

$$\cos \theta = \vec{a} \cdot \vec{b} / (\|\mathbf{a}\| \|\mathbf{b}\|)$$

$$\text{comp}_{\mathbf{b}} \mathbf{a} = \|\mathbf{a}\| \cos \theta = \mathbf{a} \cdot \hat{\mathbf{b}}$$

$$\text{proj}_{\mathbf{b}} \mathbf{a} = (\vec{a} \cdot \hat{\mathbf{b}}) \hat{\mathbf{b}}$$

$$\text{Area of a parallelogram} = \|\mathbf{a} \times \mathbf{b}\|$$

$$\text{Volume of a parallelepiped} = |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$$

Equation of a line :

$$\vec{r} = \vec{r}_2 + t(\vec{r}_2 - \vec{r}_1) = \vec{r}_2 + t\vec{a}$$

Equation of a plane: $ax + by + cz + d = 0$

$$\text{also: } [(\vec{r}_2 - \vec{r}_1) \times (\vec{r}_3 - \vec{r}_1)] \cdot (\vec{r} - \vec{r}_1) = 0$$

$$\frac{d\vec{r}(s)}{ds} = \frac{d\vec{r}}{ds} \frac{ds}{dt}$$

$$\text{Length of a curve: } s = \int_{t_1}^{t_2} |\vec{r}'(t)| dt$$

$$\kappa = \left\| \frac{d\vec{T}}{ds} \right\| = \left\| \frac{d^2\vec{r}}{ds^2} \right\| = \frac{|\vec{T}'|}{|\vec{r}'|} = \frac{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$$

$$\vec{a}(t) = \kappa v^2 \hat{\mathbf{N}} + \frac{dv}{dt} \hat{\mathbf{T}} = a_N \hat{\mathbf{N}} + a_T \hat{\mathbf{T}}$$

$$\hat{\mathbf{N}} = \frac{d\vec{T}/dt}{\|d\vec{T}/dt\|}$$

$$\hat{\mathbf{T}} = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$$

The Binormal

$$\hat{\mathbf{B}} = \hat{\mathbf{T}} \times \hat{\mathbf{N}}$$

$$a_T = \frac{dv}{dt} = \frac{\mathbf{v} \cdot \mathbf{a}}{\|\mathbf{v}\|} \quad \& \quad a_N = \kappa v^2 = \frac{\|\mathbf{v} \times \mathbf{a}\|}{\|\mathbf{v}\|}$$

$$\nabla = \mathbf{i} \frac{\partial}{\partial x} + \mathbf{j} \frac{\partial}{\partial y} + \mathbf{k} \frac{\partial}{\partial z}$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u} \frac{\partial u}{\partial x} + \frac{\partial f}{\partial v} \frac{\partial v}{\partial x} \quad \& \quad \frac{\partial f}{\partial y} = \frac{\partial f}{\partial u} \frac{\partial u}{\partial y} + \frac{\partial f}{\partial v} \frac{\partial v}{\partial y}$$

$$D_u(F) = \nabla F \cdot \hat{u}, \quad \hat{u} = \text{unit vector}$$

Equation of Tangent Plane:

$$\vec{n}_o \cdot (\vec{r} - \vec{r}_o) = 0, \quad \vec{n}_o = \nabla F \text{ at P}$$

$$W = \int_C \vec{F} \cdot d\vec{r}$$

equation of normal line to a surface: $\vec{n}_o \times (\vec{r} - \vec{r}_o) = 0, \quad \vec{n}_o = \nabla F \text{ at P}$

$$\text{Line integral: } \int_C F(x, y, z) ds = \int_a^b F(f(t), g(t), h(t)) \sqrt{[f']^2 + [g']^2 + [h']^2} dt$$

$$\text{Surface integral: } \iint_S \vec{F} \cdot \hat{n} dS = \iint_R \vec{F} \cdot \hat{n} \sqrt{1 + (z_x)^2 + (z_y)^2} dx dy$$

$$\oint_C \vec{F} \cdot d\vec{r} = \iint_S (\text{curl } \vec{F}) \cdot \hat{n} dS$$

$$\oint_S (\vec{F} \cdot \hat{n}) dS = \iiint_D (\text{div } \vec{F}) dV$$

$$\oint_C [Pdx + Qdy] = \iint_R \left[\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right] dx dy$$

$$\tilde{x} = \frac{\iiint_D x \rho(x, y, z) dV}{m}$$

$$m = \iiint_D \rho(x, y, z) dV$$

$$I_x = \iiint_D (y^2 + z^2) \rho(x, y, z) dV;$$

$$x = r \cos \theta, \quad y = r \sin \theta; \quad z = z; \quad r = \sqrt{x^2 + y^2}, \quad \theta = \tan^{-1}(y/x)$$

$$J(u, v) = \frac{\partial(x, y)}{\partial(u, v)}$$

$$x = \rho \sin \phi \cos \theta, \quad y = \rho \sin \phi \sin \theta, \quad z = \rho \cos \phi,$$

$$\rho = \sqrt{x^2 + y^2 + z^2}, \quad \theta = \tan^{-1}(y/x), \quad \phi = \tan^{-1}(\sqrt{x^2 + y^2}/z)$$

$$dV = r dr d\theta dz$$

$$dV = \rho^2 \sin \phi d\rho d\phi d\theta$$