



Digital Logic Design: Principles and Practices ELG5195 (EACJ5705), Carleton CRN: 18371

**Assignment #1**

**Question 1:**

Use Karnaugh maps to find the prime implicants (PI), essential prime implicants (EPI) and redundant prime implicants (RPI) of the following functions.

Provide the Boolean expressions of the minimized functions.

1.  $g(a,b,c,d) = \sum m(2,3,4,5,13,15) + \sum d(6,8,9,10,11)$

PI:  $ad, a'bc', bc'd, b'c$   
EPI:  $ad, a'bc', b'c$   
RPI:  $bc'd$

$f(a,b,c,d) = ad + a'bc' + b'c$

cd \ ab	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10
	d	d	d	d

$f_2(a,b,c,d) = ab + c'd' + b'd' + a'b'c$

		c			
		0	1	3	2
		4	5	7	6
		12	13	15	14
		8	9	11	10
a		x	0	1	
		d			

2.  $f(a,b,c,d) = a'c'd' + a'b'c + abd + ab'd' + d(a'bcd + ab'c'd)$

PI:  $ab, c'd', b'd', ad', a'b'c, a'cd, ab'c', ac', bcd$   
EPI:  $c'd'$   
RPI:  $ac'$

$f_1(a,b,c,d) = ab + c'd' + b'd' + a'cd$

		c			
		0	1	3	2
		4	5	7	6
		12	13	15	14
		8	9	11	10
a		x	1	1	
		d			

$f_3(a,b,c,d) = ab + c'd' + ad' + a'b'c$

		c			
		0	1	3	2
		4	5	7	6
		12	13	15	14
		8	9	11	10
a		1	1	1	
		d			

**Question 2:**

- a) Find the prime implicants of the following multiple-output function by applying the modified Quine-McCluskey method:  $Z_1(a,b,c,d) = \Sigma(0, 3, 4, 5, 7, 8, 12, 13, 15)$ ;  $Z_2(a,b,c,d) = \Sigma(1, 5, 7, 8, 9, 10, 11, 13, 14, 15)$ ;  $Z_3(a,b,c,d) = \Sigma(1, 2, 4, 5, 7, 10, 13, 14, 15)$

Class 1	a b c d	Z <sub>1</sub> Z <sub>2</sub> Z <sub>3</sub>	Class 2	a b c d	Z <sub>1</sub> Z <sub>2</sub> Z <sub>3</sub>	Class 3	a b c d	Z <sub>1</sub> Z <sub>2</sub> Z <sub>3</sub>
0	0 0000	-00 ✓	0	<del>(0,1)</del>	<del>000-000</del>	0	(0,4,8,12)	--00 -00 A
1	1 0001	0-- ✓		<del>(0,2)</del>	<del>00-0-000</del>		<del>(0,1,4,5)</del>	0-0-000
2	2 0010	00- ✓		(0,4)	0-00 -00 ✓		<del>(0,2,8,10)</del>	-0-0-000
4	4 0100	-0- ✓		(0,8)	-000 -00 ✓		<del>(0,1,8,9)</del>	-00-000
8	8 1000	--0 I	1	<del>(1,3)</del>	<del>00-1-000</del>	1	(1,5,9,13)	--01 0-0 D
2	3 0011	-00 ✓		(1,5)	0-01 0-- K		(4,5,12,13)	-10- -00 B
	5 0101	--- ✓		(1,9)	-001 0-0 ✓		(8,9,10,11)	10-- 0-0 E
	9 1001	0-0 ✓		<del>(2,3)</del>	<del>001-000</del>		<del>(8,9,12,13)</del>	1-0-000
	10 1010	0-- ✓		(2,10)	-010 00- H		<del>(1,3,5,7)</del>	0-1-000
	12 1100	-00 ✓		(4,5)	010- -0- J	2	<del>(3,7,11,15)</del>	--11-000
3	7 0111	--- ✓		(4,12)	-100 -00 ✓		(5,7,13,15)	-1-1 --- N
	11 1011	0-0 ✓		(8,10)	10-0 0-0 ✓		(9,11,13,15)	1--1 0-0 F
	13 1101	--- ✓		(8,12)	1-00 -00 ✓		(10,11,14,15)	1-1- 0-0 G
	14 1110	0-- ✓	2	(3,7)	0-11 -00 C		Class 4	
4	15 1111	--- ✓		<del>(3,11)</del>	<del>-011-000</del>		<del>(0,1,4,5,8,9,12,13)</del> --0-000	
				(5,7)	01-1 --- ✓			
				(5,13)	-101 --- ✓			
				(9,11)	10-1 0-0 ✓			
				(9,13)	1-01 0-0 ✓			
				(10,11)	101- 0-0 ✓			
				(10,14)	1-10 0-- L			
				<del>(12,14)</del>	<del>11-0-000</del>			
			3	(7,15)	-111 --- ✓			
				(11,15)	1-11 0-0 ✓			
				(13,15)	11-1 --- ✓			
				(14,15)	111- 0-- M			

- b) Using reduction methods of the *prime implicant table* find all sets of minimum sum of products expressions that covers the given set of functions.

