TYPE 01

Name ____ **Student ID** ___________________

Lecture (L01-Dr. JW Kim, L02-Dr. R. Wang) **L** _______

Tutorial (T01-Fri 2PM, T02-Tue 11AM, T03-Thu 11AM, T04-Mon 9AM) **T** _______

University of Calgary
Schulich School of Engineering
Winter 2015

ENGG 407 - Numerical Methods

Midterm

February 23 (Monday)
13:00-13:50
ICT 122 (L01) / ENE 243 (L02)

1. Check the type of your exam (**A** or **B**).
2. Mark your answers on the *bubble sheet*.
3. The exam consists of *25 multiple choice questions*.
4. Each question has equal value.
5. You have to submit *both bubble sheet and exam*.
6. Examination is *open book/notes*.
7. No calculators are allowed.
8. No wireless devices or earphones are allowed during exam.
9. All angles are in radians (for example sin(x), x is assumed to be in radians).
10. Notations and symbols follow those used during the class specified otherwise.

1. Which of the following statements best expresses the characteristics of *numerical methods*? [d]

a) Using approximation techniques to find solutions for an analytic function
b) Using iterative and/or recursive algorithms
c) Suitable for problems that cannot be or difficult to be solved analytically
d) All the above
e) None of the above

2. Which of the following is conversion of the binary number 110101.101 to decimal format? [d]
a) 53.75
b) 27.625
c) 53.25
d) 53.625
e) None of the above

3. Given A = LU = $\begin{bmatrix} 3 & 1 \\ 0 & 1.5 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix}$, what is A? [c]

a) $A = \begin{bmatrix} 2.5 & 1 \\ 1 & 4 \end{bmatrix}$

b) $A = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$.

c) $A = \begin{bmatrix} 5 & 6 \\ 3 & 0 \end{bmatrix}$

d) None of the above
e) All of the above

4. Given a general row of expression in an SLE as $L_i = (a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{in}x_n) = b_i, 1 \le i, j \le n$, and *k, k'* are a nonzero real number, what is *NOT* a valid rule that keeps the system′s solutions invariant? [d]

a) $L_i \leftrightarrow L_j$

b) $kL_i \to L_i$

c) $kL_i + k'L_j \to L_j$

d) $kL_i \to L_j$

e) None of the above

5. Which is always true about *Eigenvalue* and Eigenvector of square matrix *A*? [e]

a) Even if all elements of matrix *A* are real, its eigenvalue may be a complex
b) Matrix *A* can have more than one eigenvalue
c) Every eigenvalue must have at least one eigenvector
d) None of the above

e) All of the above

6. Among the numerical methods for solving nonlinear equations, which of the following statements is *NOT* true: [d]

a) The bisection method does not need to know the derivative of the given function
b) Newton's method may diverge
c) The secant method may leads to Newton's method when $\Delta x \to 0$
d) The regula falsi method may diverge
e) None of the above

7. Given $f(x) = x^2-1$ in [0, 2], assume $x^{(0)} = 0.5$ for root finding using Newton's method. What is the numerical result of $x^{(1)}$ after the first iteration? [a]

a) 1.25
b) 1
c) 0.75
d) 0.8
e) None of the above

8. Determine a pair of eigenvalues for the given matrix $A = \begin{bmatrix} -1 & 2 \\ \frac{5}{2} & 3 \end{bmatrix}$. [c]

a) -1, 3
b) 1, -3
c) 4, -2
d) -4, 2
e) None of the above

9. Applying the Jacobi iteration method on the SLE as given below, which of the following systems is equivalent and will converge according to the sufficient condition of convergence for iterative methods? [b]

$$3x_1 + 8x_2 - 14x_3 = 5$$
$$2x_1 + 7x_2 + 1x_3 = -9$$
$$12x_1 + 5x_2 + x_3 = 11$$

a) $\begin{bmatrix} 3 & 8 & -14 \\ 2 & 7 & 1 \\ 12 & 5 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -9 \\ 11 \end{bmatrix}$

b) $\begin{bmatrix} 12 & 5 & 1 \\ 2 & 7 & 1 \\ 3 & 8 & -14 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 11 \\ -9 \\ 5 \end{bmatrix}$

