Question 1 (30 points)

The prime implicants of function *f(w, x, y, z)* = ∑m(1, 3, 4, 6, 7, 9, 11, 12, 13, 15) are given below:

Prime Implicant	**A**	**B**	**C**	**D**	**E**	**F**	**G**
	b'd	***cd***	***ad***	***abc'***	***bc'd'***	***a'bd'***	***a'bc***
Covered minterms	**(1,3,9,11)**	**(3,7,11,15)**	**(9,11,13,15)**	**(12,13)**	**(4,12)**	**(4,6)**	**(6,7)**

Use the Prime Implicant Table to find the EPI(s) (16 pts)

		1pt		1pt		1pt		1pt		1pt		1pt		1pt	
		w x y z		$		**1** *	**3** √	**4**	**6**	**7**	**9** √	**11** √	**12**	**13**	**15**
1pt	***b'd***	**- 0 - 1**	**(1,3,9,11)**	**3**	***A***	**x**	**x**				**x**	**x**			
1pt	*cd*	- - 1 1	(3,7,11,15)	3	*B*		x			x		x			x
1pt	*ad*	1 - - 1	(9,11,13,15)	3	*C*						x	x		x	x
1pt	*abc'*	1 1 0 -	(12,13)	4	*D*								x	x	
1pt	*bc'd'*	- 1 0 0	(4,12)	4	*E*			x					x		
1pt	*a'bd'*	0 1 - 0	(4,6)	4	*F*			x	x						
1pt	*a'bc*	0 1 1 -	(6,7)	4	*G*				x	x					

Verification: K-map not required

f

ab \ cd	00	01	11	10
00	0 [0]	1* [1]	1 [3]	0 [2]
01	1 [4]	0 [5]	1 [7]	1 [6]
11	1 [12]	1 [13]	1 [15]	0 [14]
10	0 [8]	1 [9]	1 [11]	0 [10]

EPI: ***b'd'*** 1pt

Apply Petrick's method (14 pts) ***on the terms not covered by the EPI(s) to find all minimum expressions of function f;*** *Reduce the PI table by removing the rows and the columns that were covered by the EPI:*

			$		**4**	**6**	**7**	**12**	**13**	**15**
cd	- - 1 1	(3,7,11,15)	3	*B*			x			x
ad	1 - - 1	(9,11,13,15)	3	*C*					x	x
abc'	1 1 0 -	(12,13)	4	*D*				x	x	
bc'd'	- 1 0 0	(4,12)	4	*E*	x			x		
a'bd'	0 1 - 0	(4,6)	4	*F*	x	x				
a'bc	0 1 1 -	(6,7)	4	*G*		x	x			

Apply Petrick's method on the PI reduced table:

1pt	1pt	1pt	1pt	1pt	1pt
4	**6**	**7**	**12**	**13**	**15**

(E+F) (F+G) (B+G) (D+E) (C+D) (B+C) = *(F + EG)(B + CG)(D + CE) = (BF + BEG + CFG + CEG)(D + CE)*

= ***BDF*** + *BDEG + CDFG + CDEG + BCEF+ BCEG + CEFG +* ***CEG*** 8 x 0.5 pts = 4 pts

Minimum expressions: 2 x 2pts = 4 pts

BDF: f_{min1}(*a, b,* c, d) = ***b'd*** + *cd* + *abc'* + *a'bd'*

Verification only!!! Not required

f_1

ab \ cd	00	01	11	10
00	0 [0]	1 [1]	1 [3]	0 [2]
01	1 [4]	0 [5]	1 [7]	1 [6]
11	1 [12]	1 [13]	1 [15]	0 [14]
10	0 [8]	1 [9]	1 [11]	0 [10]

CEG: f_{min2}(*a, b,* c, d) = ***b'd*** + *ad* + *bc'd'* + *a'bc*

f_2

ab \ cd	00	01	11	10
00	0 [0]	1 [1]	1 [3]	0 [2]
01	1 [4]	0 [5]	1 [7]	1 [6]
11	1 [12]	1 [13]	1 [15]	0 [14]
10	0 [8]	1 [9]	1 [11]	0 [10]

Question 2 **(30 points) For the following set of functions, $Z_1(a,b,c,d) = \Sigma m(2, 3, 4, 6, 7) + \Sigma d(0, 1, 14, 15)$, $Z_2(a,b,c,d) = \Sigma m(2, 3, 5, 7, 8, 10, 11, 13) + \Sigma d(0, 1, 14, 15)$ we found the possible shared terms: *a'b'*, *a'cd*, *bcd*, *abc*. Other prime implicants of Z_1 are *a'd'*, *a'c*, *bc* and other prime implicants of Z_2 are *a'd*, *b'd'*, *bd*, *ac*.**

1. (11pts) Represent these terms on K-maps and determine what *minterms* are covered by each of them.

