**Final Exam**
**EXAMINATION**
**Summer 2013**

**DURATION: 3 HOURS**

No. of Students: **72**

Department Name & Course Number: **Physics 1004A**
Course Instructor(s) **Dr. Andrew Robinson**

AUTHORIZED MEMORANDA

Pen, pencil, calculator

**Students MUST count the number of pages in this examination question paper before beginning to write, and report any discrepancy to a proctor. This question paper has 12 pages including the cover page.**

**This examination question paper may not be taken from the examination room.**

| | | |
|---|---|---|
| **In addition to this question paper, students require:** | **an examination booklet** | **no** |
| | **a Scantron sheet** | **no** |

**Please print your name and student number in the boxes below:**

| Family Name | First Name | Student Number |
|---|---|---|
| | | |

This exam is out of 60 marks and consists of two parts:

Section A: 10 multiple choice questions, each worth one mark. Please circle the correct answer on the paper.

Section B: Answer 5 out of the 6 questions and tick the check boxes provided to indicate your choice of the questions to be marked. Each question is worth 10 marks. Write your answers under the questions. You may use the other side of the sheet if you require more space. Useful formulae are provided on the last three pages of the examination paper. These three sheets may be removed from the paper.

| | | |
|---|---|---|
| | **Section A** | |
| ☐ | **B1** | |
| ☐ | **B2** | |
| ☐ | **B3** | |
| ☐ | **B4** | |
| ☐ | **B5** | |
| ☐ | **B6** | |
| | **Total** | |

## Section A

Answer all questions. Each question is worth 1 mark. Please circle your answer in the box.

### A1

A negatively-charged object is released from rest in a region containing a uniform magnetic field. Which one of the following statements concerning the subsequent motion of the object is **correct**?

(A) The object will remain motionless.
(B) The object will experience a constant acceleration and move in the direction of the magnetic field.
(C) The object will experience a constant acceleration and move in the direction opposite that of the magnetic field.
(D) The object will move at constant speed in a circle defined by the right hand rule.

Answer A is correct – a stationary particle feels no magnetic force

### A2

A wave is described by the equation:

$$y(x,t) = (0.45\ m)sin[(8\pi\ rad/s)t + (\pi\ rad/m)x]$$

Find the frequency of the wave.

| | | | |
|---|---|---|---|
| (A) 1.0 Hz | (B) 2.0 Hz | (C) 4.0 Hz | (D) 8.0 Hz |

$$\omega = 8.0\pi \text{ rad/s} \qquad \omega = 2\pi f$$

$$f = \frac{8\pi \text{ rad/s}}{2\pi \text{ rad}} \Rightarrow f = 4.0 \text{ Hz}$$

### A3

Calculate the **magnitude** of the magnetic force on an electron which is moving at $1.00 \times 10^{6}$ m/s at an angle of $60.0^{o}$ to a magnetic field of 1.00 mT.

| | | | |
|---|---|---|---|
| (A) $3.5 \times 10^{-16}$ N | (B) $1.39 \times 10^{-16}$ N | (C) $9.43 \times 10^{-17}$ N | (D) $2.13 \times 10^{-17}$ N |

Solution:

$$F = qvBsin\theta = (1.60 \times 10^{-19} \times 1.00 \times 10^{6} \times 1.0 \times 10^{-3}T \ \times \sin 60.0° = 1.39 \times 10^{-16}N$$

## A4

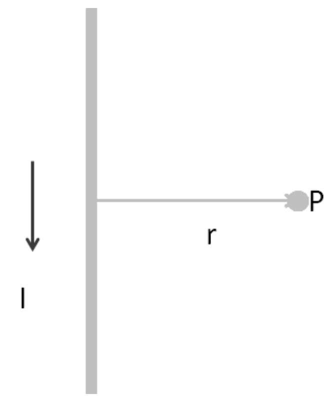
If a proton is accelerated in a $2.0\times10^{4}$ N/C electric field, what is the magnitude of the acceleration? The mass of a proton is $1.6726 \times 10^{-27}$ kg.

| | | | |
|---|---|---|---|
| (A) $1.9\times10^{12}$ m/s | (B) $2.8\times10^{2}$ m/s | (C) $5.4\times10^{7}$ m/s | (D) $8.4\times10^{10}$ m/s |

$$F = ma = qE = 1.6 \times 10^{-19} C \times 2.00 \times \frac{10^4\, N}{C}$$
$$F = 1.9 \times 10^{12} m/s$$