c) $\begin{bmatrix} 12 & 5 & 1 \\ 2 & 7 & 1 \\ 3 & 8 & -14 \end{bmatrix} \begin{bmatrix} x_3 \\ x_2 \\ x_1 \end{bmatrix} = \begin{bmatrix} 11 \\ -9 \\ 5 \end{bmatrix}$

d) $\begin{bmatrix} 3 & 8 & -14 \\ 12 & 5 & 1 \\ 2 & 7 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 11 \\ -9 \end{bmatrix}$

e) None of the above

10. Given an SLE $Ax = b$, the condition number Cond(A) = $||A|| \, ||A^{-1}||$ can be an indicator for which of the following? [d]

a) If Cond(A) >>1, the SLE is ill-conditioned
b) If Cond(A) ≈ 1 and Cond(A) $\not< 1$, the LSE is normal
c) $\forall A$, Cond(A) ≥ 1
d) All of above
e) None of the above

11. What is the first order Taylor expansion of the function $f_1(x,y) = \sin(xy)$ at the given expansion point (x_0, y_0)? [c]

a) $f_1(x,y) = \sin(x_0 y_0) + \cos(x_0 y_0)(x - x_0) + \cos(x_0 y_0)(y - y_0)$
b) $f_1(x,y) = \sin(x_0 y_0) + y_0 \cos(xy)(x - x_0) + x_0 \cos(xy)(y - y_0)$
c) $f_1(x,y) = \sin(x_0 y_0) + y_0 \cos(x_0 y_0)(x - x_0) + x_0 \cos(x_0 y_0)(y - y_0)$
d) $f_1(x,y) = \sin(xy) + y\cos(xy)(x - x_0) + x\cos(xy)(y - y_0)$
e) None of the above

12. According to Taylor's theorem for an arbitrary continued function f(x) as given below:

$$f(x) = \sum_{i=0}^{n} \frac{f^{(i)}(a)}{i!}(x-a)^i + R_n \text{ where the } \textit{remainder } R_n = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-a)^{n+1}, \xi \in (a,x)$$

what is the order (maximum) of the truncation error for a 3rd order expansion of f(x)? [d]

a) $R_3 = \dfrac{f^{(3)}(x)}{3!}(x-a)^3$

b) $R_3 = \dfrac{f^{(4)}(a)}{4!}(x-a)^4$

c) $R_3 = \dfrac{f^{(3)}(\xi)}{3!}(x-a)^3, \xi \in (a,x)$

d) $R_3 = \frac{f^{(4)}(\xi)}{4!}(x-a)^4, \xi \in (a,x)$

e) None of the above

13. In an experiment, a set of data for three variables x, y, and z were collected at various levels of accuracy with different numbers of significant decimal digits as [x y z] = [23578.3 0.1892 75.22]. Assume the data are calculated as

$$\begin{aligned} f &= (x + y) \times z \\ &= (23{,}578.3 + 0.1892) \times 75.22 \\ &= 1{,}773{,}573.95823 \end{aligned}$$

in order to determine f. What is the most accurate result obtained according to engineering data and error processing rules? [c]

a) f = 1,773,573.95823
b) f = 1,773,573.96
c) f = 1,773,574.0
d) f = 1,773,573.9582
e) None of the above

14. It is guaranteed that a root of a continuous nonlinear function $f(x)$ exists in [*a*, *b*] when *f(a)* • *f(b)* < *0*. However, when *f(a)* • *f(b)* > *0*, there would be *N* roots existing in [*a*, *b*]. In this case, which of the following possibilities may be ruled out? [e]

a) N = 0
b) N = 1
c) N ≥ 2 and are even numbers
d) N > 3 and are odd numbers
e) None of the above

15. One of the strategies for solving an SLE can be described as x = [A, b] = [I, b′] ⇒ x = b′. What is the name of this method? [c]

a) Newton
b) Gauss-Seidel
c) Gauss-Jordan
d) LU decomposition
e) None of the above

16. The convergent condition for solving root(s) for a nonlinear functions *f(x)* in [*a*, *b*] is determined by the follows, *except*: [b]

