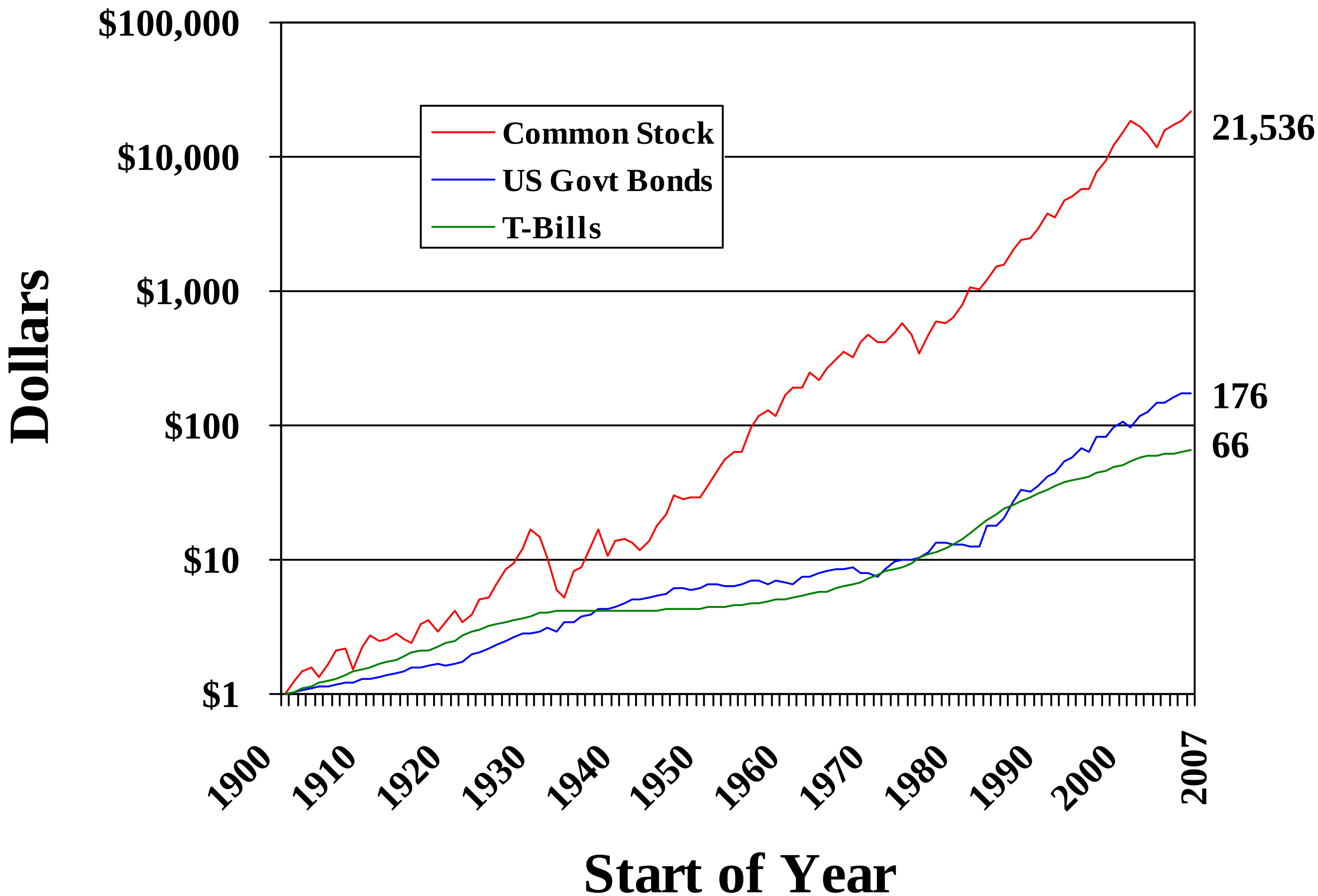
Risk and Return

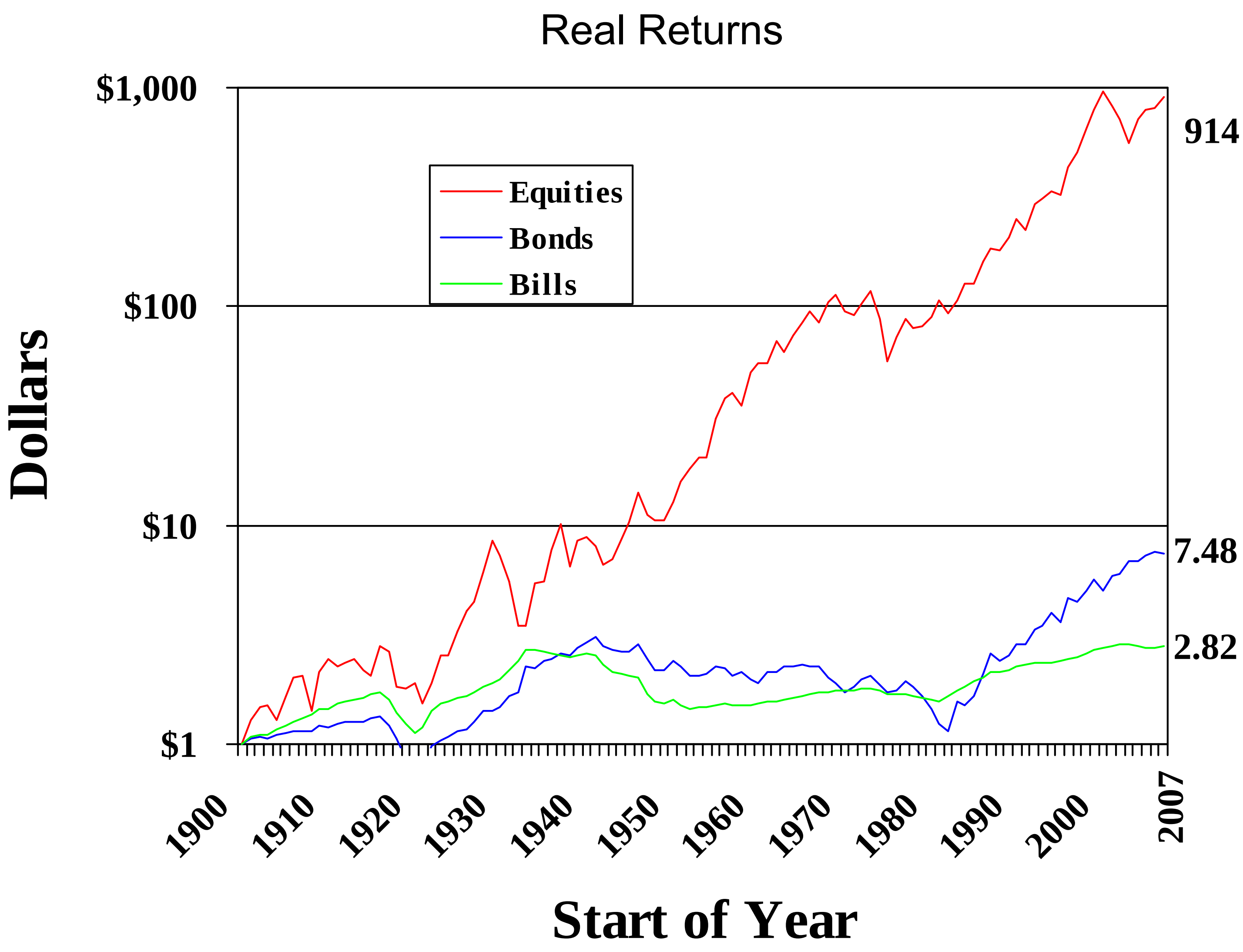
Topics Covered

- Over a Century of Capital Market History
- Measuring Portfolio Risk
- Calculating Portfolio Risk
- How Individual Securities Affects Portfolio Risk
- Diversification & Value Additivity

