

CVG2140 – Solutions to Assignment No. 7 (Shear & Torsion)

Problem 1. The wood beam shown in Fig. 1 has an allowable shear stress of $\tau_{allow} = 7 \text{ MPa}$. Determine the maximum shear force V that can be applied to the cross section.

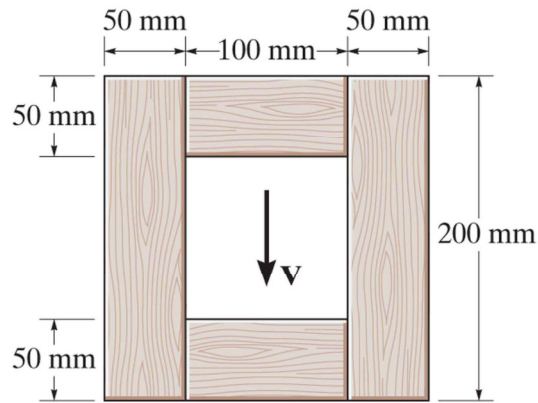


Fig. 1

$$I = \frac{200 \times 200^3}{12} - \frac{100 \times 100^3}{12} = 125 \times 10^6 \text{ mm}^4$$

$$\tau_{allow} = \frac{VQ_{max}}{Ib} \Rightarrow V = \frac{\tau_{allow} I b}{Q_{max}} = \frac{7 \times 125 \times 10^6 \times 100}{(200 \times 50 \times 75) + (2 \times 50 \times 50 \times 25)} = \underline{100 \text{ kN}}$$

Problem 2. Determine the absolute maximum shear stress in the T-beam shown in Fig. 2, as well as the maximum shear stress in the cross section that passes through point C.

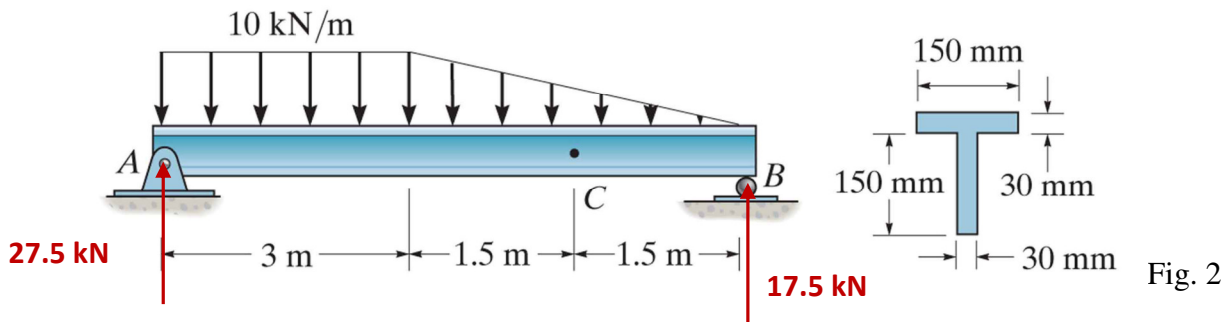
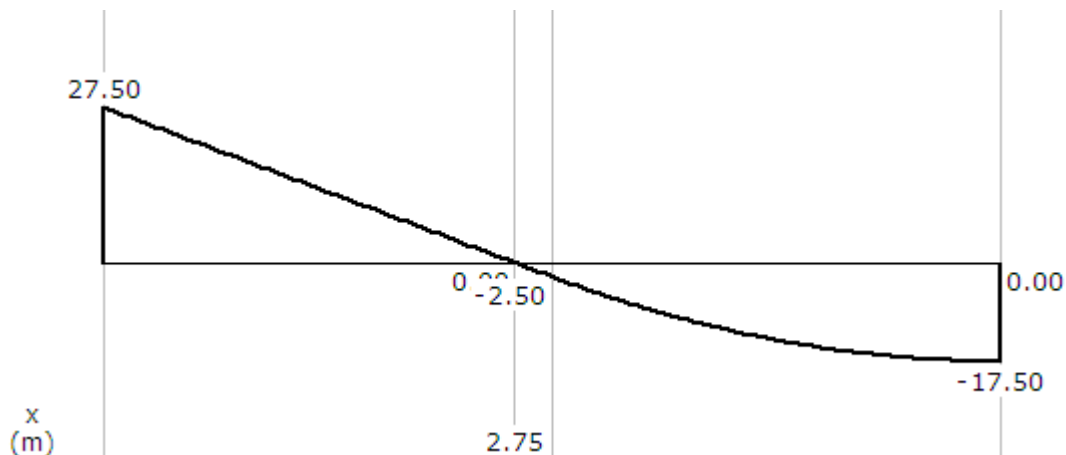
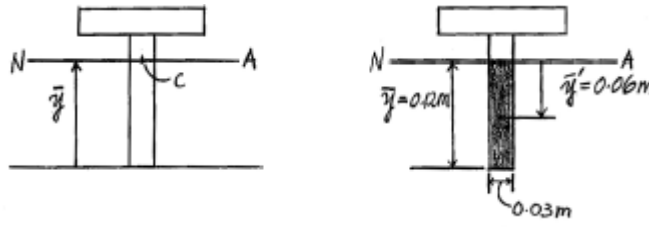