**Iteration #1**

1) Find EPI				0	3	4	5	7	8	12	13	15	1	5	7	8	9	10	11	13	14	15	1	2	4	5	7	10	13	14	15		
$c'd'$	(0,4,8,12)	-00-00	A	3	1*	1			1	1													1										
$bc'$	(4,5,12,13)	-10--00	B	3		1	1			1	1																						Z <sub>1</sub>
$a'cd$	(3,7)	0-11-00	C	4	1*			1																									
$c'd$	(1,5,9,13)	--01-0-0	D	3									1	1			1						1										Z <sub>2</sub>
$ab'$	(8,9,10,11)	10--0-0	E	3												1	1	1	1														
$ad$	(9,11,13,15)	1--10-0	F	3													1		1	1	1	1											
$ac$	(10,11,14,15)	1-1-0-0	G	3														1	1			1	1										
$b'cd'$	(2,10)	-0100-	H	4																				1*				1					Z <sub>3</sub>
$abc'd'$	8	1000-0	I	5					1							1																	Z <sub>1</sub> Z <sub>2</sub>
$a'bc'$	(4,5)	010-0-	J	4		1	1																		1*	1							Z <sub>1</sub> Z <sub>3</sub>
$a'c'd$	(1,5)	0-01-0-	K	4									1	1									1*			1							
$acd'$	(10,14)	1-10-0-	L	4														1									1		1				Z <sub>2</sub> Z <sub>3</sub>
$abc$	(14,15)	111-0-	M	4																									1	1			
$bd$	(5,7,13,15)	-1-1---	N	3			1	1			1	1*		1	1*						1	1					1	1*		1*	1		Z <sub>1</sub> Z <sub>3</sub> Z <sub>3</sub>

EPI:  $Z_1 = bd + c'd' + a'cd$  - done!  $Z_2 = bd + \dots$   $Z_3 = bd + b'cd' + a'bc' + a'c'd$

2) No dominating columns

3) Eliminate dominated rows				Z <sub>2</sub>						Z <sub>3</sub>
			cu	1	8	9	10	11	14	14
c'd	(1,5,9,13)	--01 0-0	D	3	1	1				
ab'	(8,9,10,11)	10-- 0-0	E	3	1	1	1	1		
ad	(9,11,13,15)	1-1 0-0	F	3		1	1	1		
ac	(10,11,14,15)	1-1- 0-0	G	3			1	1	1	
abc'd'	8	1000- 0	H	5	1					Z <sub>1</sub> Z <sub>2</sub>
a'c'd	(1,5)	0-01 0--	K	1	1					
acd'	(10,14)	1-10 0--	L	4			1		1	Z <sub>2</sub> Z <sub>3</sub>
abc	(14,15)	111 0--	M	4					1	1

Iteration #2

1) Find SEPI			cu	1	8	9	10	11	14	14
					•	•	•	•		•
c'd	(1,5,9,13)	--01 0-0	D	3	1	1				
ab'	(8,9,10,11)	10-- 0-0	E	3	1*	1	1	1		
ac	(10,11,14,15)	1-1- 0-0	G	3			1	1	1	
a'c'd	(1,5)	0-01 0--	K	1	1					
acd'	(10,14)	1-10 0--	L	4			1		1	1*

{SEPI} : Z<sub>2</sub> = bd + {ab'} +  
 Z<sub>3</sub> = bd + b'cd' + a'bc' + a'c'd + {acd'} -> done!!!

2) No dominating columns			cu	1	14
3) Dominated rows					
c'd	(1,5,9,13)	--01 0-0	D	3	1
ac	(10,11,14,15)	1-1- 0-0	G	3	1
a'c'd	(1,5)	0-01 0--	K	1	1
acd'	(10,14)	1-10 0--	L	1	1

Z<sub>2</sub> = bd + ab' + a'c'd + acd' -> done!!!