Shared terms: *a'b'*, *a'cd*, *bcd*, *abc*

$Z_1 Z_2$

ab \ cd	00	01	11	10
00	x	x	1	1
01			1	
11			x	x
10				

4pts

Minterms covered by shared terms:

Shared terms	Minterms covered by shared term
a'b'	0,1,2,3
a'cd	3,7
bcd	7,15
abc	14,15= dc

Other Z_1 PIs: *a'd'*, *a'c*, *bc*

Z_1

ab \ cd	00	01	11	10
00	x	x	1	1
01	1		1	1
11			x	x
10				

3pts

Minterms covered by other Z_1 PIs:

Other Z_1 PIs	Minterms covered by other Z_1 PIs
a'd'	0,2,4,6
a'c	2,3,6,7
bc	6,7,14,15

Other Z_2 PIs: *a'd*, *b'd'*, *bd*, *ac*

Z_2

ab \ cd	00	01	11	10
00	x	x	1	1
01		1	1	
11		1	x	x
10	1		1	1

4pts

Minterms covered by other Z_2 PIs:

Other Z_2 PIs	Minterms covered by other Z_1 PIs
a'd	1,3,5,7
b'd'	0,2,8,10
bd	5,7,13,15
ac	10,11

2. (12pts) Find EPI of the functions *f* and *g*, by using the Prime Implicant Table 4pts.

	PI	m	CU	$m(Z_1)$ 2	3	4	6	7	$m(Z_2)$ 2	3	5	7	8	10	11	13	Eliminate EPI columns and rows:
				√		*	√		√		√	√	*	√	*	*	
$Z_1 Z_2$	*a'b'*	0,1,2,3	3	1	1				1	1							
	a'cd	3,7	4		1			1		1		1					
	bcd	7,15	4					1				1					
Z_1	~~*a'd'*~~	~~0,2,4,6~~	~~3~~	~~1~~		~~1~~	~~1~~										1pt
	a'c	2,3,6,7	3	1	1		1	1									
	bc	6,7,14,15	3				1	1									
Z_2	*a'd*	1,3,5,7	3							1	1	1					
	~~*b'd'*~~	~~0,2,8,10~~	~~3~~						~~1~~				~~1~~	~~1~~			1pt
	~~*bd*~~	~~5,7,13,15~~	~~3~~								~~1~~	~~1~~				~~1~~	1pt
	~~*ac*~~	~~10,11~~	~~3~~											~~1~~	~~1~~		1pt

EPI Z_1 = *a'd'* + … 1pt EPI Z_2 = *b'd'* + *bd* + *ac*… 3pts

3. (7pts) Using reduction methods of the Prime Implicant Table find a *set* of minimum sum of products expressions that covers the given *set* of functions. Reduced Prime Implicant Table 2pts

	PI	m	CU	Z_1 3	7	Z_2 3
$Z_1 Z_2$	*a'b'*	0,1,2,3	1	1		1
	a'cd	3,7	4	1	1	1
	~~*bcd*~~	~~7,15~~	4		~~1~~	
Z_1	*a'c*	2,3,6,7	3		1	
	~~*bc*~~	~~6,7,14,15~~	~~3~~		~~1~~	
Z_2	~~*a'd*~~	~~1,3,5,7~~	~~3~~			~~1~~

Eliminate dominated rows

a'cd > *bcd* 1pt

a'c = *bc* 1pt

a'b' > *a'd* 1pt

	PI	m	CU	Z_1 3	7	Z_2 3
$Z_1 Z_2$	*a'b'*	0,1,2,3	1	1		1
	a'cd	3,7	4	1	1	1
Z_1	*a'c*	2,3,6,7	3		1	

NO SEPI

Z_1 = *a'd'* + *a'cd* 1pt

Z_2 = *b'd'* + *bd* + *a'cd* + *ac* 1pt

Question 3 **(40 points)**

Use the iterated consensus method to find all prime implicants of the incompletely defined function:

$$V(a,b,c,d) = \bar{a}b\bar{c}\bar{d} + \bar{a}b\bar{c}d + abd + \bar{a}\bar{b}c$$

given that $a\bar{b}d + a\bar{b}\bar{d} = 0$, **i,e,** $dc\{a\bar{b}d, a\bar{b}\bar{d}\}$