Fig. 2

The shear force diagram, as shown below, indicates that $V_{max} = 27.50 \text{ kN}$ at support A.



The neutral axis passes through the centroid of the cross section as shown in the figure:



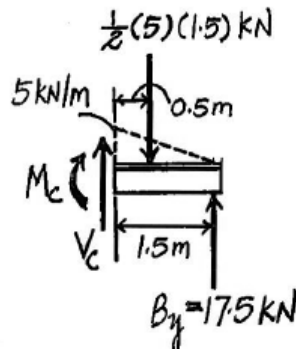
$$\bar{y} = \frac{\sum \bar{y}_i A_i}{\sum A_i} = \frac{(30 \times 150 \times 75) + (150 \times 30 \times 165)}{(30 \times 150) + (150 \times 30)} = 120 \text{ mm (from the bottom)}$$

$$I_{NA} = \frac{150 \times 30^3}{12} + (150 \times 30 \times 45^2) + \frac{30 \times 150^3}{12} + (150 \times 30 \times 45^2) = 27 \times 10^6 \text{ mm}^4$$

The absolute maximum shear stress occurs at the neutral axis where Q is maximum, i.e.,

$$\tau_{\max} = \frac{VQ_{\max}}{Ib} = \frac{27.5 \times 10^3 \times (120 \times 30 \times 60)}{27 \times 10^6 \times 30} = 7.33 \text{ MPa}$$

The shear force at C is determined from the following FBD:



$$\sum F_y = 0; \quad V_c + 17.5 - \frac{5 \times 1.5}{2} = 0 \Rightarrow V_c = -13.75 \text{ kN}$$

The maximum shear stress at section C is therefore given by:

$$\tau_{\max}^C = \frac{VQ_{\max}}{Ib} = \frac{13.75 \times 10^3 \times (120 \times 30 \times 60)}{27 \times 10^6 \times 30} = 3.67 \text{ MPa}$$

Problem 3. The 60-mm-diameter solid shaft in Fig. 3 is subjected to the distributed and concentrated torsional loadings shown. Determine the absolute maximum and minimum shear stresses on the outer surface of the shaft and specify their locations, measured from the fixed end A.