Z<sub>1</sub> = bd + c'd' + a'cd

Z<sub>2</sub> = bd + ab' + a'c'd + acd'

Z<sub>3</sub> = bd + b'cd' + a'bc' + a'c'd + {acd'}

Verification with K-maps:

Z<sub>1</sub> = bd + c'd' + a'cd

cd	00	01	11	10
ab				
00	1	0	1	0
01	1	1	1	0
11	1	1	1	0
10	1	0	0	0

Z<sub>1</sub> = bd + c'd' + a'cd

Z<sub>2</sub> = bd + ab' + a'c'd + acd'

cd	00	01	11	10
ab				
00	0	1	0	0
01	0	1	1	0
11	0	1	1	1
10	1	1	1	1

Z<sub>3</sub> = bd + b'cd' + a'bc' + a'c'd + acd'

cd	00	01	11	10
ab				
00	0	1	0	1
01	1	1	1	0
11	0	1	1	1
10	0	0	0	1

**Question 3:**

- a) Find the prime implicants of the following function by applying the Quine-McCluskey method:  
 $G(v,w,x,y,z) = \Sigma m(0, 2, 4, 5, 6, 7, 8, 9, 10, 11, 13, 15, 21, 23, 26, 28, 29, 30, 31)$

Class 0				Class 1				Class 2			
Group	Min term	vwxyz		Group	Dec	vwxyz		Group	Dec	vwxyz	
0	0	00000	✓	0	(0, 2)	000-0	✓	0	(0, 2, 4, 6)	00--0	C
1	2	00010	✓		(0, 4)	00-00	✓		(0, 2, 8, 10)	0-0-0	D
	4	00100	✓		(0, 8)	0-000	✓		<del>(0, 4, 2, 6)</del>	<del>00--0</del>	
	8	01000	✓	1	(2, 6)	00-10	✓		<del>(0, 8, 2, 10)</del>	<del>0-0-0</del>	
2	5	00101	✓		(2, 10)	0-010	✓	1	(4, 5, 6, 7)	001--	E
	6	00110	✓		(4, 5)	0010-	✓		<del>(4, 6, 5, 7)</del>	<del>001--</del>	
	9	01001	✓		(4, 6)	001-0	✓		(8, 9, 10, 11)	010--	F
	10	01010	✓		(8, 9)	0100-	✓		<del>(8, 10, 9, 11)</del>	<del>010--</del>	
3	7	00111	✓	2	(5, 7)	001-1	✓	2	(5, 7, 13, 15)	0-1-1	✓
	11	01011	✓		(5, 13)	0-101	✓		(5, 7, 21, 23)	-01-1	✓
	13	01101	✓		(5, 21)	-0101	✓		<del>(5, 13, 7, 23)</del>	<del>0-1-1</del>	
	21	10101	✓		(6, 7)	0011-	✓		(5, 13, 21, 29)	--101	✓
	26	11010	✓		(9, 11)	010-1	✓		<del>(5, 21, 7, 23)</del>	<del>-01-1</del>	
	28	11100	✓		(9, 13)	01-01	✓		<del>(5, 21, 13, 29)</del>	<del>--101</del>	
4	15	01111	✓		(10, 11)	0101-	✓		(9, 11, 13, 15)	01--1	G
	23	10111	✓		(10, 26)	-1010	A		<del>(9, 13, 11, 15)</del>	<del>01--1</del>	
	29	11101	✓	3	(7, 15)	0-111	✓	3	(7, 15, 23, 31)	--111	✓
	30	11110	✓		(7, 23)	-0111	✓		<del>(7, 23, 15, 31)</del>	<del>--111</del>	
5	31	11111	✓		(11, 15)	01-11	✓		(13, 15, 29, 31)	-11-1	✓
					(13, 15)	011-1	✓		<del>(13, 29, 15, 31)</del>	<del>-11-1</del>	
					(13, 29)	-1101	✓		(21, 23, 29, 31)	1-1-1	✓
					(21, 23)	101-1	✓		<del>(21, 29, 23, 31)</del>	<del>1-1-1</del>	
					(21, 29)	1-101	✓		(28, 29, 30, 31)	111--	H
					(26, 30)	11-10	B		<del>(28, 30, 29, 31)</del>	<del>111--</del>	
					(28, 29)	1110-	✓				
					(28, 30)	111-0	✓				
				4	(15, 31)	-1111	✓				
					(23, 31)	1-111	✓				
					(29, 31)	111-1	✓				
					(30, 31)	1111-	✓				