## A5

Calculate the magnitude of the magnetic field at point P, created when a current of 0.500 amps flows through the long straight wire shown in the figure. The distance r = 1.00 mm




| | | | |
|---|---|---|---|
| (A) $1.0\times10^{+2}$ T | (B) 1.0 T | (C) $1.0\times10^{-2}$ T | (D) $1.0\times10^{-4}$ T |

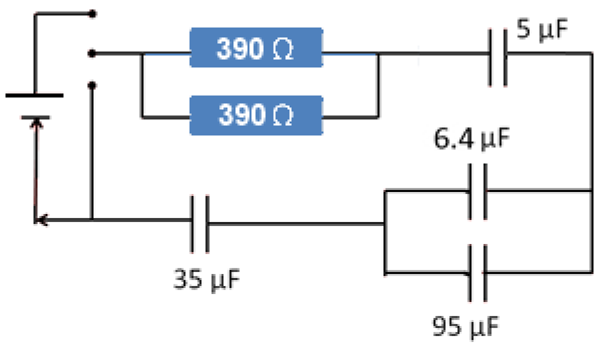
$$B = \frac{\mu_0 I}{2\pi R}$$

Solution

$$B = \frac{4\pi \times 10^{-7} \mathrm{V} \cdot \frac{\mathrm{s}}{\mathrm{A \cdot m}} \times 0.5\mathrm{A}}{2\pi \times 1 \times 10^{-3}} = 1.0 \times 10^{-4}\, T$$

## A6

What is the time constant of this RC circuit?




| | | | |
|---|---|---|---|
| (A) 16 µs | (B) 0.82 ms | (C) 1.2 ms | (D) 0.15 s |

Resistors in parallel:

1/R = 1/390 + 1/390   R = 390/2 = 195 ohm

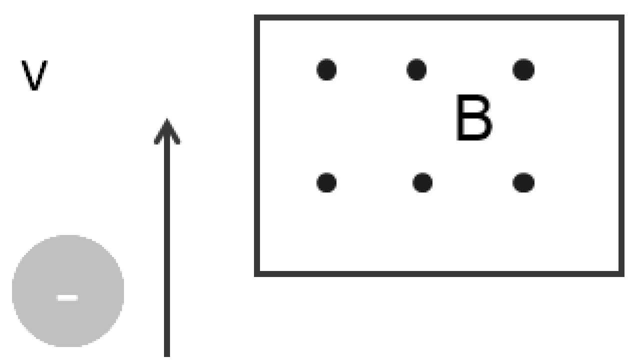
Capacitors in parallel : add   6.4 + 95 = 101.4 uF

Capacitors in series 1/C = 1/35 + 1/5 + 1/101.4    C = 4.19 uF

Time constant t = RC = 195 x 4.19 uF = 0.82 ms

## A7

A negatively charged particle enters a uniform magnetic field with a velocity vector perpendicular to the direction of the magnetic field. In what direction is the magnetic force exerted on the particle?




| | | | |
|---|---|---|---|
| (A) Left | (B) Right | (C) Into the page | (D) Out of the page |

DO RHR  - thumb points right, but that's for a positive charge, so the force is to the left

## A8

Length should be 1.0 cm

A solenoid of length 15 cm, and with 45 turns, has a current of 1.25 amps flowing in it. What is the magnitude of the magnetic field inside the solenoid?

| | | | |
|---|---|---|---|
| (A) $1.0\times10^{-5}$ T | (B) $1.2\times10^{-4}$ T | (C) $3.8\times10^{-3}$ T | (D) $7.0\times10^{-3}$ T |

B = $B = \mu_0 in = 4\pi \times 10^{-7} \times 1.25 \times \frac{45}{0.01} = 7.1 \times 10^{-3}\ T$

## A9

A coil is connected in series with a 10.0 kΩ resistor. An ideal 50.0V battery is connected across the two devices, and the current reaches 2.00 mA after 5.00 milliseconds. Find the inductance of the coil.