The Value of an Investment of $1 in 1900

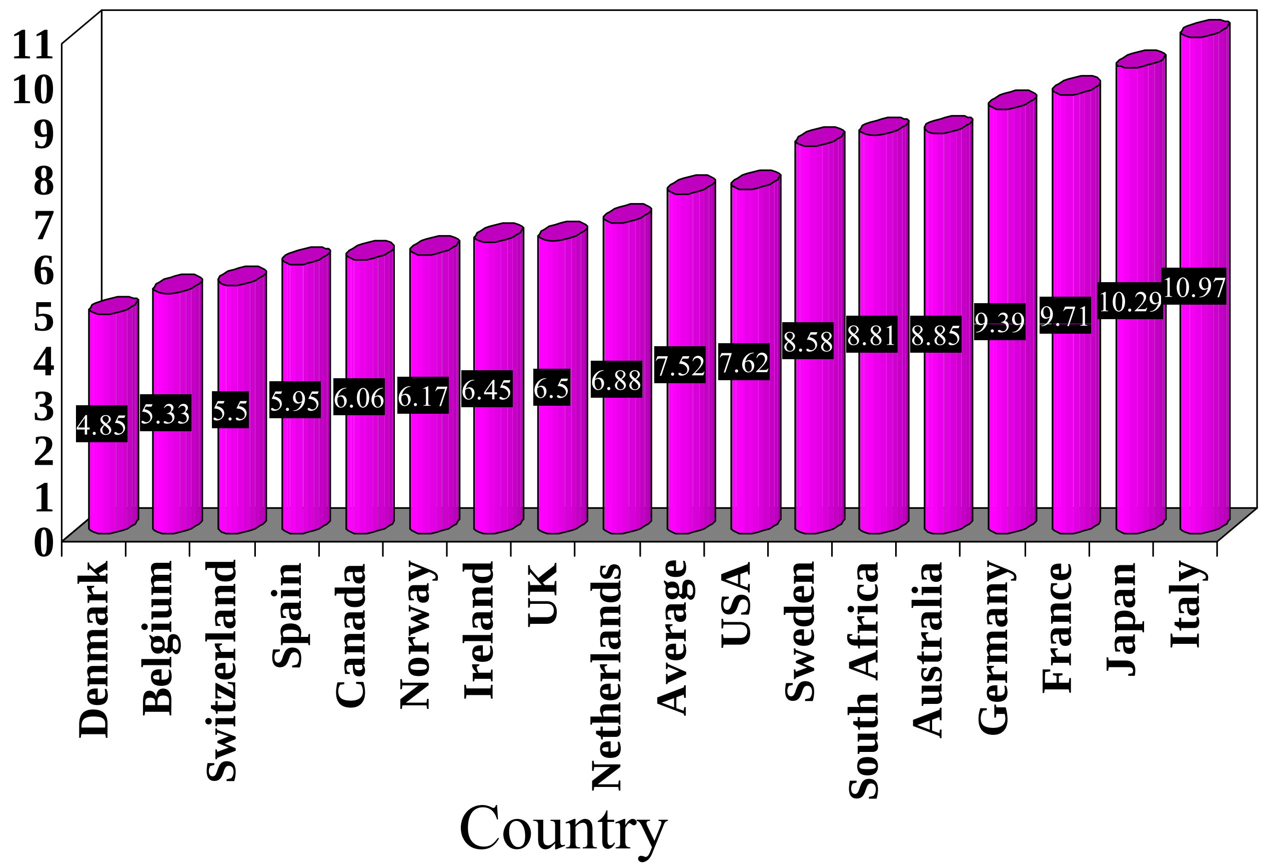


The Value of an Investment of $1 in 1900

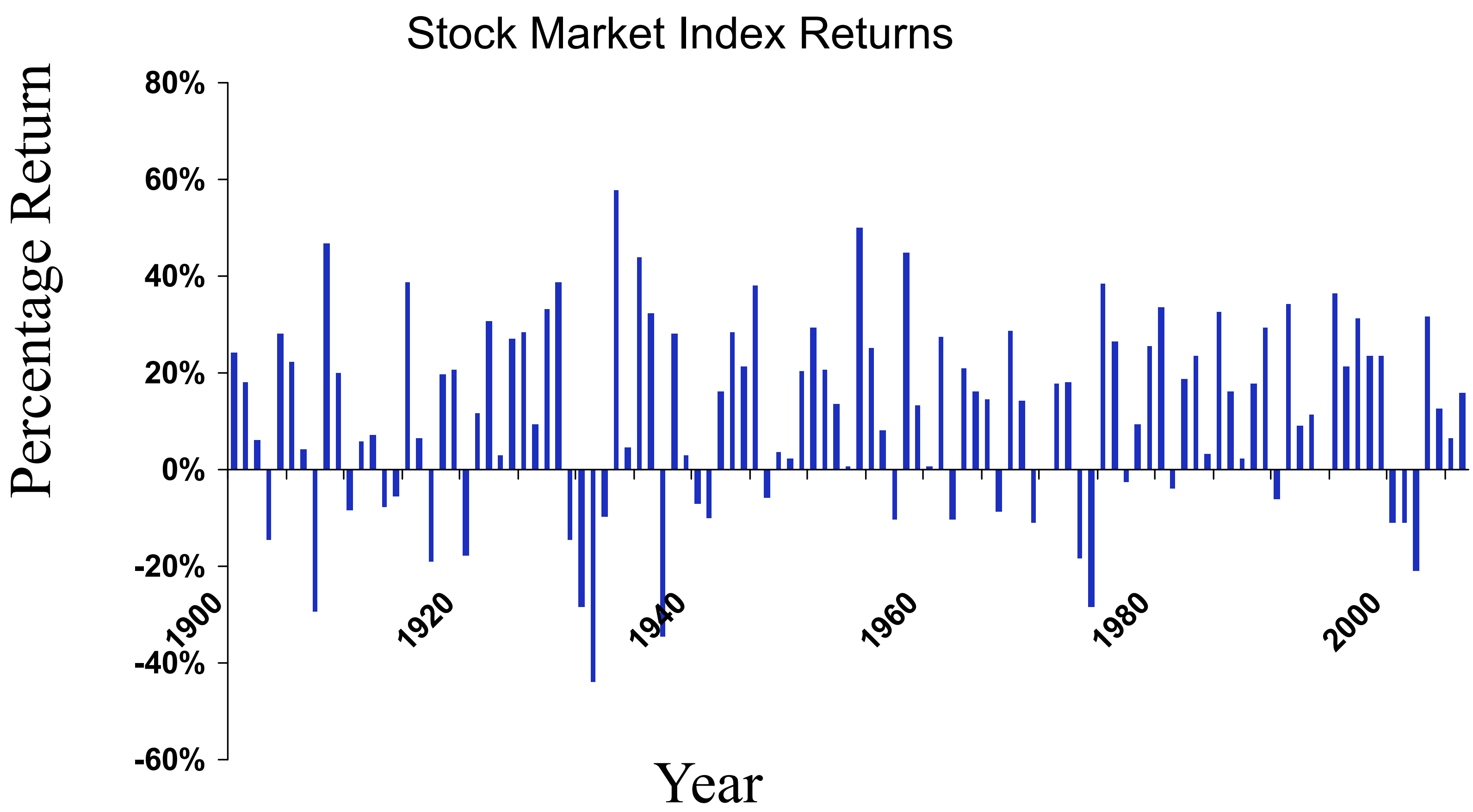


Average Market Risk Premia (by country)

Risk premium, %

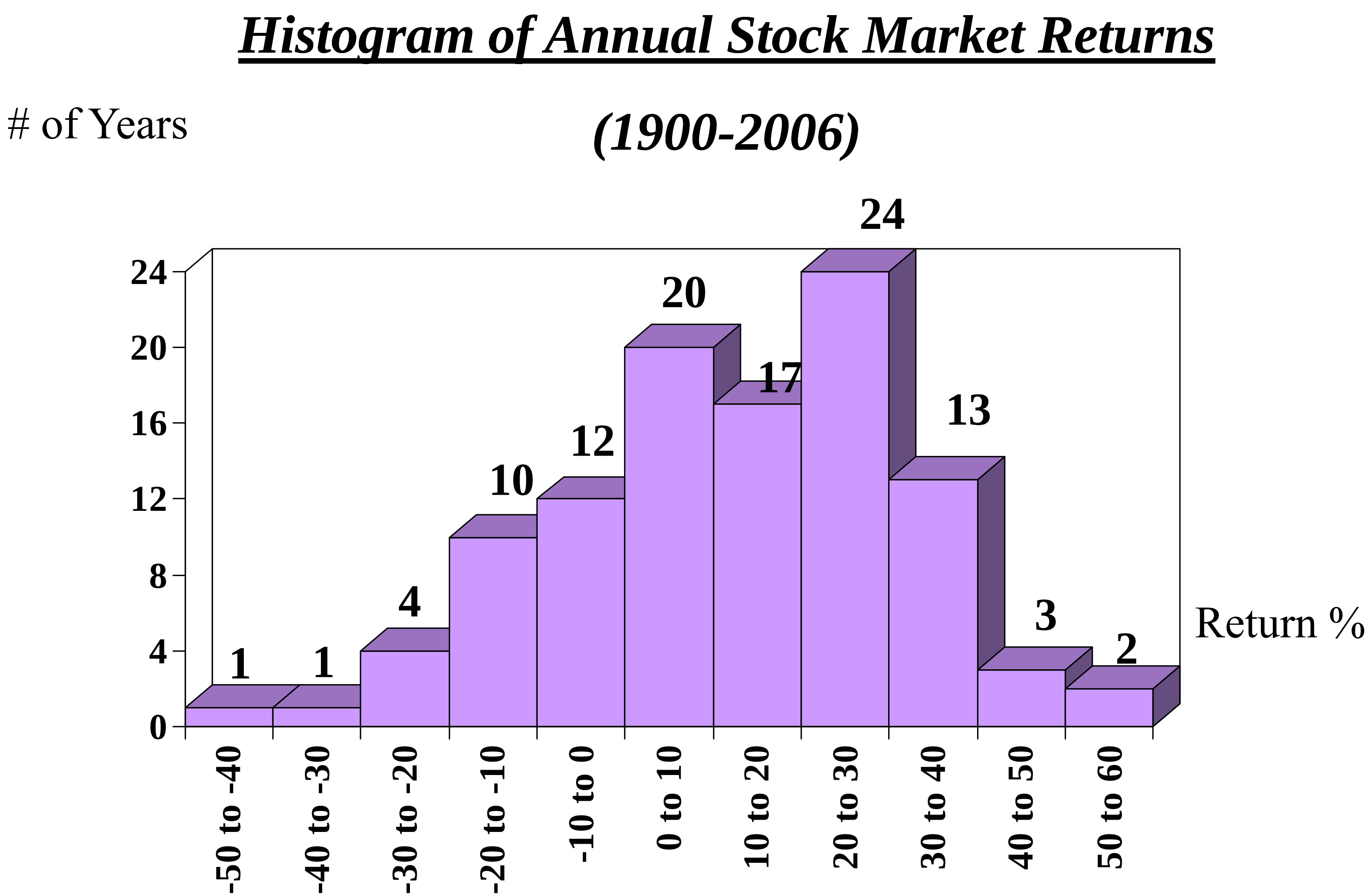


Rates of Return 1900-2006

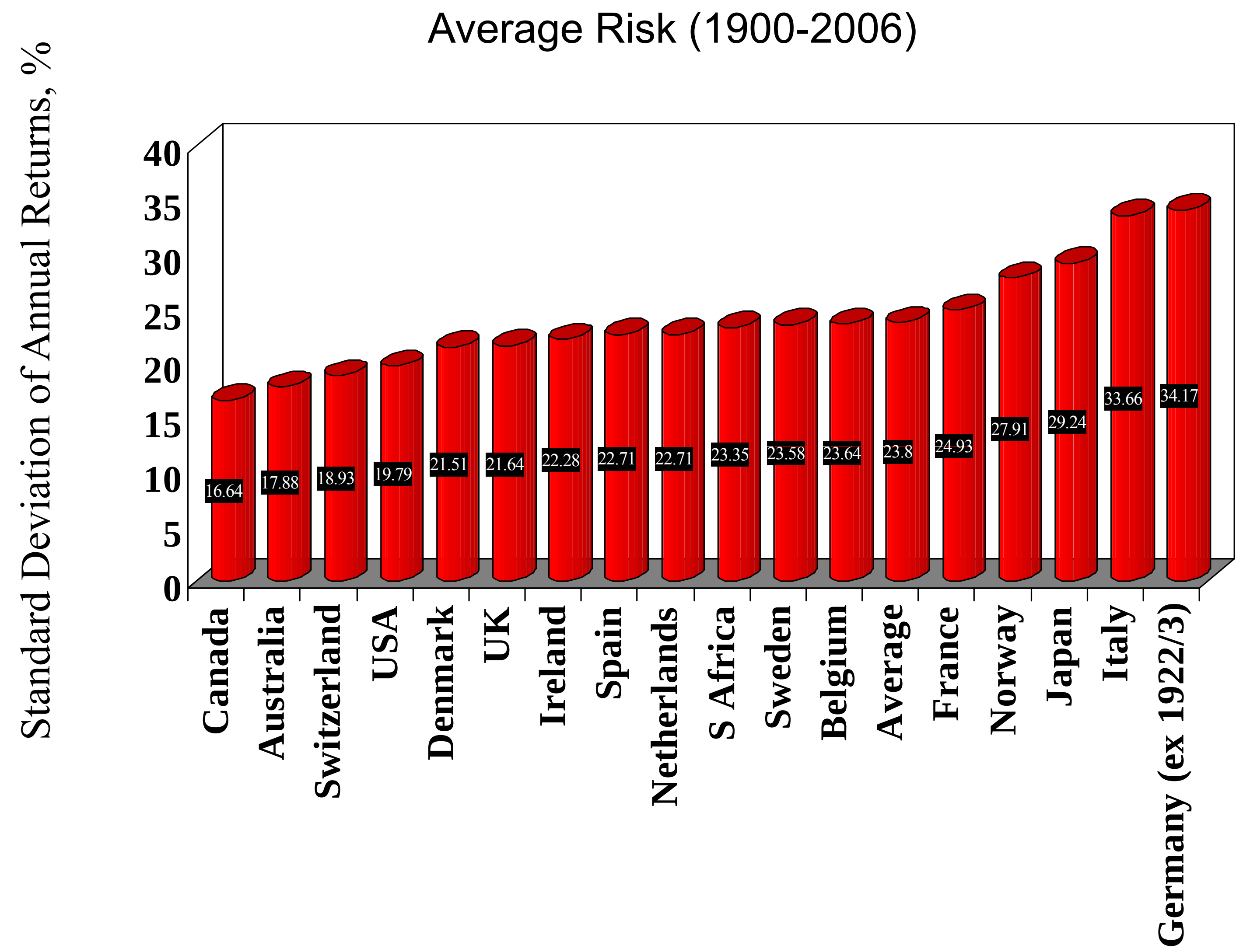


Source: Ibbotson Associates

Measuring Risk



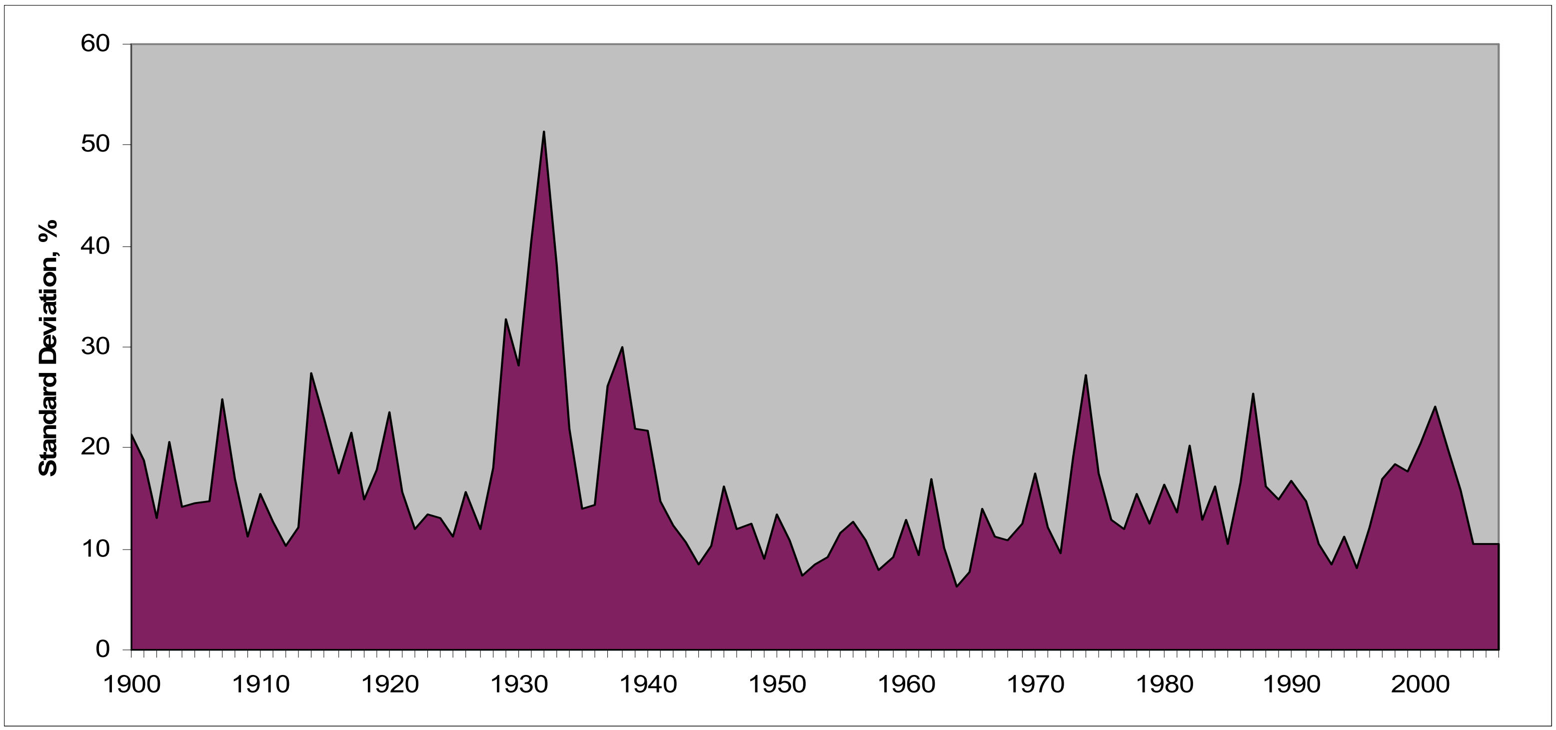
Equity Market Risk (by country)



Dow Jones Risk

Annualized Standard Deviation of the DJIA over the preceding 52 weeks

(1900 – 2006)



Measuring Risk

Variance - Average value of squared deviations from mean. A measure of volatility.