Class 3			
Group	Dec	vwxyz	
2	(5, 7, 13, 15, 21, 23, 29, 31)	--1-1	I
	<del>(5, 7, 21, 23, 13, 15, 29, 31)</del>	<del>--1-1</del>	
	<del>(5, 13, 21, 29, 7, 15, 23, 31)</del>	<del>--1-1</del>	

Since no other implicants can be formed, this is the end of the procedure.

$$PI = \{A, B, C, D, E, F, G, H, I\}$$

$$PI = \{wx'yz', vwyz', v'w'z', v'x'z', v'w'x, v'wx', v'wz, vwx, xz\}$$

b) Find the essential prime implicant(s) from the corresponding *Prime Implicant Table*.

PI		CU	0	2	4	5	6	7	8	9	10	11	13	15	21	23	26	28	29	30	31
						√		√					√	√	√	√		√	√	√	√
(10,26)	A	4								x							x				
(26,30)	B	4															x			x	
(0,2,4,6)	C	3	x	x	x		x														
(0,2,8,10)	D	3	x	x					x		x										
(4,5,6,7)	E	3			x	x	x	x													
(8,9,10,11)	F	3							x	x	x	x									
(9,11,13,15)	G	3								x		x	x	x							
(28,29,30,31)	H*	3																X	x	x	x
(5,7,13,15,21,23,29,31)	I*	1				x		x					x	x	X	X			x		x

EPI: H\*, I\* => G = H\* + I\* + ...

Eliminate the EPI's rows and the columns of their corresponding minterms:

PI		CU	0	2	4	6	8	9	10	11	26
(10,26)	A	4			X	X			x		x
(26,30)	B	4			X	X					x
(0,2,4,6)	C	3	X	X	X	X					
(0,2,8,10)	D	3	X	X	X	X	X		X		
(4,5,6,7)	E	3			X	X					
(8,9,10,11)	F	3			X	X	X	X	X	X	
(9,11,13,15)	G	3			X	X		X		X	

Eliminate double columns:

0=2, 4=6, 8=10, 9=11

PI		CU	0	4	8	9	26
(10,26)	A	4					x
(26,30)	B	4					x
(0,2,4,6)	C	3	X	X			
(0,2,8,10)	D	3	X		X		
(4,5,6,7)	E	3		X			
(8,9,10,11)	F	3			X	X	
(9,11,13,15)	G	3				X	

Remove dominated rows implemented at the same cost: A=B, C>E, F>G

PI		CU	0	4	8	9	
			√	√	√	√	√
(10,26)	A**	4					X*
(0,2,4,6)	C**	3	X	X*			
(0,2,8,10)	D	3	X		X		
(8,9,10,11)	F**	3			X	X*	

Secondary EPI: A\*\*, C\*\*, F\*\* cover all the minterms remained uncovered by EPI's =>

=> A minimum function is

$$f = H^* + I^* + A^{**} + C^{**} + F^{**} = (vwx + xz) + (wx'yz' + v'w'z' + v'wx')$$

c) Apply the Petrick method to determine ALL the minimal covers of the function G.

After finding the EPI's, the Petrick method is used to find all the function's covers:

PI		CU	0	2	4	6	8	9	10	11	26
(10,26)	A	4							x		x
(26,30)	B	4									x
(0,2,4,6)	C	3	X	X	X	X					
(0,2,8,10)	D	3	X	X			X		X		
(4,5,6,7)	E	3			X	X					
(8,9,10,11)	F	3					X	X	X	X	
(9,11,13,15)	G	3					X		X		

$$P = (C+D)(C+E)(D+F)(A+D+F)(F+G) (A+B)$$

$$P = (C+CD+CE+DE)(D+F)(F+G) (A+B)$$

$$P = (C+DE)(F+DG)(A+B) = ACF + BCF + ACDG + BCDG + AFDE + BFDE + ADEG + BDEG$$

$$G_1 = (H + I) + (A + C + F) =$$

$$= (vwx + xz) + (wx'yz' + v'w'z' + v'wx')$$

$$G_2 = (H + I) + (B + C + F) =$$

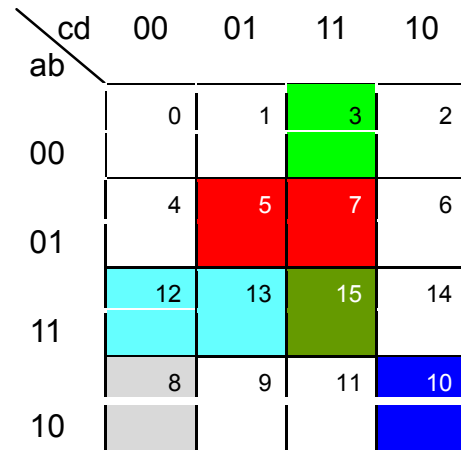
$$= (vwx + xz) + (vwy'z' + v'w'z' + v'wx')$$

