CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number		Section(s)
Mathematics	208/2		All
Examination	**Date**	**Time**	**Pages**
Midterm	October 2011	1 Hour 30 minutes	2
Instructors			**Course Examiner**
B. Rhodes, C. Santana, D. Sen, E. Smith, F. romanelli, H. Greenspan, J. Mcsweeney, R. Mearns, T. Hughes, T. Zaihra			D. Sen

FORMULAE:

$$A = P(1+i)^n, \quad A = Pe^{rt}, \quad FV = PMT\,\frac{(1+i)^n - 1}{i}, \quad PV = PMT\,\frac{1-(1+i)^{-n}}{i}$$

Special Instructions:

- ▷ Answer all questions.
- ▷ **Only approved calculators are allowed.**

MARKS

[10] **1.** At a price of \$2.28 per bushel, the supply of barley is 7,500 million bushels and the demand is 7,900 million bushels. At a price of \$2.37 per bushel, the supply of barley is 7,900 million bushels and the demand is 7,800 million bushels.

(A) Find a price-supply equation of the form $p = mx + b$.

(B) Find a price-demand equation of the form $p = mx + b$.

(C) Find the equilibrium point.

[10] **2.** Solve for x in the following equations:

(A) $64^{x^2} = 256^x$

(B) $3^{\log_2 x} = 3^5$

(C) $(e)^{-x^2-1} = (e)^{3x+1}$

(D) $\log_{10}(x+1) - \log_{10}(10x-3) = 1$

PLEASE TURN OVER

[10] **3.**

(A) If the 1st and 15th terms of an arithmetic sequence are -5 and 23, respectively, find the 73rd term of the sequence.

(B) If the 1st and 10th terms of a geometric sequence are 3 and 30, respectively, find the 40th term (to three decimal places) of the sequence.

[10] **4.** For services rendered, an attorney accepts a 90 day note for $\$5,500$ at 8% simple interest from a client (both interest and principal are repaid at the end of 90 days). Wishing to use her money sooner, the attorney sells the note to a third party for $\$5,560$ after 30 days. What annual interest rate will the third party receive for the investment?

[10] **5.** If \$500 is deposited each quarter into an account paying 8% compounded quarterly for 3 years, find the interest earned during each of the 3 years.

[10] **6.** A family has a $\$240,000$, 20–year mortgage at 5.75% compounded monthly.

(A) Find the monthly payment and the total interest paid.

(B) Find the unpaid balance after 8 years.

1

(supply, price): (7,500, 2.28) & (7,900, 2.37)

(demand, price): (7,900, 2.28) & (7,800, 2.37)

a) $m = \frac{2.37 - 2.28}{7,900 - 7,500} = \frac{0.09}{400} = 0.000225$

$P = 2.28 + 0.000225(X - 7,500) = 0.000225X + 0.5925$

b) $m = \frac{2.37 - 2.28}{7,800 - 7,900} = \frac{0.09}{-100} = -0.0009$

$P = 2.28 - 0.0009(X - 7,900) = -0.0009X + 9.39$

c) $-0.0009X + 9.39 = 0.000225X + 0.5925$

$0.001125X = 8.7975$

$X = 7820 \Rightarrow P(7820) = 2.352$

Ans: (7820, 2.352)

2

a) $64^{x^2} = 256^x$ $\quad 64 = 4^3$ & $256 = 16^2 = 4^4$

$\Rightarrow (4^3)^{x^2} = (4^4)^x \Rightarrow 4^{3x^2} = 4^{4x} \Rightarrow 3x^2 = 4x$

$\Rightarrow 3x^2 - 4x = 0 \Rightarrow x(3x-4) = 0 \Rightarrow x_1 = 0$ & $x_2 = 4/3$.

b) $3^{\log_2 x} = 3^5 \Rightarrow \log_2 x = 5 \Rightarrow x = 2^5 = 32$.

c) $e^{-x^2-1} = e^{3x+1} \Rightarrow -x^2-1 = 3x+1 \Rightarrow x^2+3x+2 = 0$

$(x+1)(x+2) = 0 \Rightarrow x_1 = -1$, $x_2 = -2$.

d) $\log_{10}(x+1) - \log_{10}(10x-3) = 1 \Rightarrow \log_{10} \frac{x+1}{10x-3} = 1$

$\Rightarrow \frac{x+1}{10x-3} = 10 \Rightarrow x+1 = 10(10x-3) = 100x-30$

$\Rightarrow 99x = 31 \Rightarrow x = \frac{31}{99}$

3

A) $a_1 = -5$, $a_{15} = 23$, $a_{73} = ?$

$a_{15} = a_1 + 14d \Rightarrow 23 = -5 + 14d \Rightarrow 28 = 14d \Rightarrow d = 2$.

$a_{73} = a_1 + 72d = -5 + 72 \cdot 2 = -5 + 144 = 139$.