| | | | |
|---|---|---|---|
| (A) 97.9 H | (B) 108 H | (C) 979 H | (D) $2.61\times10^{3}$ H |

$$\mathrm{i} = \frac{\mathrm{V}}{\mathrm{R}}\left(1 - \mathrm{e}^{-\frac{\mathrm{Rt}}{\mathrm{L}}}\right)$$

$$\frac{\mathrm{iR}}{\mathrm{V}} = 1 - \mathrm{e}^{-\frac{\mathrm{Rt}}{\mathrm{L}}}$$

$$1 - \frac{\mathrm{iR}}{\mathrm{V}} = \mathrm{e}^{-\frac{\mathrm{Rt}}{\mathrm{L}}}$$

$$\ln\left(1 - \frac{\mathrm{iR}}{\mathrm{V}}\right) = -\frac{\mathrm{Rt}}{\mathrm{L}}$$

$$\mathrm{L} = -\frac{\mathrm{Rt}}{\ln\left(1 - \frac{\mathrm{iR}}{\mathrm{V}}\right)} = 97.9\ \mathrm{H}$$

## A10 ignore

In an oscillating LC circuit with L = 65 mH and C = 4.0μF. The current is initially a maximum. How long will it take before the capacitor is fully charged for the first time?

| | | | |
|---|---|---|---|
| (A) $7.1\times10^{-7}$ s | (B) $2.6\times10^{-6}$ s | (C) $4.0\times10^{-5}$ s | (D) $8.0\times10^{-4}$ s |

# Section B

Answer **5** questions out of **6**. Use the check boxes on the front page to indicate which questions you want marked. If the check boxes are not filled in, the first five questions encountered will be marked. All questions are worth 10 marks. **Show all work**. Equations not on the formula sheet must be derived from first principles. The appropriate number of significant figures must be used in the final answer.

**Section B questions continue on the next page.**

## B1

A $1.20\times10^{-14}$ N force with a fixed orientation does work on a particle of mass $2.50\times10^{-26}$ kg, as the particle moves through a displacement d = (2.00 m)i - (4.00 m)j + (3.00 m)k. The change in the particle's kinetic energy is $+3.00\times10^{-14}$ Joules. You may assume that no non-conservative forces act on the particle.

(a) Calculate the angle between the force and the displacement (4 marks)
(b) Derive an equation for the final speed in terms of mass m , change in kinetic energy ΔK and initial speed $v_i$ (4 marks)
(c) If the particle has an initial speed of $1.00\times10^{5}$ m/s, calculate the final speed. (2 marks)

*Solution:*
*(a)*
*Using the work-kinetic energy theorem, (1 mark for work kinetic energy theorem mentioned in words NOT as an equation)*

$$\Delta K = W = \vec{F}\cdot\vec{d} = Fd\cos\phi$$

*. 1 mark for this equation*

*In addition, F = $1.20\times10^{-14}$ N and*

$$d = \sqrt{(2.00\text{ m})^2 + (-4.00\text{ m})^2 + (3.00\text{ m})^2} = 5.39\text{ m}$$
.

*If ΔK = $3.00\times10^{-17}$ J*

*then 1 mark for the values of F,d and ΔK*

$$\varphi = \cos^{-1}\left(\frac{\Delta K}{Fd}\right) = \cos^{-1}\left(\frac{3.0\times10^{-14}}{1.20\times\ 10^{-14}\,N\times 5.39m}\right) =$$

$$\phi = \cos^{-1}\left(\frac{\Delta K}{Fd}\right) = \cos^{-1}\left(\frac{30.0\text{ J}}{(12.0\text{ N})(5.39\text{ m})}\right) = 62.3°$$

*. 1 mark for the final answer*

Deduct a mark if the final answer does not have three significant figures!

(b) *1 mark for the final equation, and three marks for showing the work. If the work does not show a clear logical progression, deduct a mark. If there is no work, but just the answer, they get 1 mark*

$$\boldsymbol{\Delta K = K_f - K_i}$$

$$\boldsymbol{\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}$$

$$\boldsymbol{\frac{2\Delta K}{m} = v_f^2 - v_i^2}$$

$$\boldsymbol{\frac{2\Delta K}{m} + v_i^2 = v_f^2}$$

$$\boldsymbol{|v_f| = \sqrt{\frac{2\Delta K}{m} + v_i^2}}$$

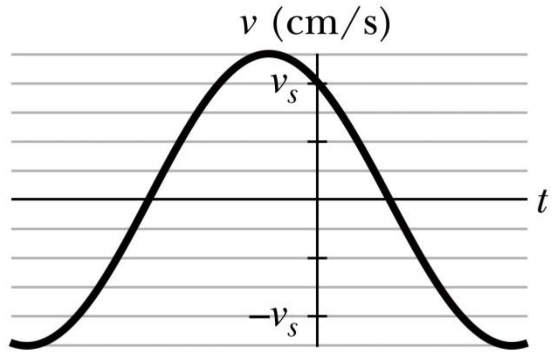
(c) Put the correct numbers into the equation. To get 2 marks, there must be the right answer, with units and with 3 sig figs. If the sig figs are incorrect, or the units are wrong or missing, then deduct 1 mark. Zero marks if the number is right, but there are the wrong number of significant figures and wrong/no units.