**Question 4:**

a) Find PI's of the following function using Iterated Consensus :

$$F(a,b,c,d) = a'b'cd + a'bd + abc' + ab'c'd' + ab'cd' + abcd$$

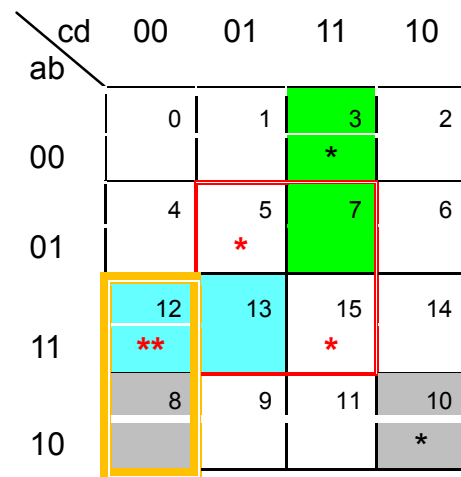
Consensus	Term	Implicants	Covered by	Covers & Removes
	T1	a'b'ed	<T7	
	T2	a'bd	<T13	
	T3	abc'		
	T4	ab'e'd'	<T9	
	T5	ab'ed'	<T11	
	T6	abcd	<T10	
T2ϕT1	T7	a'cd		>T1
T3ϕT2	T8	be'd	<T13	
T4ϕT3	T9	ac'd'		>T4
T5ϕT3		-		
T5ϕT2		-		
T6ϕT5		-		
T6ϕT3	T10	abd	<T13	>T6
T7ϕT5		-		
T7ϕT3		-		
T7ϕT2		-		
T8ϕT7		<del>a'bd = T2</del>		
T8ϕT5		-		
T8ϕT3		-		
T8ϕT2		-		
T9ϕT8		<del>abc' = T3</del>		
T9ϕT7		-		
T9ϕT5	T11	ab'd'		>T5
T9ϕT3		-		
T9ϕT2		-		
T10ϕT9		<del>abc' = T3</del>		
T10ϕT8		-		
T10ϕT7	T12	bed	<T13	
T10ϕT3		-		
T10ϕT2	T13	bd		>T2,T10
T11ϕT9		-		
T11ϕT7		-		
T11ϕT3		<del>ac'd' = T9</del>		
T13ϕT11		-		
T13ϕT9		<del>abc' = T3</del>		
T13ϕT3		-		



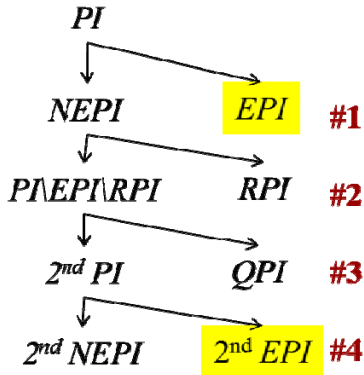
**K-maps are for verification only:**

PI : { T3,T7,T9,T11, T13}

PI : {abc', ab'd', a'cd, ac'd', bd}



b) Find a minimum set of Prime Implicants that covers F:



**#1. Find Essential Prime Implicants (EPI's) by evaluating  $E_{PI \setminus P_j}(P_j) = \sum P_k, P_k \in PI \setminus P_j$**

If  $E_j = 1 \rightarrow P_j$  implies  $PI \setminus P_j \rightarrow P_j \in NEPI =$  set of Non-Essential PI's  
 If  $E_j \neq 1 \rightarrow P_j$  does not imply  $PI \setminus P_j \rightarrow P_j \in EPI =$  set of Essential PI's

$$PI: \{abc', ab'd', a'cd, ac'd', bd\}$$

$$PI: \{T3, T11, T7, T9, T13\}$$

**T3 = abc'**: a=1 b=1 c=0

$$E_{PI \setminus T3}(T3) = (ab'd' + a'cd + ac'd' + bd)|_{abc'=1} = 0 + 0 + d' + d = 1 \rightarrow T3 \in NEPI$$