B) $b_1 = 3$, $b_{10} = 30$, $b_{40} = ?$

$b_{10} = b_1 q^9 \Rightarrow 30 = 3 \cdot q^9 \Rightarrow q^9 = 10 \Rightarrow q = \sqrt[9]{10}$

$b_{40} = b_1 \cdot q^{39} = 3 \cdot q^{39} = 3 \cdot q^{36} \cdot q^3 = 3 \cdot 10^4 \cdot \sqrt[3]{10} =$

4

90 days, $5,500, 8% simple (annual) interest rate sells for $5,560 after 30 days.

$$A_{90\text{ days}} = P\left(1+\frac{0.08}{4}\right) = 5,500\left(1+\frac{0.08}{4}\right) = 5,500 \cdot 1.02 =$$

$$= 55 \cdot 102 = 5610$$

$$\Rightarrow \quad I = 5,610 - 5,560 = \$50$$

$$I = Prt = 5,560 \cdot \frac{1}{6} \cdot r \Rightarrow 50 = \frac{5,560}{6} \cdot r$$

$$r = \frac{300}{5,560} = \frac{30}{556} = 0.054 \Rightarrow \text{Ans.: } 5.4\%$$

5

$500 each quarter at 8% compounded quarterly for 3 years.

$$FV_3 = PMT\,\frac{(1+i)^n - 1}{i} = 500\,\frac{1.02^{12} - 1}{0.02} = 6,706.05$$

The total investment is $500 \cdot 4 \cdot 3 = 500 \cdot 12 = 6,000$.

$$\widetilde{I}_3 = FV - 6,000 = \$706.05 \text{ . (total interest earned)}$$

$$FV_1 = PMT\,\frac{(1+i)^4 - 1}{i} = 500 \cdot \frac{1.02^4 - 1}{0.02} = 2060.8$$

$$\boxed{I_1 = 2060.8 - 2000 = \$60.8}$$

$$FV_2 = PMT\,\frac{(1+i)^8 - 1}{i} = 500 \cdot \frac{1.02^8 - 1}{0.02} = 4,291.48$$

$$\widetilde{I}_2 = 4,291.48 - 4,000 = \$291.48 \Rightarrow I_2 = 291.48 - 60.8 = 230.68$$

$$I_3 = \widetilde{I}_3 - 291.48 = \$414.57$$

(6)

$240,000 20-year mortgage at 5.75% compounded monthly.

$$PMT = PV\frac{i}{1-(1+i)^{-n}} = 240{,}000 \cdot \frac{0.0575/12}{1-\left(1+\frac{0.0575}{12}\right)^{-240}}$$

$$= 240{,}000 \cdot \frac{0.004791(6)}{0.68249241(2)} = \$1{,}685.$$

a) $A = 1{,}685 \cdot 240 = \$404{,}400. \Rightarrow I = 404{,}400 - 240{,}000 = 164{,}400.$

b) is it remains to pay 500 12 more years with PMT = 1,685 the present value is

$$PV = PMT\frac{1-(1+i)^{-144}}{i} = 174{,}180.97.$$

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Instructors **Course Examiner**

A. Kokotov, F. Romanelli, H. Greenspan, M. Padamadan, R. Rodriguez, U. Tiwari, V. Kalvin — D. Sen

FORMULAE:

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MARKS

[4+3+3] **1.** Given the quadratic function $f(x) = -0.15x^2 - 0.90x + 3.3$

(A) Find x and y intercepts algebraically.

(B) Find the vertex form for f.

(C) Find the vertex and the maximum or minimum.

[$2\frac{1}{2} * 4$] **2.** Solve for x in the following equations:

(A) $5^{x^2+4x-9} = 125^{-2x+5}$

(B) $\ln(x+3) + \ln(2x+4) = 2\ln 2$

(C) $e^{x^2} = \left(\frac{1}{e^2}\right) e^{3x}$

(D) $\log_3(x^2+1) = 2$

PLEASE TURN OVER

[6+4] **3.**

(A) Given an arithmetic sequence $-60, -56, -52, -48, -44, \ldots\ldots$ Find the 200th term and the sum of the first 200th terms of the arithmetic sequence.

(B) Find the 16th term of the geometric sequence $\frac{1}{64}, \frac{-1}{32}, \frac{1}{16}, \ldots\ldots$

[10] **4.** For services rendered, an attorney accepts a 90 day note for $\$5,500$ at 8% simple interest from a client (both interest and principal are repaid at the end of 90 days). Wishing to use her money sooner, the attorney sells the note to a third party for $\$5,560$ after 30 days. What annual interest rate will the third party receive for the investment?

[5+5] **5.** A company estimates that it will have to replace a piece of equipment at a cost of $\$800,000$ in 5 years. To have this money available in 5 years, a sinking fund is established by making equal monthly payments into an account paying 6.6% compounded monthly.

(A) How much should each payment be?

(B) How much interest is earned during the last year?