Standard Deviation – The square root of the variance

Measuring Risk

Coin Toss Game-calculating variance and standard deviation

(1) Percent Rate of Return	(2) Deviation from Mean	(3) Squared Deviation
+ 40	+ 30	900
+ 10	0	0
+ 10	0	0
- 20	- 30	900

Variance = average of squared deviations = 1800 / 4 = 450

Standard deviation = square of root variance = $\sqrt{450}$ = 21.2%

Measuring Risk

Diversification - Strategy designed to reduce risk by spreading the portfolio across many investments.

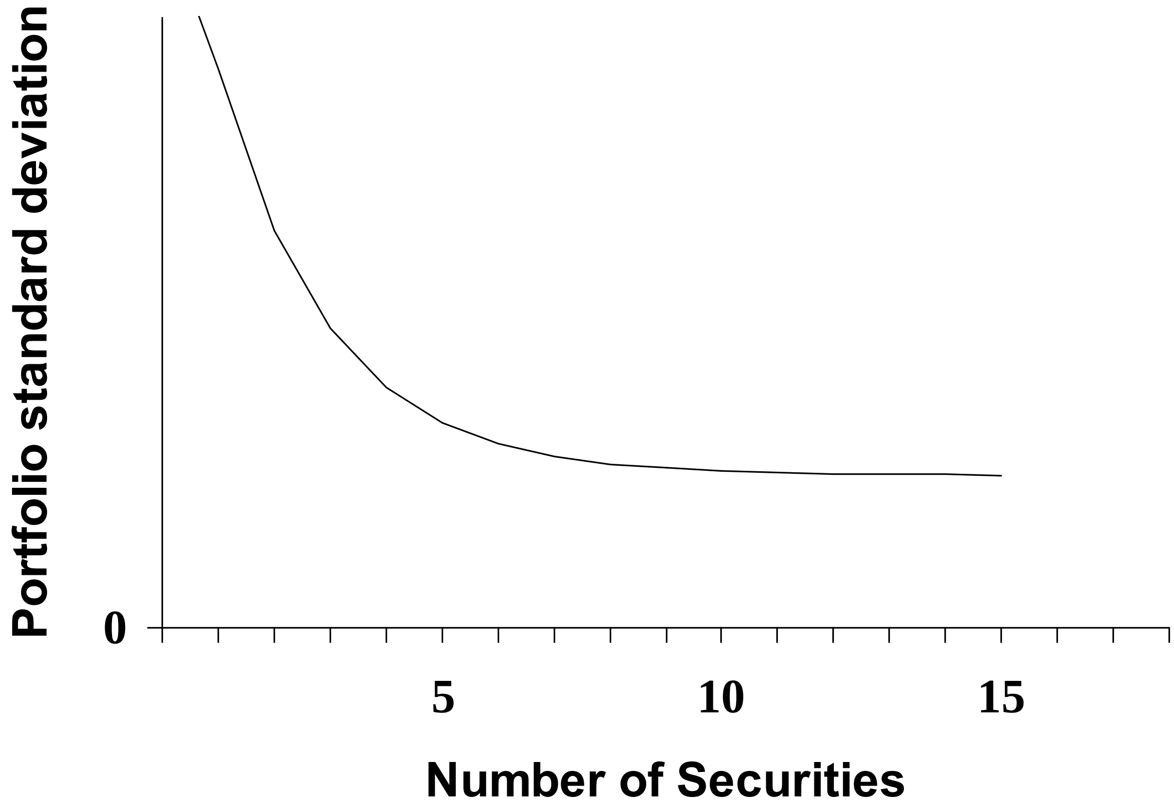
Unique Risk - Risk factors affecting only that firm. Also called "diversifiable risk."

Market Risk - Economy-wide sources of risk that affect the overall stock market. Also called "systematic risk."

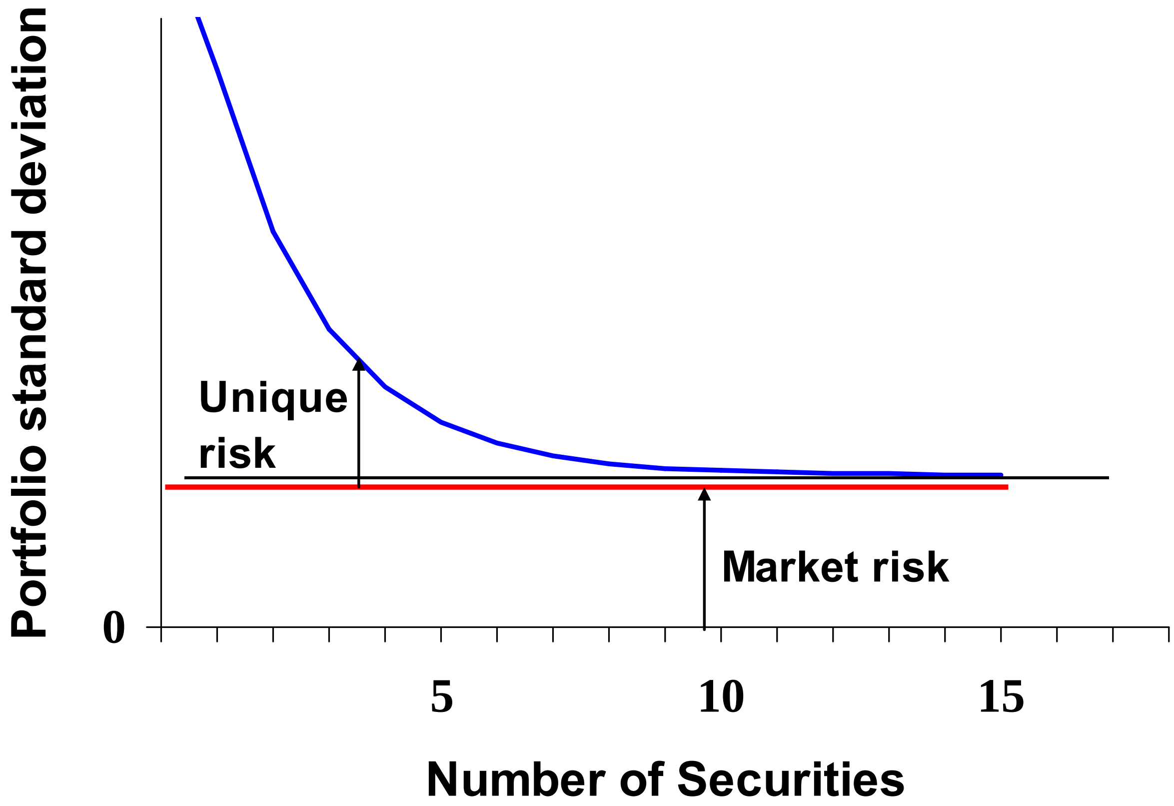
Measuring Risk

$$\text{Portfolio rate of return} = \left(\begin{array}{l}\text{fraction of portfolio}\\ \text{in first asset}\end{array}\right) \times \left(\begin{array}{l}\text{rate of return}\\ \text{on first asset}\end{array}\right) + \left(\begin{array}{l}\text{fraction of portfolio}\\ \text{in second asset}\end{array}\right) \times \left(\begin{array}{l}\text{rate of return}\\ \text{on second asset}\end{array}\right)$$

Measuring Risk



Measuring Risk



Portfolio Risk

The variance of a two stock portfolio is the sum of these four boxes

	Stock 1	Stock 2
Stock 1	$x_1^2\sigma_1^2$	$x_1 x_2 \sigma_{12} = x_1 x_2 \rho_{12} \sigma_1 \sigma_2$
Stock 2	$x_1 x_2 \sigma_{12} = x_1 x_2 \rho_{12} \sigma_1 \sigma_2$	$x_2^2\sigma_2^2$

Portfolio Risk

Example

Suppose you invest 60% of your portfolio in Wal-Mart and 40% in IBM. The expected dollar return on your Wal-Mart stock is 10% and on IBM is 15%. The expected return on your portfolio is:

$$\text{Expected Return} = (.60 \times 10) + (.40 \times 15) = 12\%$$

Portfolio Risk

Example

Suppose you invest 60% of your portfolio in Wal-Mart and 40% in IBM. The expected dollar return on your Wal-Mart stock is 10% and on IBM is 15%. **The standard deviation of their annualized daily returns are 19.8% and 29.7%, respectively. Assume a correlation coefficient of 1.0 and calculate the portfolio variance.**

	Wal-Mart	IBM
Wal-Mart	$x_1^2\sigma_1^2 = (.60)^2 \times (19.8)^2$	$x_1 x_2 \rho_{12} \sigma_1 \sigma_2 = .40 \times .60 \times 1 \times 19.8 \times 29.7$
IBM	$x_1 x_2 \rho_{12} \sigma_1 \sigma_2 = .40 \times .60 \times 1 \times 19.8 \times 29.7$	$x_2^2\sigma_2^2 = (.40)^2 \times (29.7)^2$

Portfolio Risk

Example

Suppose you invest 60% of your portfolio in Wal-Mart and 40% in IBM. The expected dollar return on your Wal-Mart stock is 10% and on IBM is 15%. The standard deviation of their annualized daily returns are 19.8% and 29.7%, respectively. Assume a correlation coefficient of 1.0 and calculate the portfolio variance.

$$\text{Portfolio Variance} = [(.60)^2 \times (19.8)^2] + [(.40)^2 \times (29.7)^2] + 2(.40\text{x}.60 \times 19.8 \times 29.7) = 564.5$$

$$\text{Standard Deviation} = \sqrt{564.5} = 23.8\ \%$$

Portfolio Risk

Example

Suppose you invest 60% of your portfolio in Exxon Mobil and 40% in Coca Cola. The expected dollar return on your Exxon Mobil stock is 10% and on Coca Cola is 15%. The expected return on your portfolio is:

$$\text{Expected Return} = (.60 \times 10) + (.40 \times 15) = 12\%$$

Portfolio Risk

Example

Suppose you invest 60% of your portfolio in Exxon Mobil and 40% in Coca Cola. The expected dollar return on your Exxon Mobil stock is 10% and on Coca Cola is 15%. **The standard deviation of their annualized daily returns are 18.2% and 27.3%, respectively. Assume a correlation coefficient of 1.0 and calculate the portfolio variance.**

	Exxon - Mobil	Coca - Cola
Exxon - Mobil	$x_1^2\sigma_1^2 = (.60)^2 \times (18.2)^2$	$x_1 x_2 \rho_{12}\sigma_1\sigma_2 = .40 \times .60 \times 1 \times 18.2 \times 27.3$
Coca - Cola	$x_1 x_2 \rho_{12}\sigma_1\sigma_2 = .40 \times .60 \times 1 \times 18.2 \times 27.3$	$x_2^2\sigma_2^2 = (.40)^2 \times (27.3)^2$

Portfolio Risk

Example

Suppose you invest 60% of your portfolio in Exxon Mobil and 40% in Coca Cola. The expected dollar return on your Exxon Mobil stock is 10% and on Coca Cola is 15%. The standard deviation of their annualized daily returns are 18.2% and 27.3%, respectively. Assume a correlation coefficient of 1.0 and calculate the portfolio variance.

$$\text{Portfolio Variance} = [(.60)^2 \times (18.2)^2] + [(.40)^2 \times (27.3)^2] + 2(.40\text{x}.60 \times 18.2 \times 27.3) = 477.0$$

$$\text{Standard Deviation} = \sqrt{477.0} = 21.8\ \%$$

Portfolio Risk

$$\text{Expected Portfolio Return} = (x_1 r_1) + (x_2 r_2)$$

$$\text{Portfolio Variance} = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2(x_1 x_2 \rho_{12}\sigma_1\sigma_2)$$

Portfolio Risk

<u>Example</u> Correlation Coefficient = .4

Stocks	σ	% of Portfolio	Avg Return
ABC Corp	28	60%	15%
Big Corp	42	40%	21%

Standard Deviation = weighted avg = <u>33.6</u>

Standard Deviation = Portfolio = <u>28.1</u>

Real Standard Deviation:

$= (28^{2)}(.6^2) + (42^2)(.4^2) + 2(.4)(.6)(28)(42)(.4)$

= <u>28.1</u> CORRECT

Return : r = (15%)(.60) + (21%)(.4) = 17.4%

Portfolio Risk

Example Correlation Coefficient = .4

Stocks	σ	% of Portfolio	Avg Return
ABC Corp	28	60%	15%
Big Corp	42	40%	21%

Standard Deviation = weighted avg = 33.6

Standard Deviation = Portfolio = 28.1

Return = weighted avg = Portfolio = 17.4%

Let's Add stock New Corp to the portfolio

Portfolio Risk

Example Correlation Coefficient = .3

Stocks	σ	% of Portfolio	Avg Return
Portfolio	28.1	50%	17.4%
New Corp	30	50%	19%

NEW Standard Deviation = weighted avg = 31.80

NEW Standard Deviation = Portfolio = 23.43

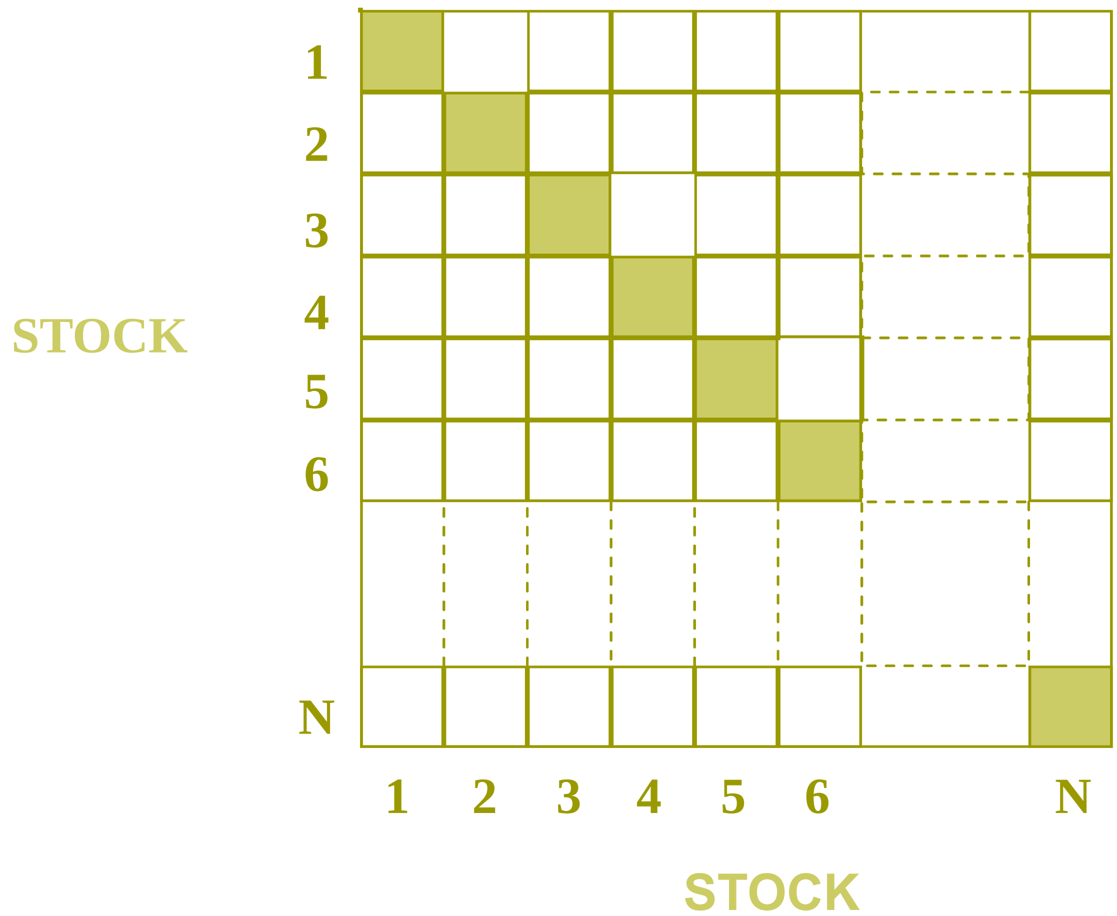
NEW Return = weighted avg = Portfolio = 18.20%

NOTE: Higher return & Lower risk

How did we do that? DIVERSIFICATION

Portfolio Risk

The shaded boxes contain variance terms; the remainder contain covariance terms.



To calculate portfolio variance add up the boxes

Portfolio Risk

- Limits to Diversification

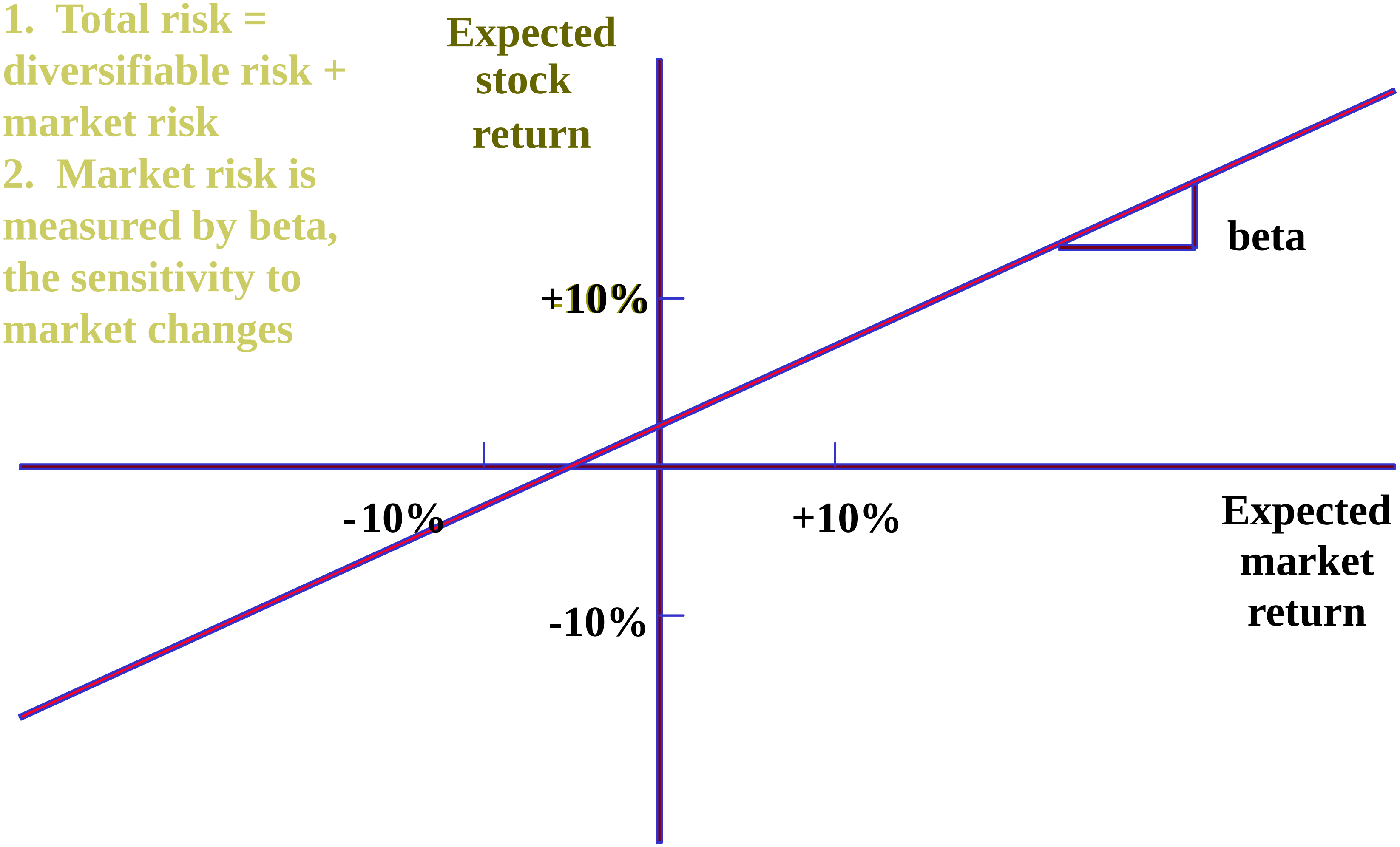
$$\text{Portfolio Variance} = N\left(\frac{1}{N}\right)^2 \times \text{average variance}$$

$$+(N^2 - N)\left(\frac{1}{N}\right)^2 \times \text{average covariance}$$

$$= \left(\frac{1}{N}\right) \times \text{average variance} + (1 - \frac{1}{N}) \times \text{average covariance}$$

Beta and Unique Risk

1. Total risk = diversifiable risk + market risk
2. Market risk is measured by beta, the sensitivity to market changes



Beta and Unique Risk

Market Portfolio - Portfolio of all assets in the economy. In practice a broad stock market index, such as the S&P Composite, is used to represent the market.

Beta - Sensitivity of a stock's return to the return on the market portfolio.

Beta and Unique Risk

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2}$$

Covariance with the market

Variance of the market

Beta

Calculating the variance of the market returns and the covariance between the returns on the market and those of Anchovy Queen. Beta is the ratio of the variance to the covariance (i.e., $\beta = \sigma_{im}/\sigma_m^2$)

(1) Month	(2) Market return	(3) Anchovy Q return	(4) Deviation from average market return	(5) Deviation from average Anchovy Q return	(6) Squared deviation from average market return	(7) Product of deviations from average returns (cols 4 x 5)
1	-8%	-11%	-10%	-13%	100	130
2	4	8	2	6	4	12
3	12	19	10	17	100	170
4	-6	-13	-8	-15	64	120
5	2	3	0	1	0	0
6	8	6	6	4	36	24
Average	2	2		Total	304	456

Variance = σ_m^2 = 304/6 = 50.67

Covariance = σ_{im} = 456/6 = 76

Beta (β) = σ_{im}/σ_m^2 = 76/50.67 = 1.5

Topics Covered

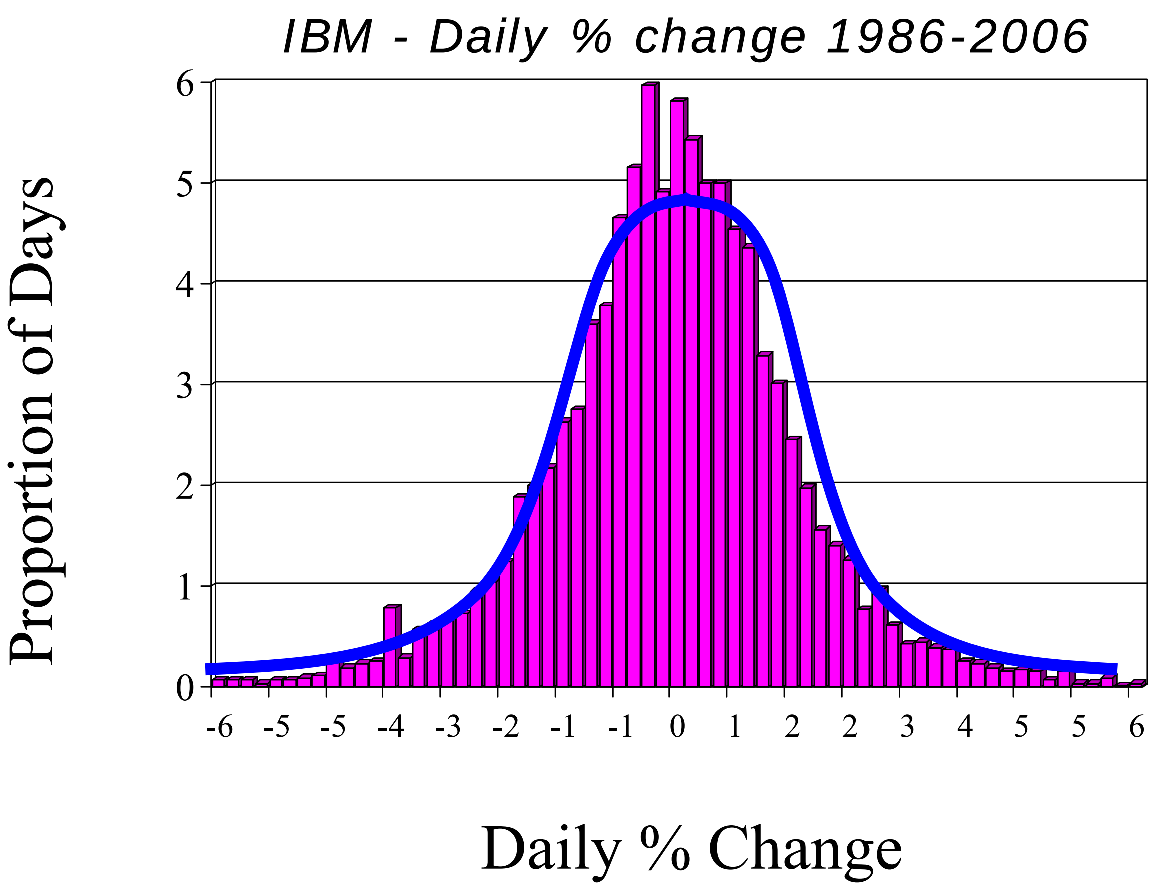
- Markowitz Portfolio Theory
- The Relationship Between Risk and Return
- Validity and the Role of the CAPM
- Some Alternative Theories