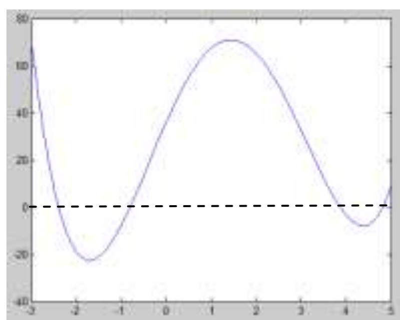
a) *(b-a)/2* < tolerance
b) $f'(x_i) = 0$
c) $f(x_i) = 0$

d) $f(x_i)$ < tolerance
e) None of the above

17. The *Jocobi iterative method* for solving SLEs is a special case of the *Gauss-Siedel iterative method*. Given the general iteration formula of the latter as $x_i = \frac{1}{a_{ii}}(b_i - \sum_{j=1 \wedge j \neq i}^{n} a_{ij} x_j)$, what is the iterative expression of the former? [c]

a) $x_i^{(k+1)} = \frac{1}{a_{ii}}(b_i - \sum_{j=1 \wedge j \neq i}^{n} a_{ij} x_j)$
b) $x_i^{(k)} = \frac{1}{a_{ii}}(b_i - \sum_{j=1 \wedge j \neq i}^{n} a_{ij} x_j^{(k)})$
c) $x_i^{(k+1)} = \frac{1}{a_{ii}}(b_i - \sum_{j=1 \wedge j \neq i}^{n} a_{ij} x_j^{(k)})$
d) None of the above

18. The plot of a function $f(x) = x^4 - 5.5x^3 - 7.2x^2 + 43x + 36 = 0$ is as follows. What is *NOT* a suitable strategy to find the roots of *f(x)* in [-3, 5] using any single root finding method? [a]



a) Using the *f(a) • f(b) < 0* criterion
b) Using a spline or a section for screening
c) Using Newton`s method around the four points
d) Using a random method for root finding
e) None of the above

19. A disadvantage of Newton's iterative method for root finding is that it: [c]

a) Only applicable to force related problems
b) Requires the second order numerical derivative
c) Requires the algebraic form of the derivative of the given function
d) Only applicable when the Jacobian is used
e) None of the above

20. For a generic SLE as $\begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n) = b_1 \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n) = b_2 \\ ... \\ a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n) = b_m \end{cases}$, what is the condition for obtaining unique solutions? [d][e]

a) $m > n$
b) $n \neq m$
c) $n \geq m$
d) $m \geq n$
e) None of the above

21. How does the trend of a nonlinear function *f(x)* as being descending at x_0 towards $x_0+\Delta x$ be detected where Δx is a small positive real number close to machine epsilon? [a]

a) $f(x_o) > f(x_o + \Delta x)$
b) $f(x_o) < f(x_o + \Delta x)$
c) $|f(x_o)| > |f(x_o + \Delta x)|$
d) $|f(x_o)| < |f(x_o + \Delta x)|$
e) None of the above

22. In the following characteristics of iterative methods, such as those of Gauss-Seidel and Jacobi, for solving an SLE, which statement is *NOT* true? [c]

a) The sufficient convergent condition needs always to be tested
b) May not converge in iteration when the sufficient convergent condition is not met
c) May be slower than those of direct methods if the coefficient matrix with many zero elements
d) Cannot be carried out when zero pivoting elements are involved without pivoting treatment
e) None of the above

23. Given a function $f(x) = 2\sin(x) + 1$, what is the first order Taylor expansion of the function at the point $x_0 = \pi$? [b]

a) $f(x) = f(x_0) + 2\cos(x_0)(x - x_0)$
b) $f(x) = 1 - 2(x - \pi)$
c) $f(x) = f(x_0) + 2\cos(x)(x - x_0)$
d) $f(x) = 1 + 2(x - \pi)$
e) None of the above

24. In the *bisection* method, the root of *f(x)* is to be determined given an initial bracket of $[a,b] = [0,10]$. After 3 iterations, what is the length of the bracket ($L_3 = b_3 - a_3$)? [b]

a) 2.50
b) 1.25
c) 5.00
d) 1.75
e) None of the above

25. For a given SLE as $\begin{cases} 3x_1 + x_2 + x_3 = 1 \\ 2x_1 + 5x_2 - 2x_3 = 0 \\ -x_1 + 4x_2 - 6x_3 = 1 \end{cases}$, the Jacobi method will: [a]

a) definitely converge
b) definitely diverge
c) undeterminable
d) depend on the initial values
e) None of the above

- End of Exam -