$$|v_f| = \sqrt{\frac{2 \times (3.00 \times 10^{-14}\, J)}{(2.5 \times 10^{-26} kg)} + (1.0 \times 10^{5}\ m/s)^2} = 1.55 \times 10^{6}\,\frac{m}{s}\ \ to\ 3\ s\ f$$

## B2

A harmonic oscillator has the position function

$$x(t) = x_m \cos(\omega t + \phi)$$




halliday_9e_fig_15_30

The velocity function, *v(t),* is pictured on the right. The vertical scale has a value of $v_s$ $4.0\times10^6$ cm/s at t = 0

(a) Calculate the phase constant ϕ (in radians) for the harmonic oscillator with the velocity function v(t) given in the figure (6 marks)

(b) Calculate the position of the particle at t = 1.0 seconds, assuming $x_m = 1.0\times10^{-7}$ cm. If you did not find a value in part (a), use ϕ = -0.8 instead. (4 marks)

**Solution**

We have the graph of the velocity function v(t), and the value of v at time t = 0.

If we start with the position function, and differentiate with respect to time, we will get the velocity function:

$$x(t) = x_m \cos(\omega t + \phi)$$

If you are comfortable with differentiation then you will know that this has the result

$$v(t) = \frac{dx}{dt} = -\omega x_m \sin(\omega t + \phi)$$

We also know that $v_s = 4.0\times10^6$ cm/s at t = 0, so we substitute these values in to the expression for v(t)

$$v_s = -\omega x_m \sin(\phi)$$

From the graph, the maximum value of the velocity function is $5.0\times10^6$ cm/s, and this must be equal to $-\omega x_m$

$$v_s = -v_m \sin(\phi)$$

$$\sin\phi = -\frac{v_s}{v_m}$$

$$\phi = \sin^{-1}\left(-\frac{v_s}{v_m}\right)$$

When you evaluate this function you MUST have your calculator set on the radian settings, so that you return a value of ϕ in radians (if it is on the degree settings, you will get the wrong answer!).

$$\phi = -0.93\ radians\ to\ 2\ significant\ figures$$

The value for phi is one of a series obtained by adding or subtracting 2π radians to this base value. Thus the smallest possible positive phase constant is

$$\phi = -0.93 + 2\pi = 5.4\, radians\ to\ 2\ significant\ figures$$

(b) Using $v(t) = \frac{dx}{dt} = -\omega x_m \sin(\omega t + \phi)$

The maximum value of the velocity function is ±5.0×10[6] cm/s, and this must be equal to $-\omega x_m$ and with the information that $x_m = 1.0\times10^{-7}$ cm

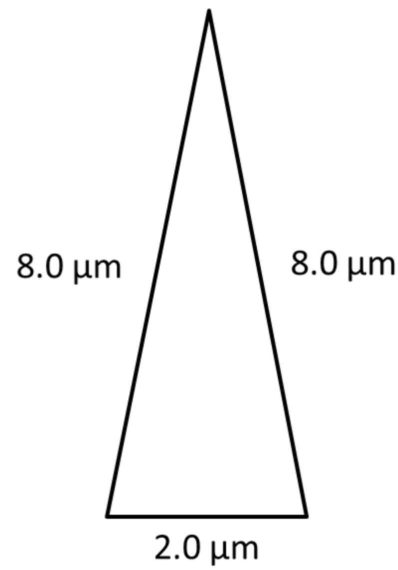
$$v_m = \pm\omega x_m$$

Hence $\omega = \left|\frac{v_m}{x_m}\right| = \frac{(5\times10^6\ cm/s)}{(1.0\times10^{-7} cm)} = 5 \times 10^{13} rad/s$

$$x(t = 1) = x_m \cos\left(5 \times \frac{10^{13} rad}{s} \times 1s + 5.36\right)$$

$$x(t = 1) = (1.0 \times 10^{-7} cm)\cos(5 \times 10^{13}) = 8.2 \times 10^{-8} cm$$

**B3** Calculate the potential energy in electron volts, stored by three protons, when they are arranged in an isosceles triangle with short side 2.0 μm, and equal sides of 8.0 μm. You may assume that the potential at infinite separation is zero.



