1.645 \times \frac{2.56}{\sqrt{64}} = 0.5264$

31. The following data were obtained in an experiment designed to check whether there is a systematic difference in the weights obtained with two different scales:

Rock Specimen	Weight in Grams from Scale 1	Weight in grams from Scale II	
1	11.23	11.27	-0.04
2	14.36	14.41	-0.05
3	8.33	8.35	-0.02
4	10.50	10.52	-0.02
5	23.42	23.41	0.01
6	9.15	9.17	-0.02
7	13.47	13.52	-0.05
8	6.47	6.46	0.01
9	12.40	12.45	-0.05
10	19.38	19.35	0.03

Find the confidence intervals for the difference of the means of the weights obtained with the two scales at the 0.05 confidence level.

$\bar{D} = -0.02 \quad s_D = 0.029$

$n = 10$

(a) 5.20, -5.24
(b) 4.83, -4.87
(c) 1.52, -1.55
(d) -0.0407, 0.0007

$\bar{D} \pm t_{0.025,9} \times \frac{0.029}{\sqrt{10}}$

$t_{0.025,9} = 2.262$

32. A process for making steel pipe is under control if the diameter of the pipe has a mean of 3.0000 inches with a standard deviation of 0.0250 inches. To check whether the process is under control, a random sample of size n=30 is taken each day and the null hypothesis of H_o: $\mu = 3.0000$ is rejected if the sample average is less than 2.9960 or greater than 3.0040. Find the probability of a Type I error. Assume the standard deviation remains constant.

(a) 0.38
(b) 0.62
(c) ≈ 0
(d) 0.89

P(Type I Error)

P(Reject H_o | H_o is true)

$P(\bar{x} < 2.9960 \text{ or } \bar{x} > 3.0040 \mid \mu = 3.0000)$

$P\left(z < \frac{2.9960-3}{0.0250/\sqrt{30}} \text{ or } z > \frac{3.0040-3}{0.0250/\sqrt{30}}\right)$

$P(z < -0.876) + P(z > 0.876)$

$0.190 + (1 - 0.810) = 0.38$

DB

33. Suppose that the length of certain machine parts may be looked upon as a random variable having a normal distribution with a mean of 2.000 cm and a standard deviation of 0.050 cm. Specifically, we shall want to test the null hypothesis $\mu = 2.000$ against the alternative hypothesis that $\mu \neq 2.000$ on the basis of the mean of a random sample of size n = 30. It has also been determined that an important deviation in this test would be a mean of 2.010. If the probability of a Type I error is to be 0.05, what is the probability of a Type II error?

$\mu_0 = 2 \quad \sigma = 0.05 \quad n = 30 \quad \alpha = 0.05 \quad z_{0.025} = 1.96$

(a) 0.95
(b) 0.20
(c) 0.73
(d) 0.81

$L = \mu_0 - z_{0.025}\sigma/\sqrt{n} = 1.9821$

$U = \mu_0 + z_{0.025}\sigma/\sqrt{n} = 2.0179$

$P(1.9821 < \bar{x} < 2.0179 \mid \mu = 2.010) = 0.805$

34. Suppose that for a given population with standard deviation of 8.4 in^2 we want to test the null hypothesis that the population mean is 80 in^2 against the alternative hypothesis that it is less than 80 in^2. On the basis of a random sample of size n = 100, what is the acceptance region for this test for a 0.1 significance level?

$H_0: \mu = 80 \qquad n = 100$

$H_1: \mu < 80 \qquad \sigma = 8.4$

(a) $78.9 < \bar{X} < 81.1$
(b) $\bar{X} < 81.1$
(c) $\bar{X} > 78.9$
(d) $\bar{X} > 81.1$

$L = \mu_0 - z_{0.10}\sigma/\sqrt{n}$

$= 80 - 1.283 \times 8.4/\sqrt{100} = 78.9$

35. An investigation of two kinds of photocopying equipment showed that 71 failures of the first kind of equipment took on the average 83.2 minutes to repair with a standard deviation of 19.3 minutes, while 75 failures of the second kind of equipment took on the average 90.8 minutes to repair with a standard deviation of 21.4 minutes. Using 19.4 and 21.4 as very good estimates of σ_1 and σ_2, respectively, test the hypothesis that the means are equal against the alternative that they are unequal at the 95% significance level.