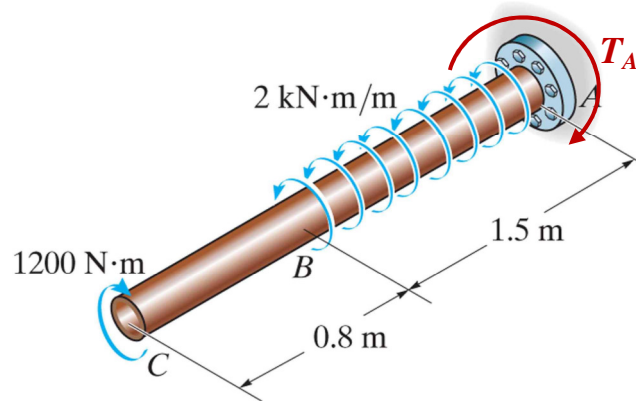
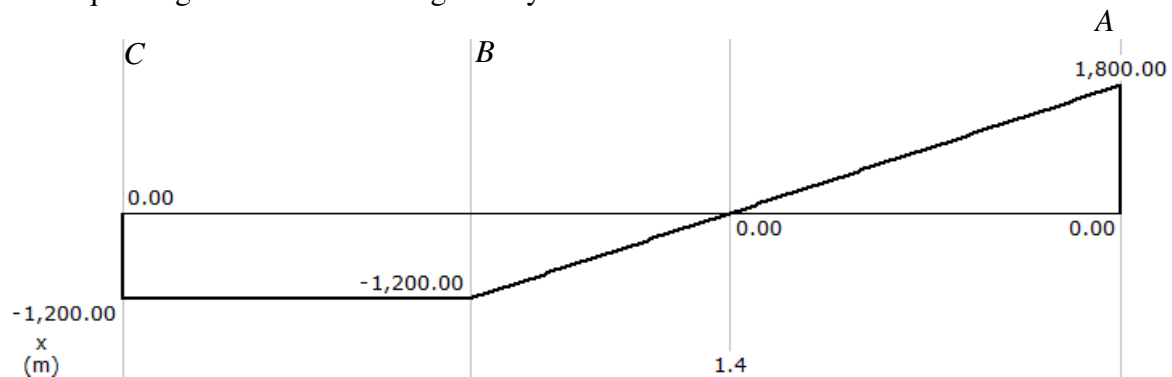


Fig. 3

By applying equilibrium:

$$\sum T = 1200 - (2000 \times 1.5) + T_A = 0 \Rightarrow T_A = 1800 \text{ N} \cdot \text{m}$$

The torque diagram of the shaft is given by:



The minimum shear stress occurs when the internal torque is zero in segment AB. By setting $T_{AB} = 0$,

$$0 = (2000x - 1200) \text{ N} \cdot \text{m} \Rightarrow x = 0.6 \text{ m}$$

Therefore, $\tau_{\min} = 0$ at a distance from A of $1.5 \text{ m} - 0.6 \text{ m} = 0.9 \text{ m}$.

The maximum shear stress occurs at A where the torque is the greatest, i.e.,

$$\tau_{\max} = \tau_A = \frac{T_A r}{J} = \frac{1800 \times 10^3 \times 30}{\frac{\pi}{2} \times 30^4} = \underline{42.4 \text{ MPa}}$$

Problem 4. The A-36 steel assembly in Fig. 4 consists of a tube having an outer radius of 1 in and a wall thickness of 0.125 in. Using a rigid plate at B , it is connected to the solid 1-in-diameter shaft AB . Determine the rotation of the tube's end C if a torque of 200 lb·in is applied to the tube at this end. The end A of the shaft is fixed. $G = 11 \times 10^6$ psi.

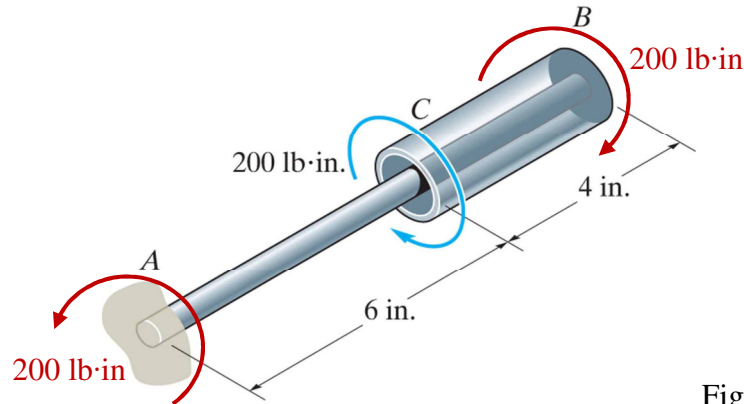


Fig. 4

$$\phi_B = \frac{T_{AB}L}{GJ} = \frac{200 \times 10}{11 \times 10^6 \times \frac{\pi}{2} \times 0.5^4} = 0.001852 \text{ rad}$$

$$\phi_{C/B} = \frac{T_{CB}L}{GJ} = \frac{-200 \times 4}{11 \times 10^6 \times \frac{\pi}{2} \times (1^4 - 0.875^4)} = -0.0001119 \text{ rad}$$

$$\phi_C = \phi_B - \phi_{C/B} = 0.001852 + 0.0001119 = \underline{0.001964 \text{ rad} = 0.113^\circ}$$