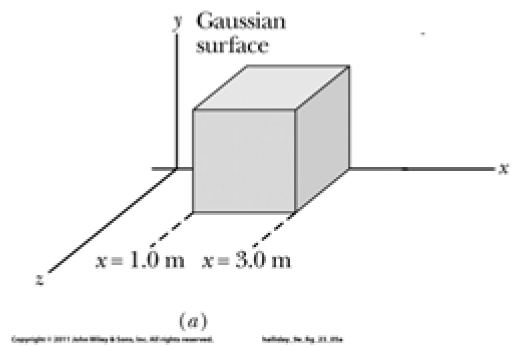
## B4

The electric field inside this Gaussian cube of side 2.0 m has the function

$\vec{E} = \left(\frac{1000}{x^2}\right)\hat{\imath}\ N/C$, where $x$ is measured in metres.

Determine

a) The charge enclosed within the cube.
b) The volume charge density
c) The number of elementary charges enclosed in the box, and whether they correspond to excess protons or excess electrons




Solution

Gauss' Law gives the enclosed charge

$$q_{enc} = \varepsilon_0 \oint \vec{E}.d\vec{A}$$

There is no electric field in the y and z directions, so the area integrals of E must equal zero. We only have to evaluate the flux into the cube from the x = 1 plane and the flux out from the x = 3 plane

$$q_{enc} = -\varepsilon_0 4\left(\frac{1000}{(x=1)^2}\right) + \varepsilon_0 4\left(\frac{1000}{(x=3)^2}\right) = \varepsilon_0 4\left[-\frac{1000}{1} + \frac{1000}{9}\right] = -\epsilon_0 \frac{8000}{9} = -32\ nC$$

(b) Volume charge density is

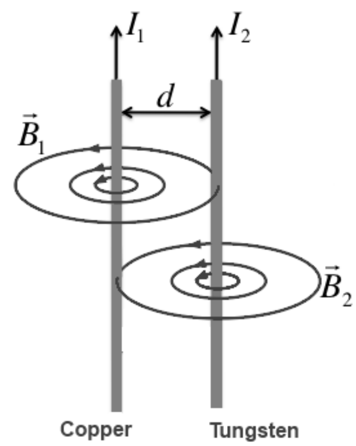
$$\rho = \frac{charge}{volume} = \frac{-32}{8\ m^3} = -4\ nC/m^3$$

(c) The charge is negative, so excess electrons

$$n = \frac{|q|}{e} = \frac{32 \times 10^{-9}}{1.60 \times 10^{-19}} = 2.0 \times 10^{11}$$

## B5

The two cylindrical wires shown in the diagram are 1.00 metres long and have a diameter of 1.00 cm. A potential difference of 1.00 Volts is applied across the ends of each wire to make the current flow. One wire is made of copper, the other from tungsten.

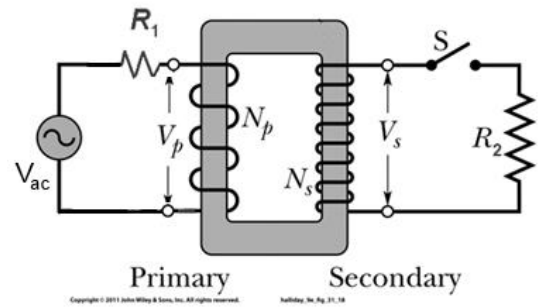



a) Calculate the current in each of the two wires.
b) Calculate the force each wire exerts on the other due to the magnetic field.

| Material | Resistivity at 20 °C (Ω.m) |
|---|---|
| Copper (Cu) | $1.68\times10^{-8}$ |
| Tungsten (W) | $5.60\times10^{-8}$ |

## B6

The step up transformer in the figure has 40 turns on its primary coil and 120 turns on its secondary coil. The primary coil is connected to a 12.0 $V_{rms}$ 60 Hz AC power supply. The resistor $R_1$ has a value of 370 Ω. A voltmeter measuring the potential across the primary coil reads 10.5 $V_{rms}$.




Calculate

(a) The rms current in the primary coil.

(b) The rms potential $V_s$ across the secondary coil

(c) The rms current passing through $R_2$ when the switch is closed

(d) The value of $R_2$

Solution

(a) Use Kirchoff's loop rule: The voltage drops across the resistor $R_1$ and primary coil must equal the voltage of the source.