**T11 = ab'd'**: a=1 b=0 d=0

$$E_{PI \setminus T5}(T11) = (abc' + a'cd + ac'd' + bd)|_{ab'd'=1} = 0 + 0 + c + 0 = c \neq 1 \rightarrow T11 \in EPI$$

**T7 = a'cd**: a=0 c=1 d=1

$$E_{PI \setminus T7}(T7) = abc' + ab'd' + ac'd' + bd = 0 + 0 + 0 + b \neq 1 \rightarrow T7 \in EPI$$

**T9 = ac'd'**: a=1 c=0 d=0

$$E_{PI \setminus T9}(T9) = abc' + ab'd' + a'cd + bd = b + b' + 0 + 0 = 1 \rightarrow T9 \in NEPI$$

**T13 = bd**: b=1 d=1

$$E_{PI \setminus T13}(T13) = abc' + ab'd' + a'cd + ac'd' = ac' + 0 + a'c + 0 = ac' + a'c \neq 1 \rightarrow T13 \in EPI$$

<b>#1 EPI*</b> : { T11, T7, T13 } = { ab'd', a'cd, bd }	<b>NEPI</b> : { T3, T9 } = { abc', ac'd' }
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**#2 Find Redundant Prime Implicants (RPI's) by evaluating each NEPI**

**T3**: a=1, b=1, c=0 =>  $E_{EPI}(T3) = (ab'd' + a'cd + bd)|_{abc'=1} = 0 + 0 + d = d \neq 1 \quad abc' \notin RPI$

**T9**: a=1, c=0, d=0 =>  $E_{EPI}(T9) = (ab'd' + a'cd + bd)|_{ac'd'=1} = b' + 0 + 0 = b' \neq 1 \Rightarrow ac'd' \notin RPI$

Hence **#2 RPI =  $\Phi$**

**#3 Find Eclipsed Prime Implicants (QPI's)  $P_q \in PI \setminus EPI \setminus RPI$  which are covered by  $EPI \cup P_s$**

with  $P_s \in PI \setminus EPI \setminus RPI$  and  $P_q \neq P_s$

$$E_{EPI \cup T3}(P_q) = (ab'd' + a'cd + bd) + abc', P_q = \{ac'd'\}$$

$$E_{EPI \cup T3}(T9)|_{a=1, c=0, d=0} = b' + 0 + 0 + b = 1$$

$\rightarrow EPI \cup T3 > T9$  i.e. T9 eclipsed by  $EPI \cup T3$

$\rightarrow T9 \in QPI$ ;  **$2^{nd} PI = SPI = \{abc'\} = \{T3\}$**

**If only 1 minimized expression is required  $\rightarrow$  #4,**

**else, find all other 2<sup>nd</sup> PI's lists as follows:**

$$E_{EPI \cup T9}(P_q) = (ab'd' + a'cd + bd) + ac'd', P_q = \{abc'\}$$

$$E_{EPI \cup T9}(T3)|_{a=1, b=1, c=0} = 0 + 0 + d + d' = 1$$

$\rightarrow EPI \cup T9 > T3$  i.e. T3 eclipsed by  $EPI \cup T9$

$\rightarrow T3 \in QPI_1$ ;  **$2^{nd} PI' = SPI_1 = \{ac'd'\} = \{T9\}$**

**#4. Find Secondary EPI's by evaluating  $E_{SPI \cup EPI \setminus P_j}(P_j) = \sum P_k + \sum P_{EPI}, P_k \in SPI \setminus P_j$**

$$SPI \cup EPI \setminus P_j = T3 + (ab'd' + a'cd + bd) \setminus T3 =$$

$$= ab'd' + a'cd + bd = EPI \rightarrow T3 = abc' = SEPI^{**}$$

$$SPI_1 \cup EPI \setminus P_j = T9 + (ab'd' + a'cd + bd) \setminus T9 =$$

$$= ab'd' + a'cd + bd = EPI \rightarrow T9 = ac'd' = SEPI_1^{**}$$

**$2^{nd} NEPI = \Phi \rightarrow STOP$**

<b><math>F(a,b,c,d) = (ab'd' + a'cd + bd) + abc'</math></b>	<b><math>F_1(a,b,c,d) = (ab'd' + a'cd + bd) + ac'd'</math></b>
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**Question 5:**

a) Using iterated consensus find all the prime implicants of the following function:

$$F(a,b,c,d) = \bar{a} \cdot \bar{b} \cdot c \cdot \bar{d} + \bar{a} \cdot b \cdot d + a \cdot b \cdot \bar{c} + a \cdot \bar{b} \cdot \bar{c} \cdot \bar{d} + a \cdot \bar{b} \cdot c \cdot \bar{d} + a \cdot b \cdot c \cdot d$$

