TEST # 1 (SOLUTION)

Long cylinder $\Rightarrow$ plane strain $(\varepsilon_z = 0)$

$$\left.\begin{array}{l} r_1 = 35\,mm \\ r_2 = 75\,mm \end{array}\right\} \Rightarrow k = \frac{r_2}{r_1} = \frac{75}{35} = 2.143$$

$\Delta T = T_1 - T_2 = 70°C$

<u>CONSIDER THERMAL LOADING.</u>

$\sigma_{r_1} = \sigma_{r_2} = 0$

$$\sigma_\theta = C_0 \left[1 - \ln\left(\frac{r_2}{r}\right) - \frac{\ln k}{k^2 - 1}\left(1 + \frac{r_2^2}{r^2}\right) \right]$$

$$C_0 = \frac{E'\alpha' \Delta T}{2 \ln k} = \frac{E\alpha \Delta T}{2(1-\nu)\ln k} = \frac{(97\times10^9)(20\times10^{-6})(70)}{2(1-0.35)\ln(2.143)}$$

$C_0 = 137.05$ MPa.

σ_z (closed ends condition)

$$\sigma_z = \underbrace{\frac{E\alpha\,\Delta T}{2(1-\nu)\ln k}}_{C_0} \left[1 - 2\ln\left(\frac{r_2}{r}\right) - \frac{2}{k^2-1}\ln k \right]$$

σ_θ at $r = r_1$ and r_2.

$$\sigma_{\theta_{r_1}} = C_0 \left[1 - \ln k - \frac{\ln k}{k^2-1}(1+k^2) \right]$$

$$\sigma_{\theta_{r_1}} = 137.05\times10^6 \left[1 - \ln(2.143) - \frac{\ln(2.143)}{2.143^2 - 1}(1 + 2.143^2) \right]$$

$\sigma_{\theta_{r_1}} = -130.03$ MPa (Compression).

$$\sigma_{\theta r_2} = C_0 \left[1 - 2\,\cancel{\ln(1)}^{\,0} - \frac{\ln k}{2.143^2 - 1} (1+1) \right]$$

$$= 137.05 \times 10^6 \left[1 - \frac{2 \ln(2.143)}{2.143^2 - 1} \right]$$

$$= 78.05 \text{ MPa (Tension)}.$$

σ_z at r_1 and r_2.

$$\sigma_{z r_1} = C_0 \left[1 - 2 \ln k - \frac{2}{k^2 - 1} \ln k \right]$$

$$= 137.05 \times 10^6 \left[1 - 2 \ln(2.143) - \frac{2}{2.143^2 - 1} \ln(2.143) \right]$$

$$= -130.04 \text{ MPa. (Compression)}$$

$$\sigma_{z r_2} = C_0 \left[1 - \frac{2}{k^2 - 1} \ln k \right]$$

$$= 137.05 \times 10^6 \left[1 - \frac{2}{2.143^2 - 1} \ln(2.143) \right]$$

$$= 78.9 \text{ MPa. (Tension)}$$

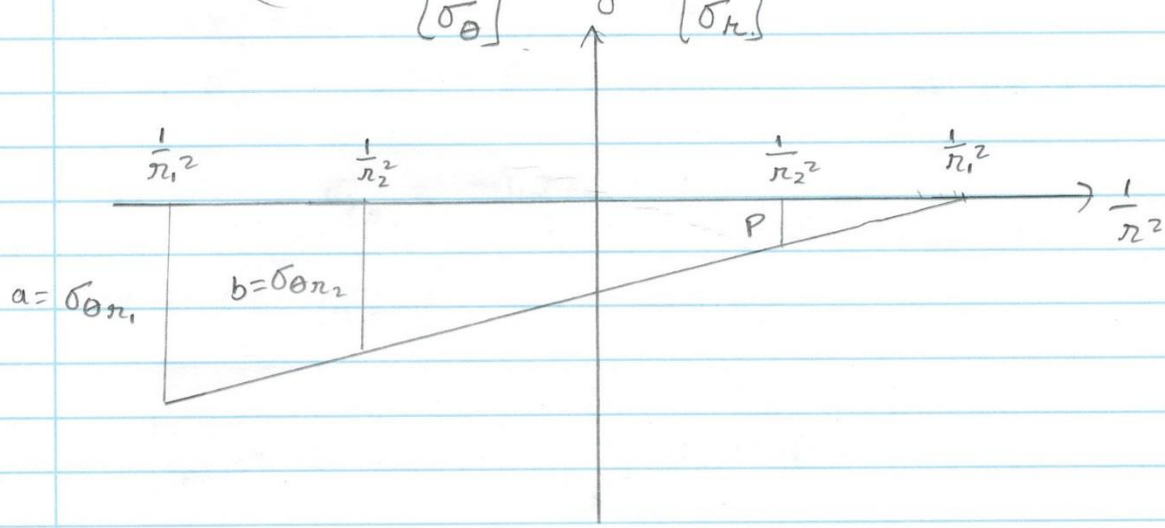
<u>SUMMARY OF THERMAL LOADS</u>

	$r = r_1$	$r = r_2$
σ_θ (MPa)	-130.04	78.9.
σ_z (MPa)	-130.04	78.9
σ_r (MPa)	0	0

PRESSURE LOAD (External pressure loading)
{Internal pressure = 0}.

$$\frac{1}{r_1^2} = \frac{1}{(35mm)^2} = 816.33$$

$$\frac{1}{r_2^2} = \frac{1}{(75mm)^2} = 177.78$$



Similiar Triangle.