Hence $V_{ac} = V_1 + V_s$

$$V_1 = i_P R_1 = V_{ac} - V_p$$

$$i_P = \frac{V_{ac} - V_P}{R_1} = \frac{12 - 10.5\ V}{370\ ohm} = 4.05 \times 10^{-3}\ A$$

(b) For the transformer, we can apply the following relationship:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

$$V_s = \frac{120}{40} \times 10.5\ V = 31.5\ V$$

(c)

$$\frac{i_p}{i_s} = \frac{N_s}{N_p}$$

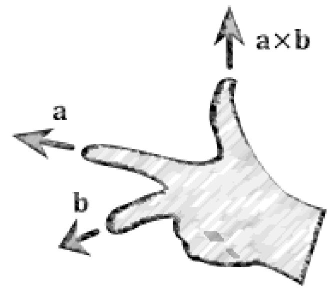
$$i_s = \frac{N_p}{N_s} i_p = \frac{40 \times 4.05 \times 10^{-3} A}{120} = 1.35 \times 10^{-3}\ A$$

(d) The resistor $R_2$ has 31.5 V across it, using Ohm's Law:

$$V = i_s R_2$$

$$R_2 = \frac{V_s}{i_s} = \frac{31.5\ V}{1.35 \times 10^{-3}} = 2.33 \times 10^4\ ohm$$

## Physical Constants and conversion factors




$G = 6.67\times10^{-11}\ \text{Nm}^2\text{kg}^{-2}$; $g = 9.81\text{m/s}^2$; $N_A = 6.02\times10^{23}\ \text{mol}^{-1}$

$c = 3.00\times10^{8}\ \text{m/s}$; $m_e = 9.31\times10^{-31}\ \text{kg}$; $m_p = 1.67\times10^{-27}\ \text{kg}$

$k_B = 1.38\times10^{-23}\ \text{J/K}$; $R = 8.314\ \text{J/mol.K}$ $\text{e} = 1.602\times10^{-19}\ \text{C}$;

$k = 8.99\times10^{9}\ \text{N.m}^2/C^2$; $\varepsilon_0 = 8.85\times10^{-12}\ C^2/\text{N.m}^2$;

$1\ \text{eV} = 1.60\times10^{-19}\ \text{J}$ $\mu_0 = 4\pi\times10^{-7}\ \text{T.m/A}$;

## Mathematics, Statistics and Geometry

$$\textit{If}\ ax^2+bx+c=0\ \textit{then}\ x=\frac{-b\pm\sqrt{b^2-4ac}}{2a};\ \frac{d}{dx}(x^n)=nx^{n-1};\ \frac{d}{dx}(\sin ax)=a\cos ax;$$

$$\frac{d}{dx}(a\cos x)=-a\sin ax;\quad \int x^n dx=\frac{x^{n+1}}{n+1}+C;\ \int\sin ax=-\frac{1}{a}\cos ax+C;\ \int\cos ax=\frac{1}{a}\sin ax+C$$

$$\vec{a}\cdot\vec{b}=a_xb_x+a_yb_y+a_zb_z=ab\cos\theta;\ \left|\vec{a}\times\vec{b}\right|=ab\sin\theta;\ \frac{dx}{dt}=\frac{dx}{du}\frac{du}{dt};$$

$$\vec{c}=\vec{a}\times\vec{b}=(a_yb_z-b_ya_z)\mathbf{i}+(a_zb_x-b_za_x)\mathbf{j}+(a_xb_y-b_xa_y)\mathbf{k}$$

$$A_{circle}=\pi r^2;\ \text{C}_{\text{circle}}=2\pi r;\ \text{V}_{\text{sphere}}=\frac{4}{3}\pi r^3;\ \text{A}_{\text{sphere}}=4\pi\,\text{r}^2;\qquad (1+x)^n\approx 1+nx\ \text{ if }\ x<<1$$

$$\sigma_f=\sqrt{\left(\frac{\partial f}{\partial x}\sigma_x\right)^2+\left(\frac{\partial f}{\partial y}\sigma_y\right)^2+\left(\frac{\partial f}{\partial z}\sigma_z^2\right)^2+\ldots}\quad;\ \sigma=\frac{H-L}{\sqrt{N}};\quad \sigma_x=\frac{H-L}{N}$$