$n_1 = 71 \qquad n_2 = 75$

(a) z = -1.645, Reject H_o
(b) z = -2.25, Reject H_o
(c) z = -2.25, Accept H_o
(d) z = -1.645, Accept H_o

$\bar{x}_1 = 83.2 \qquad \bar{x}_2 = 90.8$

$s_1 = 19.3 (= \sigma_1) \qquad s_2 = 21.4 (= \sigma_2)$

$\pm z_{0.025} = \pm 1.960$

$H_0: \mu_1 - \mu_2 = 0$

$H_1: \mu_1 - \mu_2 \neq 0$

$$z_{obs} = \frac{(83.2 - 90.8) - 0}{(19.4^2/71 + 21.4^2/75)^{1/2}} = -2.25$$

$\therefore$ Reject H_o

36. The following random samples are measurements of the heat-producing capacity (in millions of calories per ton) of specimens of coal from two mines:

Mine 1:	8,620	8,130	8,350	8,070	8,340	
Mine 2:	7,950	7,890	7,900	8,140	7,920	7,840

$s_1 = 216.956 \quad \nu_1 = 4$
$s_2 = 104.5 \quad \nu_2 = 5$

It is desired to test whether the difference between the means of these two samples is significant. Test whether it is reasonable to assume at the 0.02 level of significance that the variances of the two populations sampled are equal.

$H_0: \sigma_1^2 = \sigma_2^2$
$H_1: \sigma_1^2 \neq \sigma_2^2$
$\alpha/2 = 0.01$

(a) F = 4.31, population variances are equal
(b) F = 4.31, population variances are unequal
(c) F = 15.52, population variances are unequal
(d) F = 11.39, population variances are equal

$f_{0.01}(4,5) = 11.39$
$f_{0.99}(4,5) = 1/f_{0.01}(5,4) = 1/15.52$

$F = \frac{216.956^2}{104.5^2} = 4.31$

Questions 37 and 38 deal with the following information:

The following are measurements of the air velocity (x) in cm/sec and evaporation coefficients (y) in mm^2/sec of burning fuel droplets in an impulse engine:

$$\sum_{i=1}^{10} x_i = 2{,}000 \qquad \sum_{i=1}^{10} x_i^2 = 532{,}000$$

Note n = 10

$$\sum_{i=1}^{10} y_i = 8.35 \qquad \sum_{i=1}^{10} x_i y_i = 2{,}172.40$$

$$\sum_{i=1}^{10} y_i^2 = 9.1097$$

$S_{xx} = 132{,}000 \qquad S_{xy} = 505.40 \qquad S_{yy} = 2.13745$

37. Compute the sample coefficient of determination.

(a) 0.951
(b) 0.905
(c) 0.819
(d) 0.975

$r^2 = \frac{S_{xy}^2}{S_{xx} S_{yy}}$

$r^2 = \frac{(505.40)^2}{132000 \times 2.13745}$

$r^2 = 0.905$

D3

38. A regression model has been established such that for an air velocity of 190 cm/sec, we get an evaporation coefficient of 0.80 mm^2/sec. Compute the 90% confidence interval for the mean response of the evaporation coefficients using that velocity.

$$s^2 = \frac{S_{yy} - b S_{xy}}{n-2} \qquad s = 0.1588$$

(a) 0.700, 0.890
(b) 0.490, 1.110
(c) 0.706, 0.894
(d) 0.480, 1.120

$$\hat{y} \pm t_{0.05,8}\, s\sqrt{\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}}$$

$$0.8 \pm 1.860 \times 0.1588\left(1/10 + \frac{(190-200)^2}{132000}\right)^{1/2}$$

39. The probability of committing a Type II error in a test to determine if the mean of male weights in a university is 162 kg, is equal to 0.08. An important deviation from the conjectured mean is 165 kg. If the level of significance at which the test is being conducted is 0.05, what is the power of this test?

$\beta = 0.08$

(a) 0.95
(b) 0.92
(c) 0.08
(d) 0.05

$1 - \beta = 0.92$

40. Suppose we wish to test the hypothesis that the mean number of hurricanes spawned in the Atlantic ocean in any given year is 20, against the alternative that is greater than 20. The true standard deviation is 6 hurricanes. Find the sample size required for this test if the significance level is 0.05. The power of the test is 0.95 when the true number of hurricanes in any given year is 28.

$H_0: \mu = 20$ $\quad \sigma = 6$

$H_1: \mu > 20$ $\quad \alpha = 0.05$

$\delta = 28 - 20 = 8$ $\quad \beta = 0.05$

(a) 7
(b) 3
(c) 8
(d) 9

$$n = \frac{(z_\alpha + z_\beta)^2 \sigma^2}{\delta^2}$$

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$$n = \frac{(1.645 + 1.645)^2 \times 6^2}{8^2}$$

$n = 6.08$ say 7