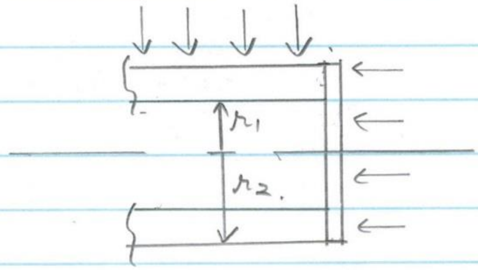
$$\frac{a}{\frac{1}{r_1^2} \times 2.} = \frac{b}{\frac{1}{r_1^2} + \frac{1}{r_2^2}} = \frac{-P}{\frac{1}{r_1^2} - \frac{1}{r_2^2}}$$

$$\frac{a}{2(816.33)} = \frac{b}{816.33 + 177.78} = \frac{-P}{816.33 - 177.78}$$

$$\frac{a}{1632.6} = \frac{b}{994.11} = \frac{-P}{638.55.}$$

$$a = -2.557P \quad - (1)$$
$$b = -1.557P \quad - (2).$$

$(\sigma_z) = \{\text{Force balance}\}$



$F_{pressure} = F_{internal}.$

$-P(\cancel{\pi} r_2^2) = \sigma_z \cancel{\pi}(r_2^2 - r_1^2)$

or, $-P(75^2) = \sigma_z(75^2 - 35^2)$

$\therefore \sigma_z = -1.278\,P \Rightarrow$ constant across thickness.

$\therefore \sigma_{z_{r_1}} = \sigma_{z_{r_2}} = -1.278\,P.$

SUMMARY OF PRESSURE LOADS.

	$r = r_1$	$r = r_2$
σ_θ (MPa)	$-2.557P$	$-1.557P$.
σ_z (MPa)	$-1.278P$	$-1.278P$
σ_r (MPa)	0	$-P$.

Total. = Pressure + Thermal.

At $r = r_1$

$\sigma_{\theta_{r_1}} = -2.557P - 130.04$

$\sigma_{z_{r_1}} = -1.278P - 130.04$

$\sigma_{r_{r_1}} = 0.$

At $r = r_2$.

$$\sigma_{\theta r_2} = -1.557P + 78.9$$

$$\sigma_{z r_2} = -1.278P + 78.9$$

$$\sigma_{r r_2} = -P.$$

a) Tresca's Criteria.

at r_1 (find the lowest and the highest)

$$\left| \frac{\sigma_{\theta r_1} - \sigma_{r_1}}{2} \right| = \frac{\sigma_y}{2}.$$

$$\left| (-130 - 2.557P) \right| = 240.$$

$$\Rightarrow P = \frac{240 - 130}{2.557} = 43.0 \text{ MPa}.$$

at r_2 $\left| \frac{\sigma_{z r_2} - (-P)}{2} \right| = \frac{\sigma_y}{2}.$

$$\left| 78.9 - 1.278P + P \right| = 240$$

$$\Rightarrow P = 579.5 \text{ MPa}.$$

Smallest one will create yeilding first.

$\therefore$ Max P according to Tresca is 43.0 MPa

b) Von-Mises Criteria.

@ $r = r_1$

$$\underbrace{\left(\sigma_{\theta r_1} - \sigma_{r r_1}\right)^2}_{a} + \underbrace{\left(\sigma_{r_1} - \sigma_{z r_1}\right)^2}_{b} + \underbrace{\left(\sigma_{\theta r_1} - \sigma_{z r_1}\right)^2}_{c} = 2\sigma_y^2.$$

$$a = (-2.557P - 130.04)^2$$
$$= 6.538P^2 + 665P + 16,910.4$$

$$b = (-(-1.278P - 130.04))^2$$
$$= 1.633P^2 + 332.4P + 16910.4.$$

$$c = (-2.557P - 130.04 - (-1.278P - 130.04))^2$$
$$= 1.636P^2$$

$$a + b + c = 2\sigma_y^2$$

or, $9.807P^2 + 997.4P + 33820 = 115200$

$\therefore P = (+53.47 \text{ or } -155.2)$ MPa.

Since pressure is +ve. use $P = 53.47$ MPa

$\Rightarrow$ Sign is already taken into account.

@ $r = r_2$.

$$\underbrace{\left(\sigma_{\theta_{r_2}} - \sigma_{z_{r_2}}\right)^2}_{a} + \underbrace{\left(\sigma_{r_2} - \sigma_{z_{r_2}}\right)^2}_{b} + \underbrace{\left(\sigma_{\theta_{r_2}} - \sigma_{r_{2}}\right)^2}_{c} = 2\sigma_y^2.$$

$$a = \left(-1.557P + \cancel{78.9} - (-1.278P + \cancel{78.9})\right)^2$$
$$= (-0.279P)^2$$
$$= 0.0778P^2$$

$$b = \left(-P - (-1.278P + 78.9)\right)^2$$
$$= (0.278P - 78.9)^2$$
$$= 0.0773P^2 - 43.87P + 6225.21$$

$$c = \left(-1.557P + 78.9 - (-P)\right)^2$$
$$= (-0.557P + 78.9)^2$$
$$= 0.310P^2 - 87.895P + 6225.21$$

$$\Rightarrow a + b + c = 2\sigma_y^2$$
$$0.4651P^2 - 131.77P + 12450.42 = 115200$$

$$\therefore P = (632.56, -349.24) \text{ MPa}.$$

Since pressure is +ve | use 632.56 MPa.
$\Rightarrow$ Sign is already taken into account.

$\therefore$ Max P according to Von Mises is 53.47 MPa