## Kinematics, Work, Energy

$$v=\frac{dx}{dt};\ a=\frac{dv}{dt}=\frac{d^2x}{dt^2};\ \ \vec{x}=\vec{x}_0+\vec{v}_0t+\frac{1}{2}\vec{a}t^2;\ \vec{v}=\vec{v}_o+\vec{a}t;\ \vec{v}_{av}=\frac{\vec{v}_0+\vec{v}}{2};\ v^2=v_0^2+2a(x-x_0);$$

$$\vec{F}_{net}=m\vec{a};\ \text{Hooke's Law}:\vec{F}=\text{-}k\vec{x}\quad;\ \text{work W}=\vec{F}\cdot\vec{d}\quad;W=\int_{r_A}^{r_B}\vec{F}(\vec{r})\cdot d\vec{r};\ \text{K}=\frac{1}{2}mv^2;\quad W=\Delta K;$$

$$\overline{P}=\frac{\Delta W}{\Delta t};\quad P=\frac{dW}{dt};\ P=\vec{F}\cdot\vec{v}\quad \Delta U=-W;\ \Delta U_g=mg\Delta y;\ U_E=\frac{1}{2}kx^2\quad;\ E=K+U;$$

$$\Delta U=-\int_{x_i}^{x_f}F(x)dx;\ F=-\frac{dU}{dx};\ W=\Delta E_{mec}+\Delta E_{th};\ \vec{F}=-\left(\frac{GMm}{r^2}\right)\hat{r};\quad U(r)=-\frac{GMm}{r};$$

## Oscillatory Motion and Waves

$$x(t) = x_m \cos(\omega t + \phi)\,;\; v(t) = -\omega x_m \sin(\omega t + \phi)\,;\; a = \frac{dv}{dt} = -\omega^2 x_m \cos(\omega t + \phi) = -\omega^2 x(t)$$

$$x(t) = Ae^{-\alpha t}\sin(\omega' t + \phi)\,;\; \omega = 2\pi f\,;\, T = \frac{1}{f};\; v = f\lambda = \frac{\lambda}{T} = \frac{\omega}{k};\; \omega = \sqrt{\frac{k}{m}}\,;\; \omega = \sqrt{\frac{g}{L}}\,;\; E = \frac{1}{2}kx_m^2;$$

$$y(x,t) = y_m \sin(kx - \omega t);\; k = \frac{2\pi}{\lambda};\; v = \sqrt{\frac{\tau}{\mu}};\; P = \frac{\mu\omega^2 A^2 v}{2};\; \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2};\; \vec{F} = k\frac{q_1 q_2}{r^2}\hat{r};\; i = \frac{dq}{dt};$$

## Electrostatics

$$n_{electrons} = \frac{q_{total}}{e};\; \vec{E} = \frac{\vec{F}}{q};\; \vec{E} = k\frac{q}{r^2}\hat{r};\; \vec{p} = q\vec{d};\; E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3};\; E_{total} = \frac{1}{4\pi\varepsilon_0}\frac{qz}{\left(z^2 + R^2\right)^{\frac{3}{2}}};$$

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{(z^2 + R^2)}}\right];\, E = \frac{\sigma}{2\varepsilon_0};\; \vec{\tau} = \vec{p}\times\vec{E};\, U = -\vec{p}.\vec{E};\; \Phi = \sum \vec{E}.\Delta\vec{A};\; \Phi = \oint \vec{E}.d\vec{A};$$

$$\varepsilon_0 \Phi = q_{enc};\; \varepsilon_0 \oint \vec{E}.d\vec{A} = q_{enc};\; E = \frac{\sigma}{\varepsilon_0};\quad \sigma = \frac{q}{A};\; E = \frac{\lambda}{2\pi\varepsilon_0 r};\quad \lambda = \frac{q}{L}\;\; E = \frac{\sigma}{2\varepsilon_0};\; E_{\parallel} = \frac{2\sigma_1}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

$$V = \frac{U}{q};\; \Delta V = \frac{\Delta U}{q} = \frac{-W}{q};\; W = q_0\int_i^f \vec{E}.d\vec{s};\; \Delta V = -\int_i^f \vec{E}.d\vec{s};\; V(r) = \frac{q}{4\pi\varepsilon_0 r};\; V = \frac{1}{4\pi\varepsilon_0}\sum \frac{q_i}{r_i};$$

$$V_{dipole} = \frac{1}{4\pi\varepsilon_0}\left[\frac{p\cos\theta}{r^2}\right];\; E_s = -\frac{\partial V}{\partial s};\quad U = W = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r};\quad C = \frac{q}{V};\quad C = \frac{\varepsilon_0 A}{d};$$