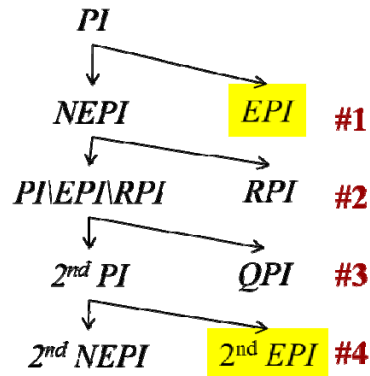
b) Find the minimum set of prime implicants (PI) that covers F by evaluating how each PI implies subsets of the other PI's. What is the expression(s) of the minimized function F?

Consensus	Term	Implicants	abcd	Covered by
	T1	a <sup>2</sup> b <sup>2</sup> cd <sup>2</sup>	0010	<T9
	T2	a <sup>2</sup> bd	01-1	<T12
	T3	abc'	110-	
	T4	ab <sup>2</sup> c <sup>2</sup> d <sup>2</sup>	1000	<T8
	T5	ab <sup>2</sup> cd <sup>2</sup>	1010	<T9
	T6	abcd	1111	<T10
T2∅T1		-		
T3∅T2	T7	bc'd	-101	<T12
T3∅T1		-		
T4∅T3	T8	ac'd'	1-00	
T5∅T3		-		
T5∅T2		-		
T5∅T1	T9	b'cd'	-010	
T6∅T3	T10	abd	11-1	<T12
T7∅T3		-		
T7∅T2		-		
T8∅T7		abc <sup>2</sup>		=T3
T8∅T3		-		
T8∅T2		-		
T9∅T8	T11	ab'd'	10-0	
T9∅T7		-		
T9∅T3		-		
T9∅T2		-		
T10∅T9		-		
T10∅T8		abc <sup>2</sup>		=T3
T10∅T7		-		
T10∅T3		-		
T10∅T2	T12	bd	-1-1	
T11∅T9		-		
T11∅T8		-		
T11∅T3		ac <sup>2</sup> d <sup>2</sup>		=T8
T12∅T11		-		
T12∅T9		-		
T12∅T8		abc <sup>2</sup>		=T3
T12∅T3		-		

PI = {a.b.c', ac'd', b'cd', ab'd', bd}

Verification with K-maps:

cd	00	01	11	10
ab				
00	0	0	0	1*
01	0	1*	1*	
11	1	1	1*	
10	1	0	0	1



#1 Find EPI's  $E_j = E_{PI \setminus P_j}(P_j) = \sum_{P_k \in PI \setminus P_j} P_k ; j = 1..N$  PI = {abc', ac'd', b'cd', ab'd', bd}

$E_{PI \setminus abc'}(abc') = ac'd' + b'cd' + ab'd' + bd = E_{PI \setminus abc'} |_{(a=1, b=1, c=0)} = d' + 0 + 0 + d = 1$  abc' => NEPI

$E_{PI \setminus ac'd'}(ac'd') = abc' + b'cd' + ab'd' + bd = E_{PI \setminus ac'd'} |_{(a=1, c=0, d=0)} = b + 0 + b' + 0 = 1$  ac'd' => NEPI

$E_{PI \setminus b'cd'}(b'cd') = abc' + ac'd' + ab'd' + bd = E_{PI \setminus b'cd'} |_{(b=0, c=1, d=0)} = 0 + 0 + a + 0 \neq 1$  b'cd' => EPI

$E_{PI \setminus ab'd'}(ab'd') = abc' + ac'd' + b'cd' + bd = E_{PI \setminus ab'd'} |_{(a=1, b=0, d=0)} = 0 + c' + c + 0 = 1$  ab'd' => NEPI

$$E_{PI \setminus bd}(bd) = abc' + ac'd' + b'cd' + ab'd' = E_{PI \setminus bd}|_{(b=1, d=1)} = ac' + 0 + c + 0 \neq 1 \quad \mathbf{bd} \Rightarrow \mathbf{EPI}$$

$$\mathbf{EPI: \{b'cd', bd\}}$$

$$\mathbf{NEPI: \{abc', ac'd', ab'd'\}}$$

**#2 Find Redundant PI's from NEPI:**  $\{abc', ac'd', ab'd'\}$ , al  $E_{EPI}(P_r) = \sum_{P_e \in EPI} P_e$  and  $P_r \in NEPI$