Markowitz Portfolio Theory

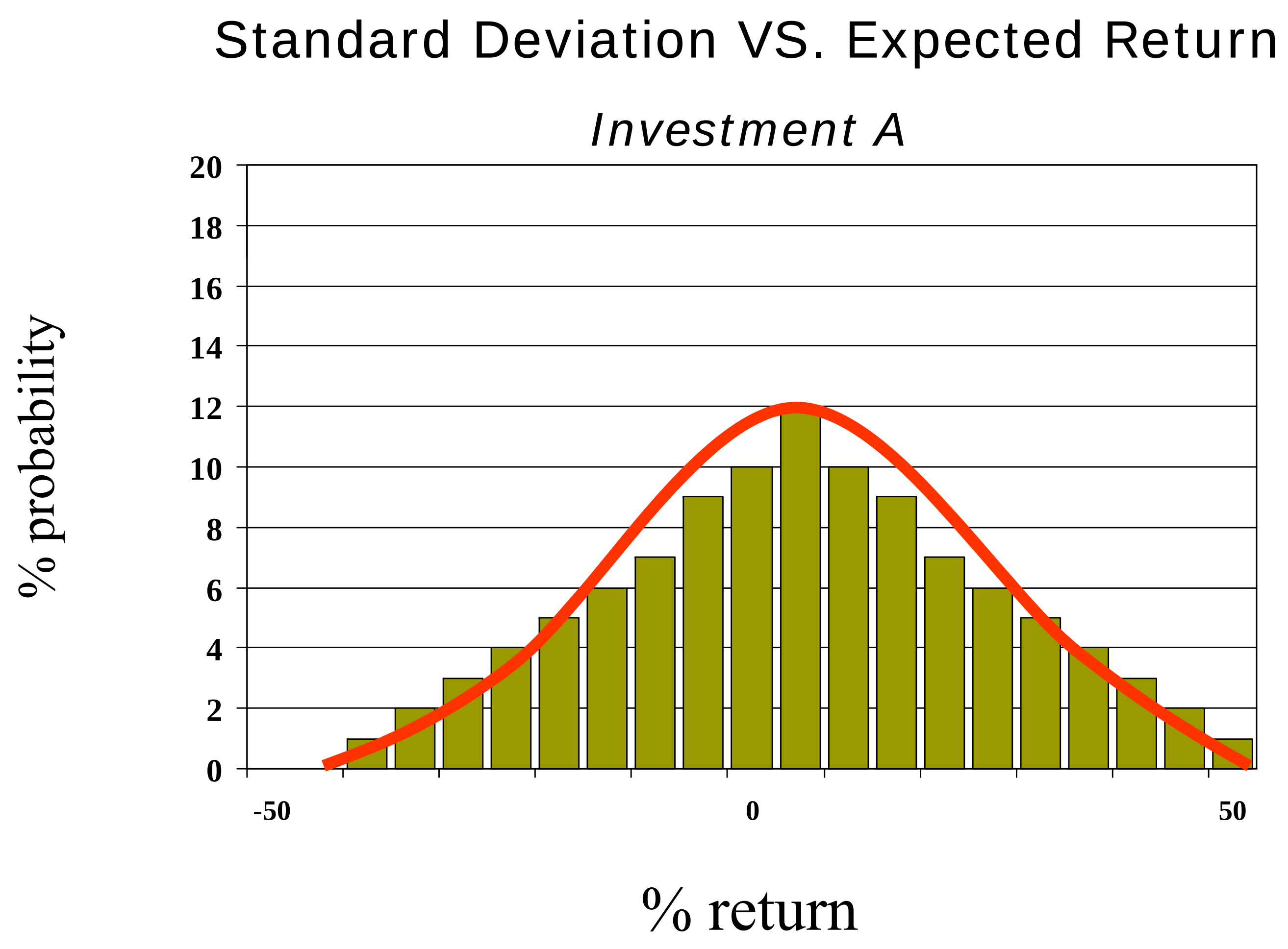
- Combining stocks into portfolios can reduce standard deviation, below the level obtained from a simple weighted average calculation.
- Correlation coefficients make this possible (see the expression in terms of *N*).
- The various weighted combinations of stocks that create this standard deviations constitute the set of ***efficient portfolios***.