$$C_{cyl} = 2\pi\varepsilon_0\frac{L}{\ln(b/a)};\; C_{sp} = 4\pi\varepsilon_0\frac{ab}{b-a};\; C_{iso} = 4\pi\varepsilon_0 R\,;\, C_{eq,p} = C_1 + C_2 + \ldots\;\; \frac{1}{C_{eq,s}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$$

$$U = \frac{q^2}{2C} = \frac{1}{2}CV^2;\; u = \frac{1}{2}\varepsilon_0 E^2;\; \varepsilon_0 \oint \kappa\vec{E}.d\vec{A} = q_{enc};\; i = \frac{dq}{dt} = \int \vec{J}\cdot d\vec{A};\quad \vec{J} = ne\vec{v}_{drift};$$

## Resistance and RC circuits

$$R = \frac{V}{i};\, R = \frac{\rho L}{A};\quad \rho - \rho_0 = \rho_0\alpha(T - T_0);\quad P = iV = i^2 R = \frac{V^2}{R};\; R_{eq,s} = R_1 + R_2 + \ldots$$

$$\frac{1}{R_{eq,p}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\;\; q = C\xi(1 - e^{\frac{-t}{RC}});\quad \tau = RC;\, i = \frac{dq}{dt} = \pm\left(\frac{\xi}{R}\right)e^{\frac{-t}{RC}};$$

## Magnetism

$$\vec{F}_B = q\vec{v}\times\vec{B}; \quad |q|vB = \frac{mv^2}{r} \; \vec{F}_B = i\vec{L}\times\vec{B}; \quad d\vec{F}_B = id\vec{L}\times\vec{B}; \; \vec{\tau} = \vec{\mu}\times\vec{B}; \quad \mu = NiA; \quad U(\theta) = -\vec{\mu}\cdot\vec{B};$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{id\vec{s}\times\hat{r}}{r^2}; \; B_{lsw} = \frac{\mu_0 i}{2\pi R}; \quad B_{arc} = \frac{\mu_0 i\phi}{4\pi R}; \quad F_{ba} = \frac{\mu_0 L i_a i_b}{2\pi d};$$

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc}; B_{sol} = \mu_0 in; \quad B_{tor} = \frac{\mu_0 iN}{2\pi}\frac{1}{r}; \; \vec{B}(z) = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3}; \; \Phi_B = \int \vec{B}.d\vec{A};$$

## Inductance

$$\xi = -\frac{d\Phi_B}{dt}; \quad \xi = -N\frac{d\Phi_B}{dt}; \; \xi = \oint \vec{E}.d\vec{s} \; L = \frac{N\Phi_B}{i}; \; \xi_L = -L\frac{di}{dt}; \; i = \frac{\xi}{R}\left(1-e^{\frac{t}{\tau_L}}\right); \; \tau_L = \frac{L}{R}; \; i = i_0 e^{\frac{-t}{\tau_L}};$$

$$U_B = \frac{1}{2}Li^2; u_B = \frac{B^2}{2\mu_0}; \; \xi_2 = -M\frac{di_1}{dt} \text{ and } \xi_1 = -M\frac{di_2}{dt};$$

## LC Circuits, LCR Circuits and Resonance

$$L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0; \; q = Q\cos(\omega t+\phi); \; \omega = \frac{1}{\sqrt{LC}};$$

$$i = -\omega Q\sin(\omega t+\phi); \; L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0; \quad q = Qe^{\frac{-Rt}{2L}}\cos(\omega' t+\phi); \; \omega' = \sqrt{\omega^2 - \left(\frac{R}{2L}\right)^2}$$

$$V_C = IX_C, \phi = -\pi/2;$$

$$V_L = IX_L, \phi = +\pi/2; \quad I = \frac{\xi_m}{Z} = \frac{\xi_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}}; \quad Z = \sqrt{R^2 + (X_L - X_C)^2};$$

$$\tan\phi = \frac{X_L - X_C}{R};$$

## AC Circuits

$$P_{avg} = I_{rms}^2 R = \xi_{rms} I_{rms}\cos\phi; \quad I_{rms} = I/\sqrt{2}; \quad V_{rms} = V/\sqrt{2}; \quad \xi_{rms} = \xi/\sqrt{2};$$

## Transformers

$$V_s = V_p\frac{N_s}{N_P}; \quad I_s = I_p\frac{N_p}{N_s}; \; R_{eq} = \left(\frac{N_p}{N_s}\right)^2 R;$$