$$E_{EPI}(abc') = b'cd' + bd = E_{EPI}|_{a=1, b=1, c=0} = 0+d \neq 1$$

$$E_{EPI}(ac'd') = b'cd' + bd = E_{EPI}|_{a=1, c=0, d=0} = 0+0 \neq 1 \quad \overline{abc} \notin RPI \quad ac'd' \Rightarrow PI/EPI/RPI$$

$$E_{EPI}(ab'd') = b'cd' + bd = E_{EPI}|_{a=1, b=0, d=0} = c+0 \neq 1 \quad a \cdot \overline{c} \cdot \overline{d} \notin RPI \quad ab'd' \Rightarrow PI/EPI/RPI$$

$$a \cdot \overline{b} \cdot \overline{d} \notin RPI$$

So, there are no RPI's (RPI =  $\Phi$ ), and all 3 NEPI's =  $PI \setminus EPI \setminus RPI$ :  $\{abc', ac'd', ab'd'\}$

**#3 Find Eclipsed PI's (QPI)  $P_q \neq P_s$  of  $PI \setminus EPI \setminus RPI$  that are covered by  $EPI \cup P_s$**

$$E_s(P_q) = \sum_{P_e \in EPI} P_e + P_s, \text{ with } P_s, P_q \in PI \setminus EPI \setminus RPI$$

Add the NEPIs ( $P_s$ ) to the set of EPIs, one at a time, to check if they eclipse the other NEPI's ( $P_q$ )

Add  $abc'$  to EPI:

$$E_{EPI \cup abc'}(ac'd') = [(b'cd' + bd) + a.b.c']|_{a=1, c=0, d=0} = 0+0+b \neq 1 \Rightarrow \text{NQPI}$$

$$E_{EPI \cup abc'}(ab'd') = [(b'cd' + bd) + a.b.c']|_{a=1, b=0, d=0} = c+0+0 \neq 1 \Rightarrow \text{NQPI}$$

Add  $ac'd'$  to EPI:

$$E_{EPI \cup ac'd'}(a.b.c') = [(b'cd' + bd) + a.c'.d']|_{a=1, b=1, c=0} = 0+d+d' = 1 \Rightarrow a.b.c' \text{ eclipsed by } EPI \cup a.c'.d'$$

$$E_{EPI \cup ac'd'}(ab'd') = [(b'cd' + bd) + a.c'.d']|_{a=1, b=0, d=0} = c+0+c' = 1 \Rightarrow ab'd' \text{ eclipsed by } EPI \cup a.c'.d'$$

$\Rightarrow$  keep  $a.c'.d'$  2<sup>nd</sup> PI and eliminate eclipsed PIs:  $\mathbf{QPI = \{a.b.c', ab'd'\}}$

Add  $ab'd'$  to EPI:

$$E_{EPI \cup ab'd'}(a.b.c') = [(b'cd' + bd) + ab'd']|_{a=1, b=1, c=0} = 0+d+0 \neq 1 \Rightarrow \text{NQPI}$$

$$E_{EPI \cup ab'd'}(ac'd') = [(b'cd' + bd) + ab'd']|_{a=1, c=0, d=0} = 0+0+b' \neq 1 \Rightarrow \text{NQPI}$$

Adding  $a.c'.d'$  to the EPI  $2^{nd} PI = SPI = \{ac'd'\}$ ; they eclipse the other 2 NEPIs  $\rightarrow \mathbf{QPI = \{a.b.c', ab'd'\}}$

**#4. Find 2nd EPI from 2<sup>nd</sup> PI ( $SPI = PI \setminus EPI \setminus RPI \setminus QPI = a.c'.d'$ )**

$$E_{SEPI} = E_{SPI \cup EPI \setminus P_j}(P_j) = \sum P_k + \sum P_{EPI}, P_k \in SPI \setminus P_j;$$

$$SPI \cup EPI \setminus P_j = [a.c'.d' + (b'cd' + bd)] \setminus a.c'.d' = (b'cd' + bd)$$

$$E_{SEPI}(ac'd') = E_{SEPI}|_{a=1, c=0, d=0} = (b'cd' + bd)|_{ac'd'=1} = 0+0 \neq 1 \rightarrow ac'd' = SEPI$$

$$\rightarrow \mathbf{F_{min} = (b'cd' + bd) + ac'd'}$$