Consens	Term	Implicant	abcd		Covered by	Covers:	
	~~T1~~	~~a'bc'd'~~	~~0100~~	√	< T7		2 pts
	~~T2~~	~~a'bc'd~~	~~0101~~	√	< T7		2 pts
	~~T3~~	~~abd~~	~~11-1~~	√	< T9		2 pts
	~~T4~~	~~a'b'c~~	~~001-~~	√	< T14		2 pts
	~~T5~~	~~ab'd~~	~~10-1~~	x√	< T9		2 pts
	~~T6~~	~~ab'd'~~	~~10-0~~	x√	< T13		2 pts
T2 ¢ T1	T7	a'bc'	010-			> T1, T2	2 pts
T4 ¢ T3	-						
T5 ¢ T4	~~T8~~	~~b'cd~~	~~-011~~	√	< T14		2 pts
T5 ¢ T3	T9	ad	1--1			> T3, T5	2 pts
T6 ¢ T4	~~T10~~	~~b'cd'~~	~~-010~~	√	< T14		2 pts
T7 ¢ T6	-						
T7 ¢ T4	-						
T8 ¢ T7	-	a'bd	01-1				
T8 ¢ T6	~~T11~~	~~ab'c~~	~~101-~~	√	< T13		2 pts
T8 ¢ T4	-						
T9 ¢ T8	-						
T9 ¢ T7	T12	bc'd	-101				2 pts
T9 ¢ T6	T13	ab'	10--	x		>T6, T11	2 pts
T9 ¢ T4		~~b'cd~~	~~-011~~	√	=T8		
T10 ¢ T9		~~ab'c~~	~~101-~~	√	<T13		
T10¢T8	T14	b'c	-01-			>T4,T8,T10	2 pts
T10 ¢ T7	-						
T12 ¢ T9	-						
T12 ¢ T7	-						
T13 ¢ T12	-	~~ac'd~~	~~1-01~~	√	<T9		
T13 ¢ T9	-						
T13 ¢ T7	-						
T14 ¢ T13	-						
T14 ¢ T12	-						
T14 ¢ T9	-						
T14 ¢ T7	-						

T13 (ab') = don't care term

2 pts

PIs: T7, T9, T12, T14

= {a'bc', ad, bc'd, b'c}

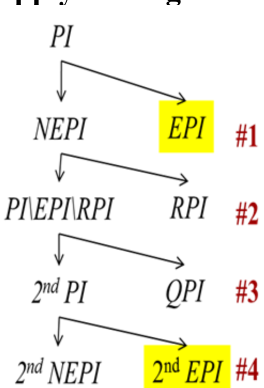
Verification (not required):

V(a,b,c,d)*= *a'bc'd'* + *bc'd* + *abd* + *b'c

+ dc{*ab'd'*, *ab'd*}

ab \ cd	00	01	11	10
00	0	0	1*	1*
01	1*	1		
11	0	1	1*	
10	x	x	x	x

Apply the Algebraic Expressions Evaluation to select PI's for function's minimum cover.



#1 Find *EPI's*

$E_{PI \setminus a'bc'}\,(a'bc') = (b'c + ad + bc'd)|_{a'bc'=1} = 0 + 0 + d \neq 1 \rightarrow a'bc' = EPI$ 2 pts

$E_{PI \setminus b'c}\,(b'c) = (a'bc' + ad + bc'd)|_{b=0,\,c=1} = 0 + ad + 0 \neq 1 \rightarrow b'c = EPI$ 2 pts

$E_{PI \setminus ad}\,(ad) = (a'bc' + b'c + bc'd)|_{a=1,\,d=1} = 0 + b'c + bc' \neq 1 \rightarrow ad = EPI$ 2 pts

$E_{PI \setminus bc'd}\,(bc'd) = (a'bc' + b'c + ad)|_{b=1,\,c=0,\,d=1} = a' + 0 + a = 1 \rightarrow bc'd = NEPI$ 2 pts

#2 Find *Redundant PI's* from *NEPI*: { *bc'd* }

$E_{EPI}(bc'd) = a'bc' + b'c + ad = E_{EPI}|_{b=1,\,c=0,\,d=1} = a' + 0 + a = 1$ $\quad b\bar{c}d \in RPI$ 2 pts

V_{min}**(a,b,c,d)** = *a'bc' + b'c + ad* since ***V*(*a,b,c,d*)** is fully covered by *EPI's*