[5+5] **6.** If you borrow \$500 that you agree to repay in six equal monthly payments at 1% interest per month on the unpaid balance, how much of each monthly payment is used for interest and how much is used to reduce the unpaid balance?

1

$$f(x) = -0.15x^2 - 0.9x + 3.3$$

A) y-intercept is $(0, 3.3)$

x-intercepts: $0.15x^2 + 0.9x - 3.3 = 0 \Rightarrow x^2 + 6x - 22 = 0$

$$x_{1,2} = \frac{-6 \pm \sqrt{36 + 4 \cdot 22}}{2} = -3 \pm \frac{\sqrt{124}}{2} = -3 \pm \sqrt{31}$$

B) Vertex: $x_v = -\frac{b}{2a} = \frac{0.9}{-0.3} = -3$

$$y_v = y(3) = -1.35 + 2.7 + 3.3 = 4.65$$

$$f(x) = -0.15(x+3)^2 + 4.65$$

C) $V(-3, 4.65)$ $\quad y_{max} = 4.65$.

2

a) $5^{x^2+4x-9} = 125^{-2x+5} \Rightarrow 5^{x^2+4x-9} = (5^3)^{-2x+5} = 5^{-6x+15}$

$\Rightarrow x^2 + 4x - 9 = -6x + 15 \Rightarrow x^2 + 10x - 24 = 0 \Rightarrow (x+12)(x-2) = 0$

$x_1 = -12$, $x_2 = 2$.

b) $\ln(x+3) + \ln(2x+4) = 2\ln 2 \Rightarrow \ln[(x+3)(2x+4)] = \ln 4$

$\Rightarrow (x+3)(2[x+2]) = 4 \Rightarrow (x+3)(x+2) = 2 \Rightarrow x^2 + 5x + 4 = 0$

$(x+1)(x+4) = 0 \Rightarrow \boxed{x_1 = -1}$, ~~$x_2 = -4$~~ extraneous.

c) $e^{x^2} = \frac{e^{3x}}{e^2} \Rightarrow e^{x^2} = e^{3x-2} \Rightarrow x^2 = 3x - 2 \Rightarrow x^2 - 3x + 2 = 0 \Rightarrow (x-2)(x-1) = 0$

$x_1 = 2$, $x_2 = 1$.

d) $\log_3(x^2+1) = 2 \Rightarrow x^2 + 1 = 3^2 \Rightarrow x^2 = 8 \Rightarrow x_{1,2} = \pm 2\sqrt{2}$

3

a) $-60, -56, -52, \ldots \qquad a_1 = -60 \quad d = 4$

$a_{200} = a_1 + 199d = -60 + 199 \cdot 4 = 736$

$S_{200} = \frac{a_1 + a_{200}}{2} \cdot 200 = (a_1 + a_{200}) \cdot 100 = 13,600$

b) $\frac{1}{64}, -\frac{1}{32}, \frac{1}{16}, \ldots \qquad b_1 = \frac{1}{64}, \; q = -2$

$b_{16} = b_1 \cdot q^{15} = \frac{1}{64}(-2)^{15} = -\frac{2^{15}}{2^6} = -2^9 = -512.$

4

90 day - note $P = \$5,500$ 8% simple interest in 30 days sells for $\$5,560$.

$A = P\left(1 + \frac{0.08}{4}\right) = 5,500 \cdot 1.02 = 55 \cdot 102 = \$5,610$

$I = 5,610 - 5,560 = \$50$

$I = 5,560 \cdot r \cdot \frac{1}{6} \Rightarrow r = \frac{5,560}{6} = 50$

$r = \frac{300}{5,560} = \frac{30}{556} = 0.054 \Rightarrow$ rate: 5.4%

5

FV = $800,000 in 5 years at 6.6% compounded monthly

a) $PMT = FV \frac{i}{(1+i)^4 - 1} = 800{,}000 \frac{\frac{0.066}{12}}{(1+\frac{0.066}{12})^{60} - 1} = 800{,}000 \frac{0.0055}{1.0055^{60} - 1}$

$PMT = \$11{,}290.42$

b) $FV_4 = PMT \frac{(1+i)^{48} - 1}{i} = 11{,}290.42 \frac{1.0055^{48} - 1}{0.0055} = 618{,}277.04$

$12 \cdot 11{,}290.42 = 135{,}485.04$

$I = 800{,}000 - 618{,}277.04 - 135{,}485.04 = \$46{,}237.92$

6

$500 for 6 months at $i = 1\%$

	Payment	interest	Bal. reduc.	Balance
1	86.27	5	81.27	418.73
2	86.27	4.19	82.08	336.65
3	86.27	3.37	82.9	253.75
4	86.27	2.54	83.73	170.02
5	86.27	1.7	84.57	85.45
6	86.27	0.85	85.42	0.03

$PMT = 500 \frac{i}{1 - (1+i)^{-6}} =$

$= 500 \frac{0.01}{1 - 1.01^{-6}} = 86.27$