Markowitz Portfolio Theory

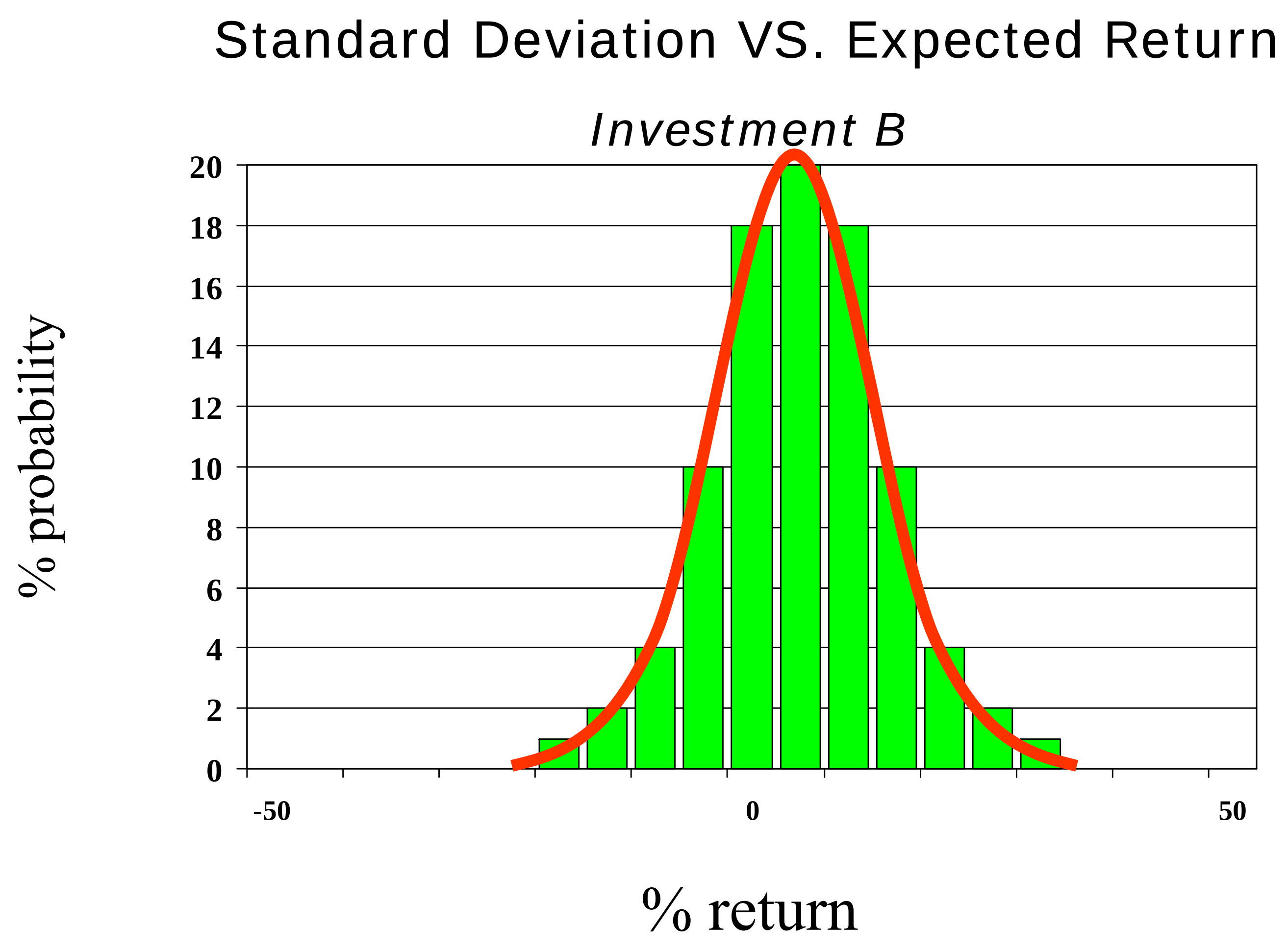
Price changes vs. Normal distribution



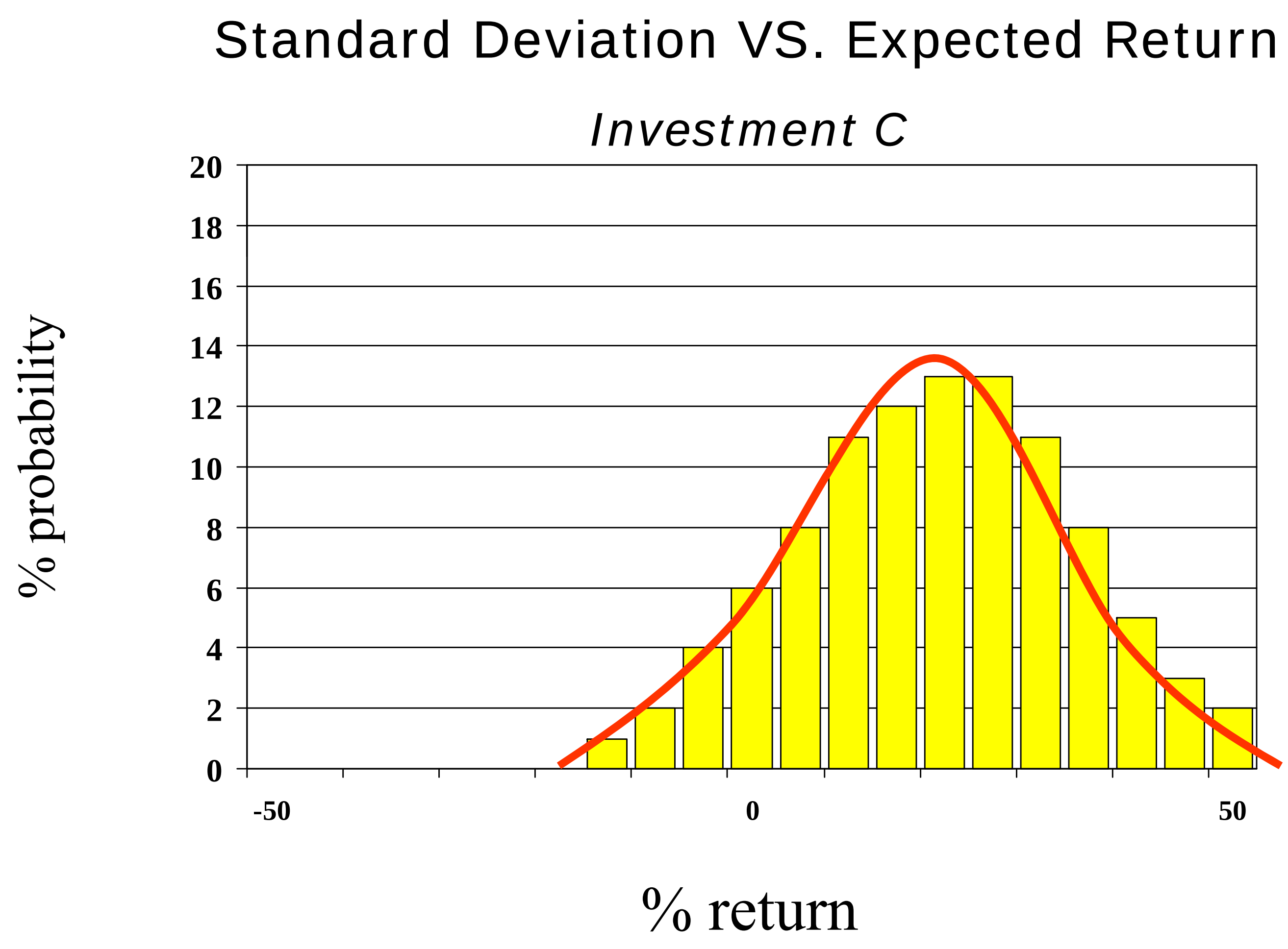
Markowitz Portfolio Theory



Markowitz Portfolio Theory



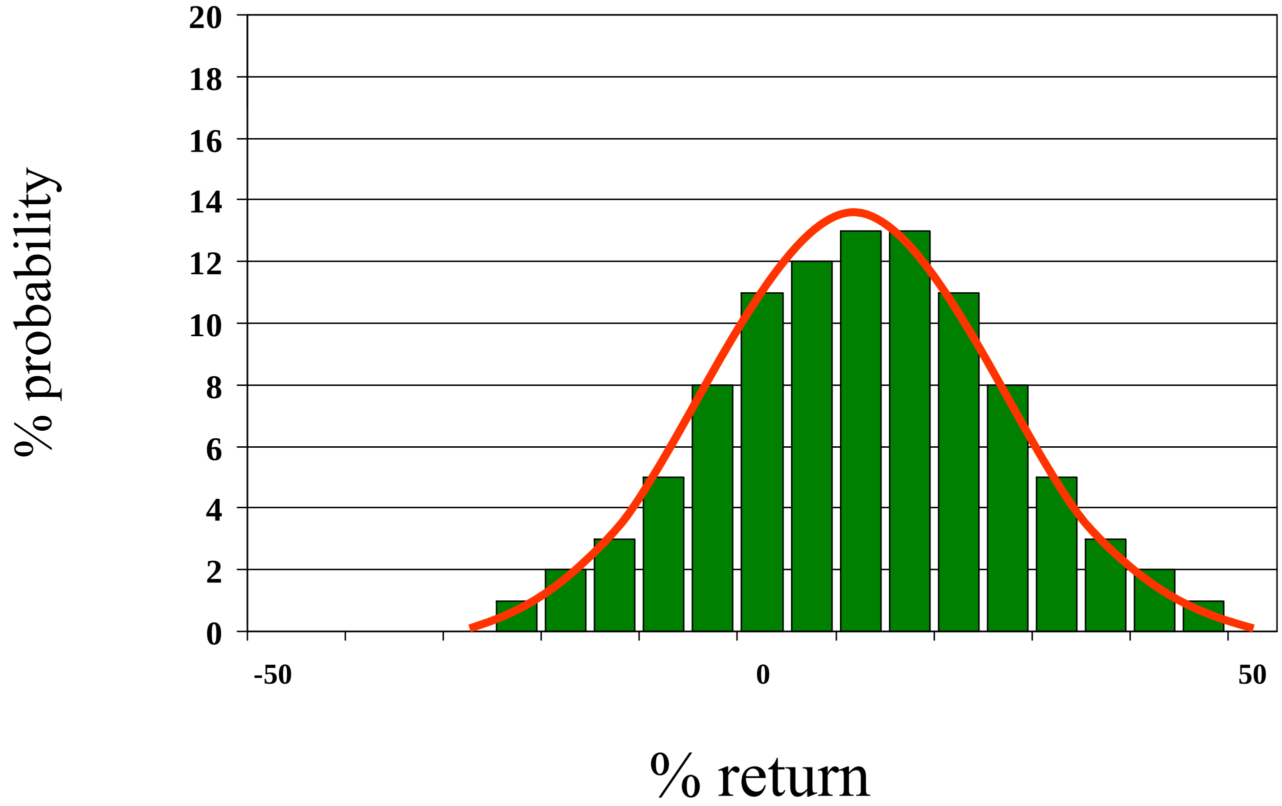
Markowitz Portfolio Theory



Markowitz Portfolio Theory

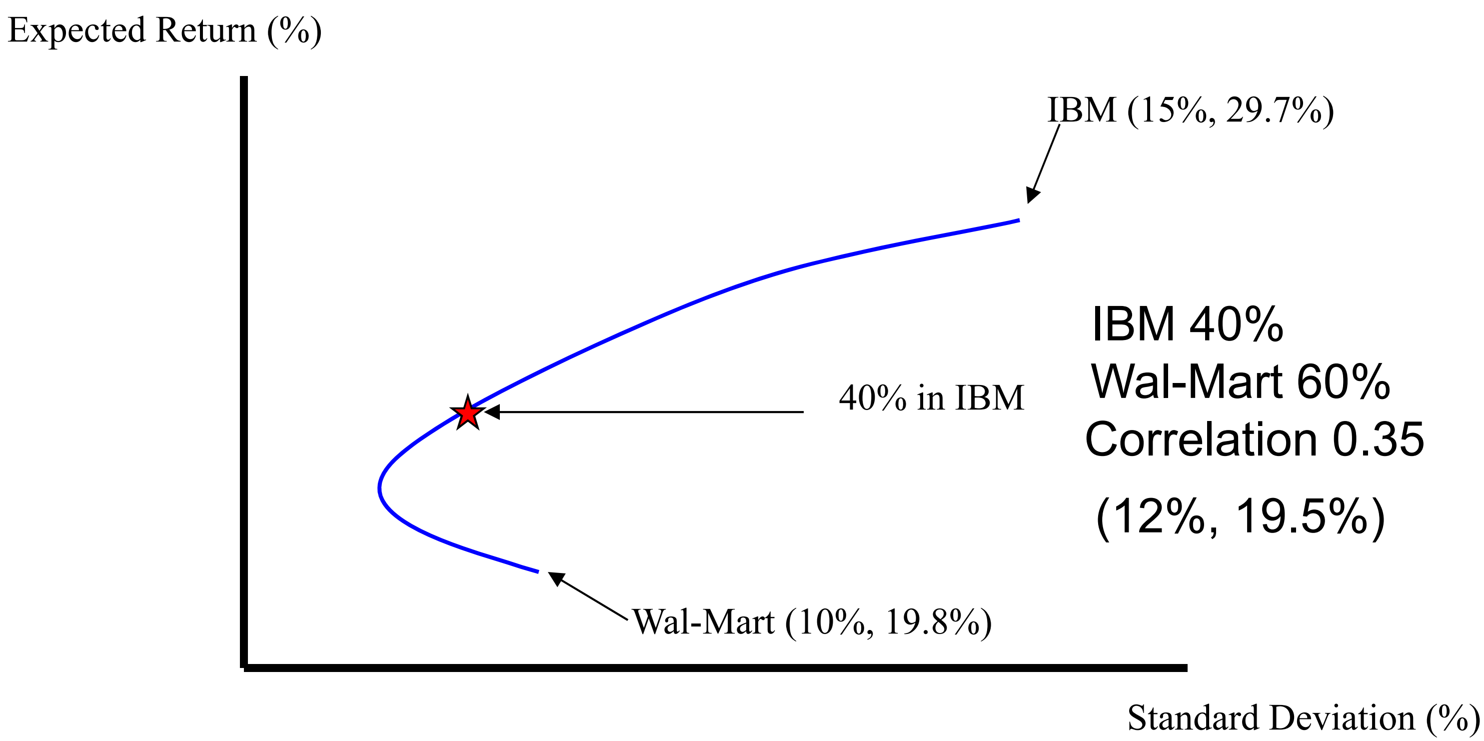
Standard Deviation VS. Expected Return

Investment D



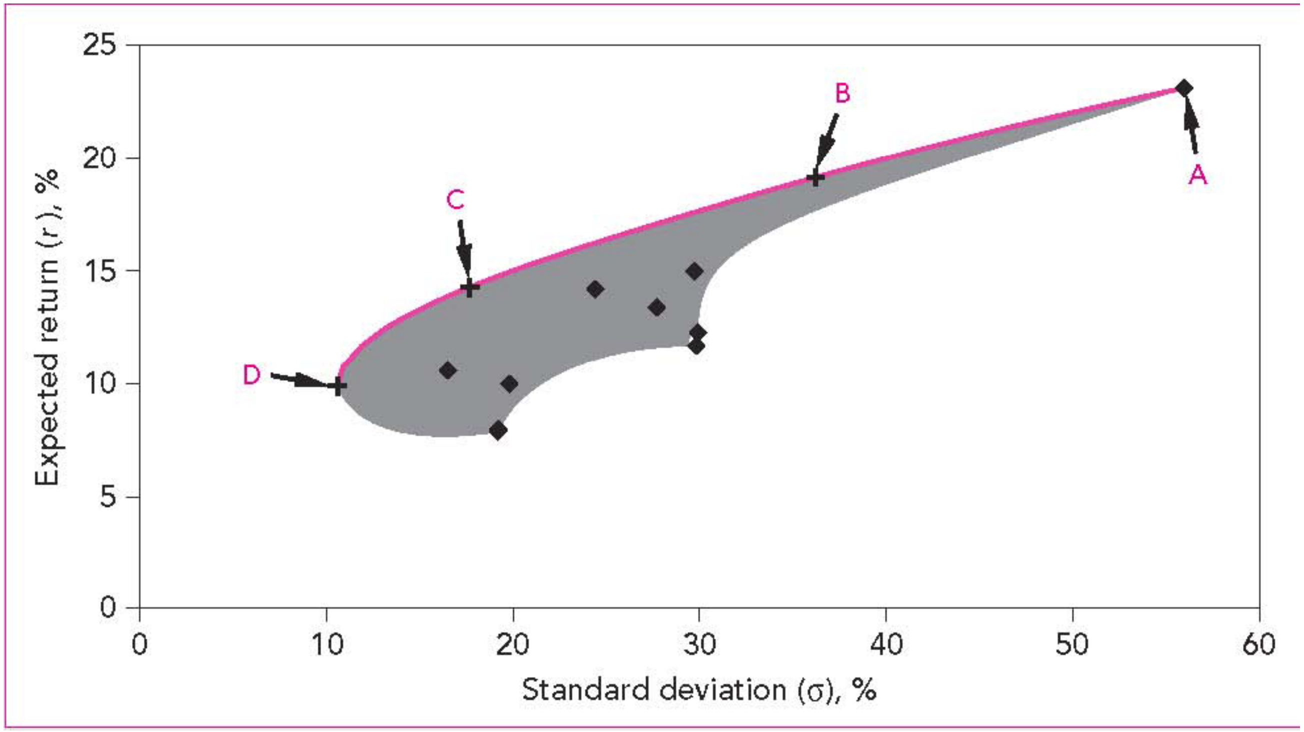
Markowitz Portfolio Theory

- Expected Returns and Standard Deviations vary given different weighted combinations of the stocks



Efficient Frontier

4 Efficient Portfolios all from the same 10 stocks

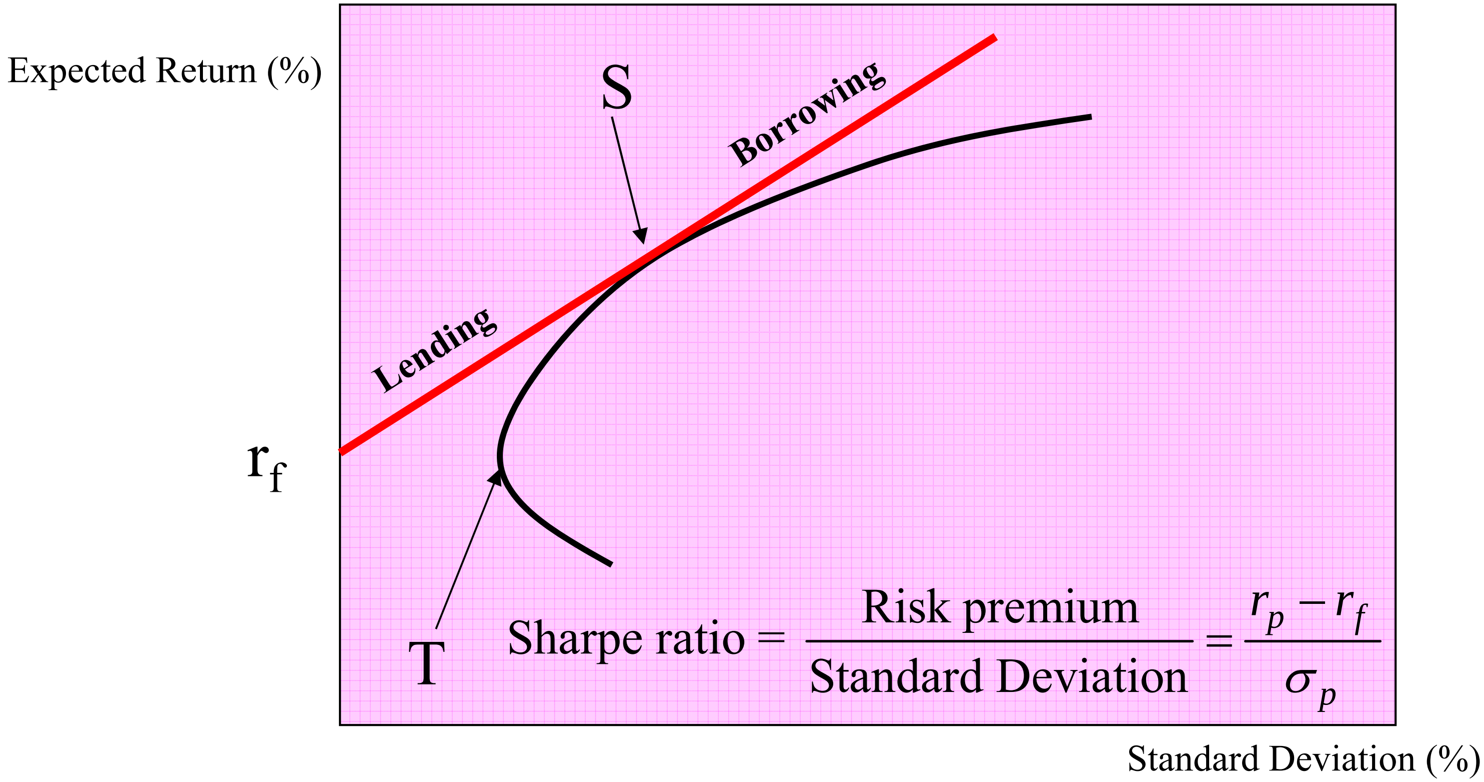


Efficient Frontier

- Portfolio *S* (15%, 16%)
- Treasury bills (5% = risk free rate)
- Lending:
- 50% on *S* & 50% Treasury bills
  - Expected return =10%
  - Std dev. = 8%
- Borrowing at treasury bill rate (equal to your initial wealth):
- Invest 100% on *S*
  - Expected return = 25%
  - Std dev. = 32%

Efficient Frontier

•Lending or Borrowing at the risk free rate (r_f) allows us to exist outside the efficient frontier.



$$\text{Sharpe ratio} = \frac{\text{Risk premium}}{\text{Standard Deviation}} = \frac{r_p - r_f}{\sigma_p}$$

Efficient Frontier

Previous Example — Correlation Coefficient = .4

Stocks	σ	% of Portfolio	Avg Return
ABC Corp	28	60%	15%
Big Corp	42	40%	21%

Standard Deviation of the Portfolio = 28.1

Return of the Portfolio = 17.4%

Efficient Frontier

Previous Example Correlation Coefficient = .4

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Let's Add stock New Corp to the portfolio

Efficient Frontier

Previous Example — Correlation Coefficient = .3

Stocks	σ	% of Portfolio	Avg Return
Portfolio	28.1	50%	17.4%
New Corp	*30*	*50%*	*19%*

NEW Standard Deviation of the Portfolio = 23.43

NEW Return of the Portfolio = 18.20%

Efficient Frontier

<u>Previous Example</u> Correlation Coefficient = .3

Stocks	σ	% of Portfolio	Avg Return
Portfolio	28.1	50%	17.4%
New Corp	30	50%	19%

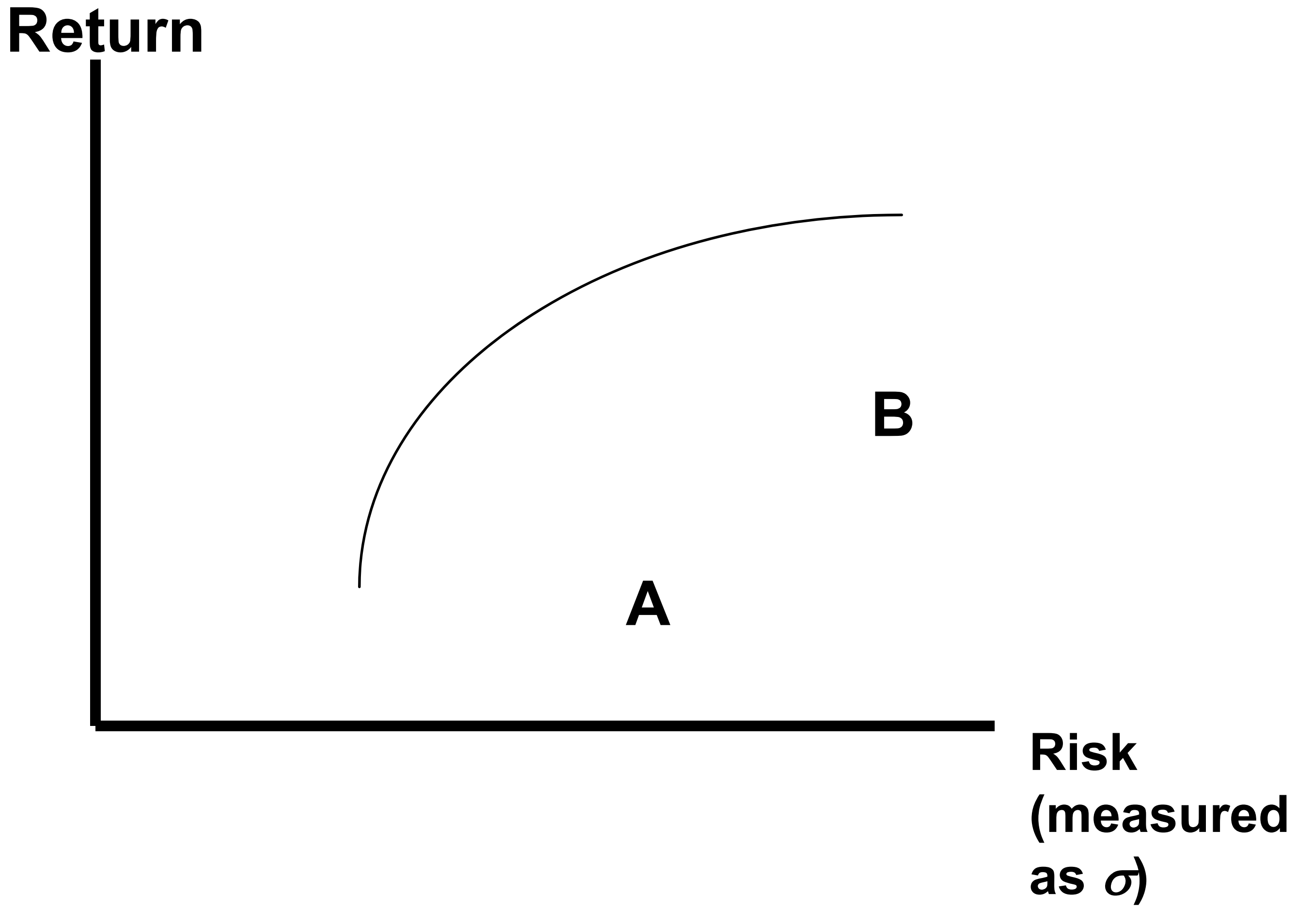
NEW Standard Deviation = Portfolio = <u>23.43</u>

NEW Return = weighted avg = Portfolio = <u>18.20%</u>

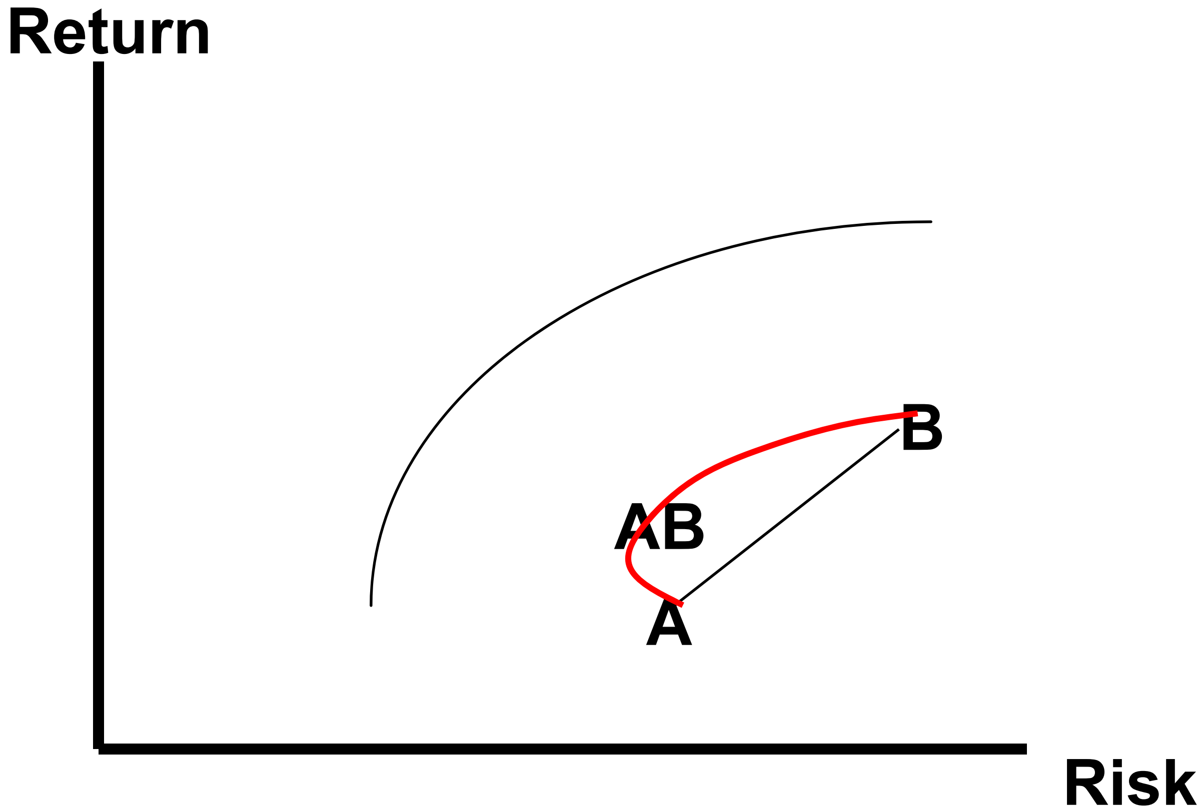
NOTE: Higher return & Lower risk

How did we do that? <u>DIVERSIFICATION</u>

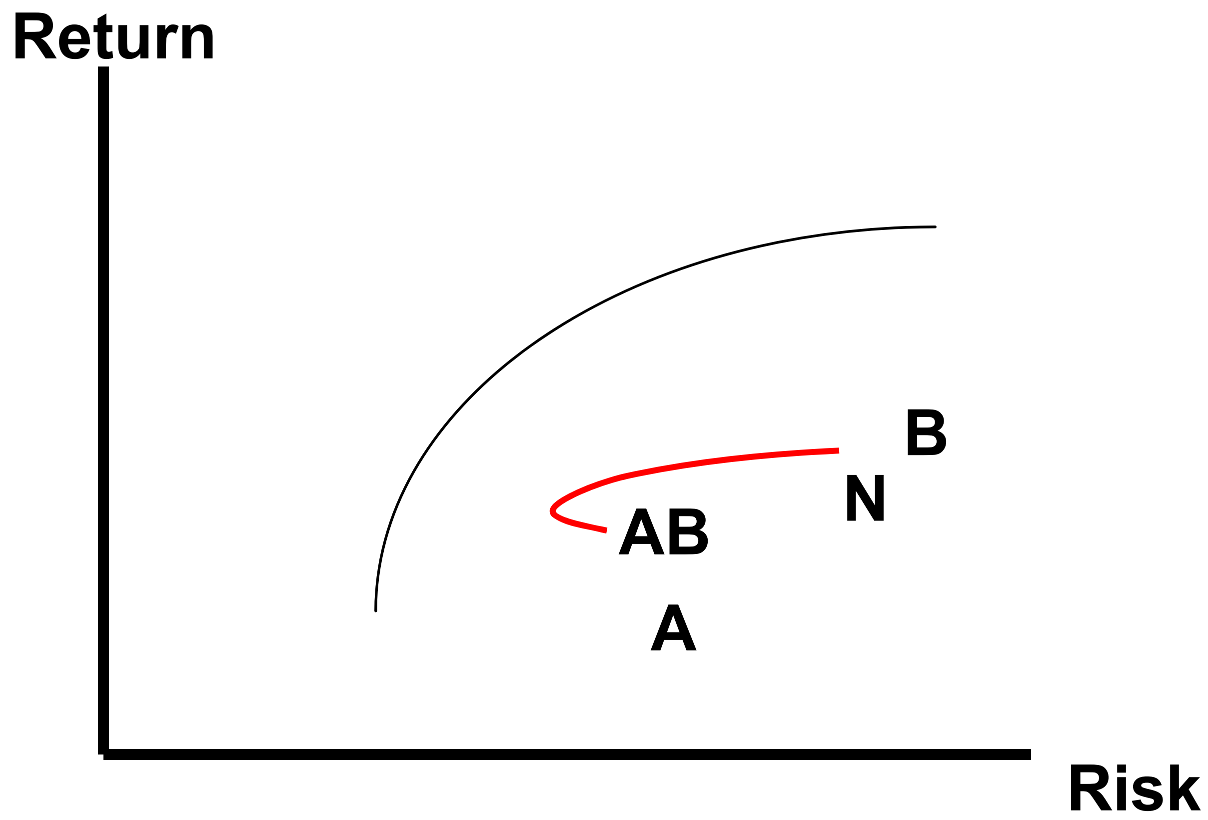
Efficient Frontier



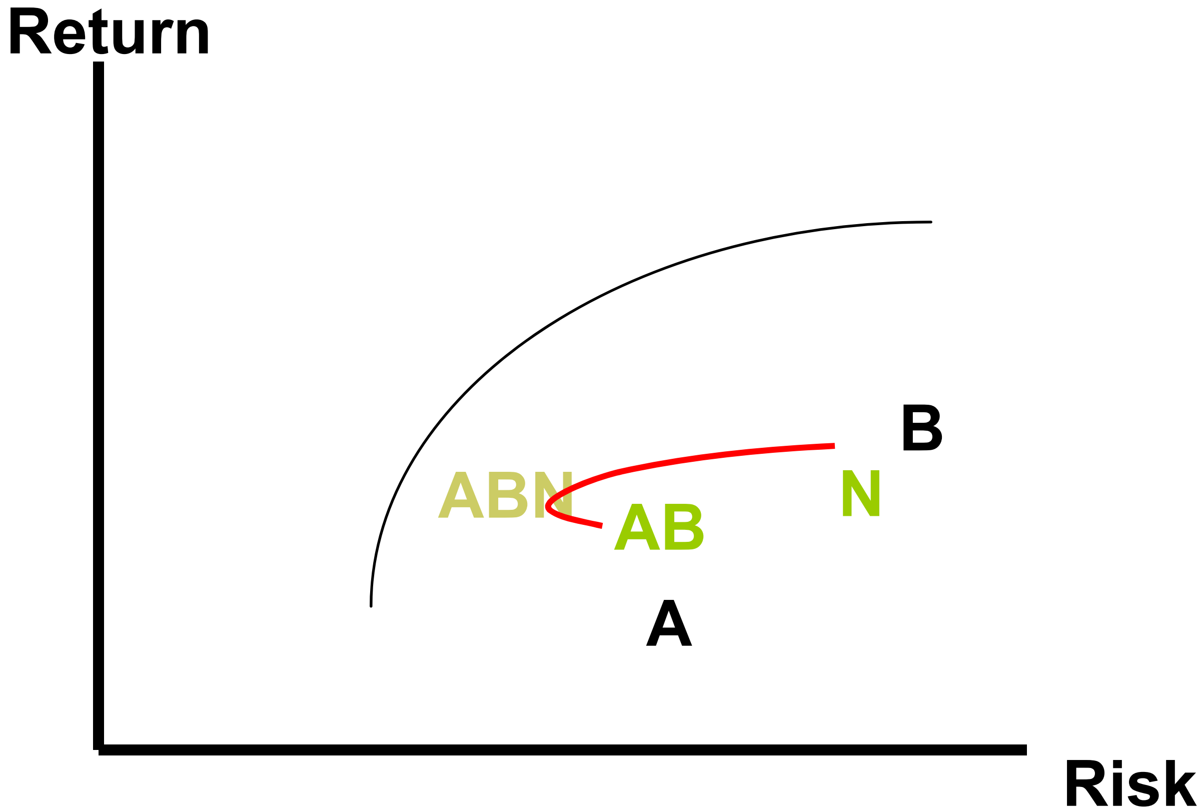
Efficient Frontier



Efficient Frontier



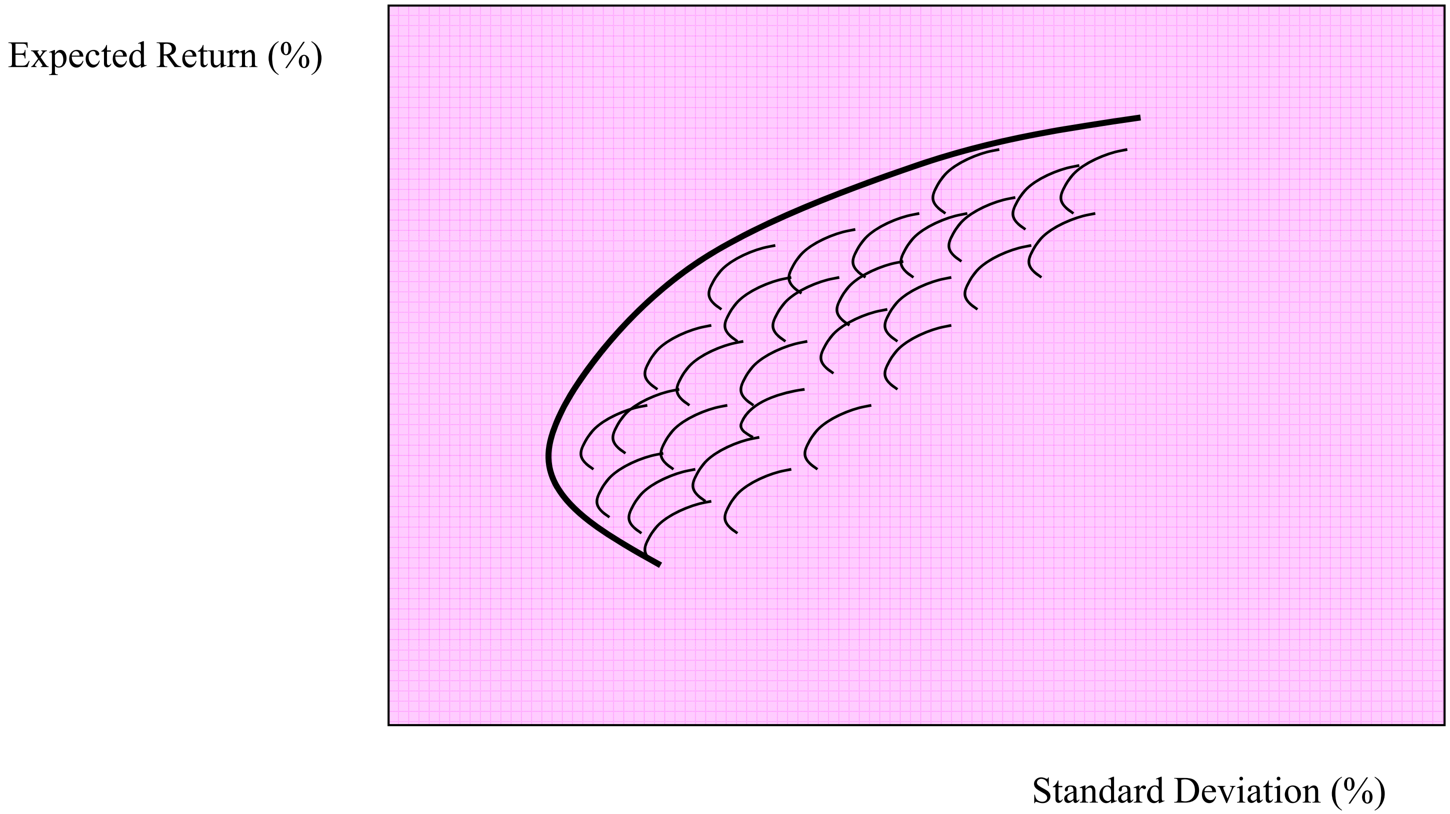
Efficient Frontier



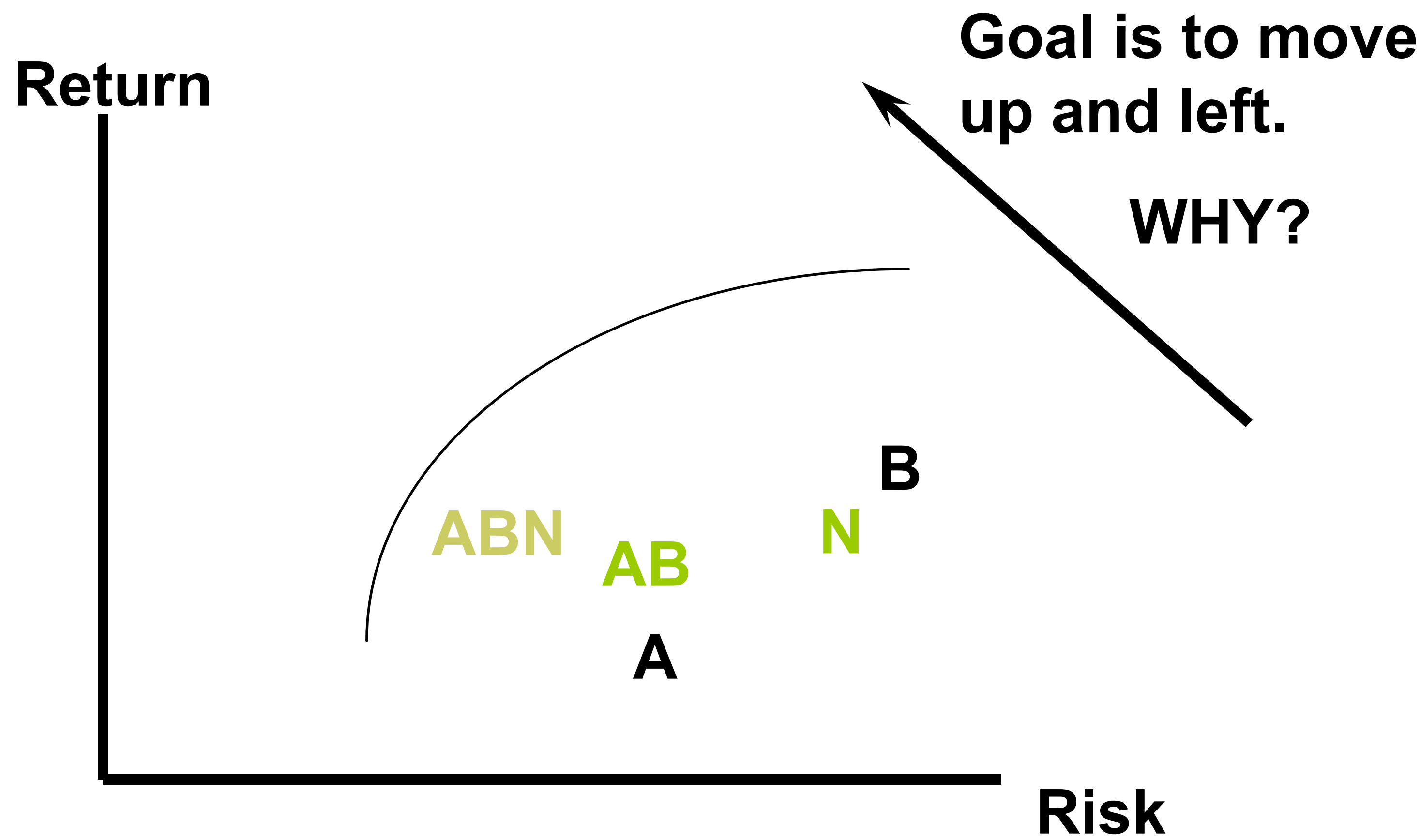
Efficient Frontier

•Each half egg shell represents the possible weighted combinations for two stocks.

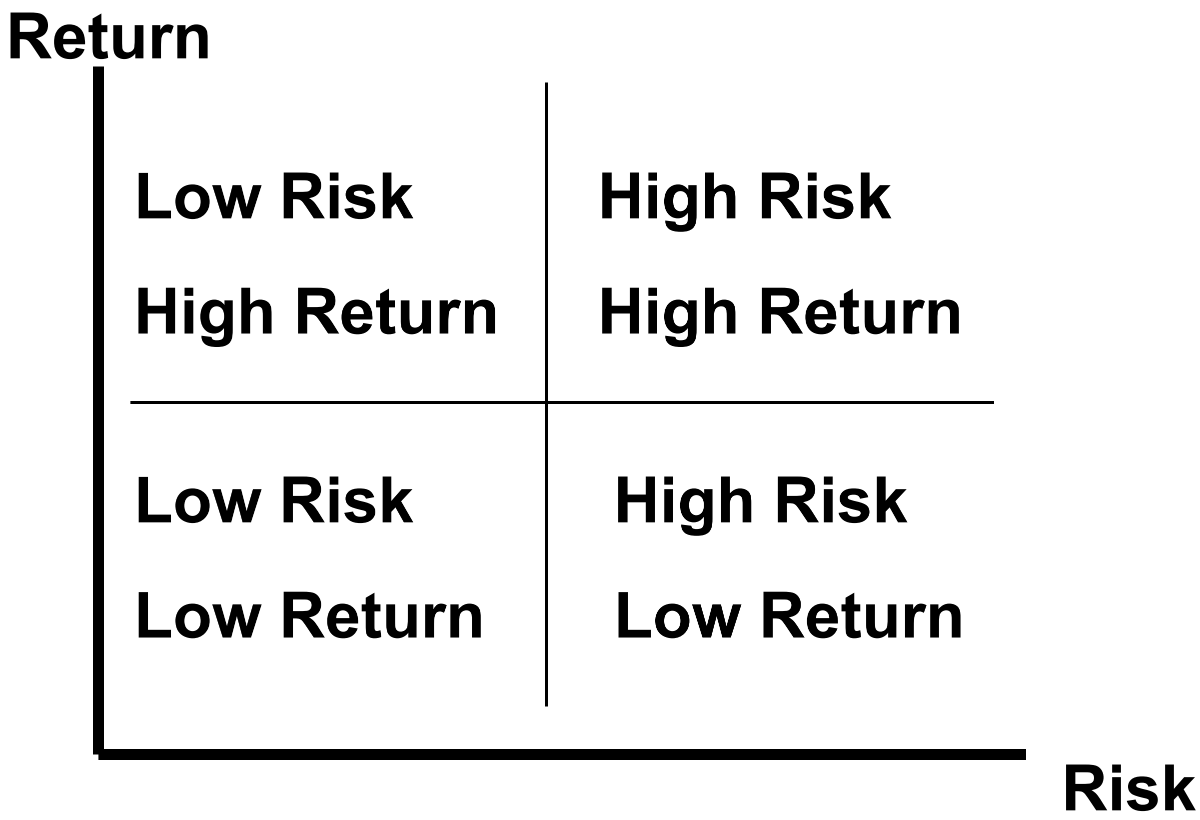
•The composite of all stock sets constitutes the efficient frontier



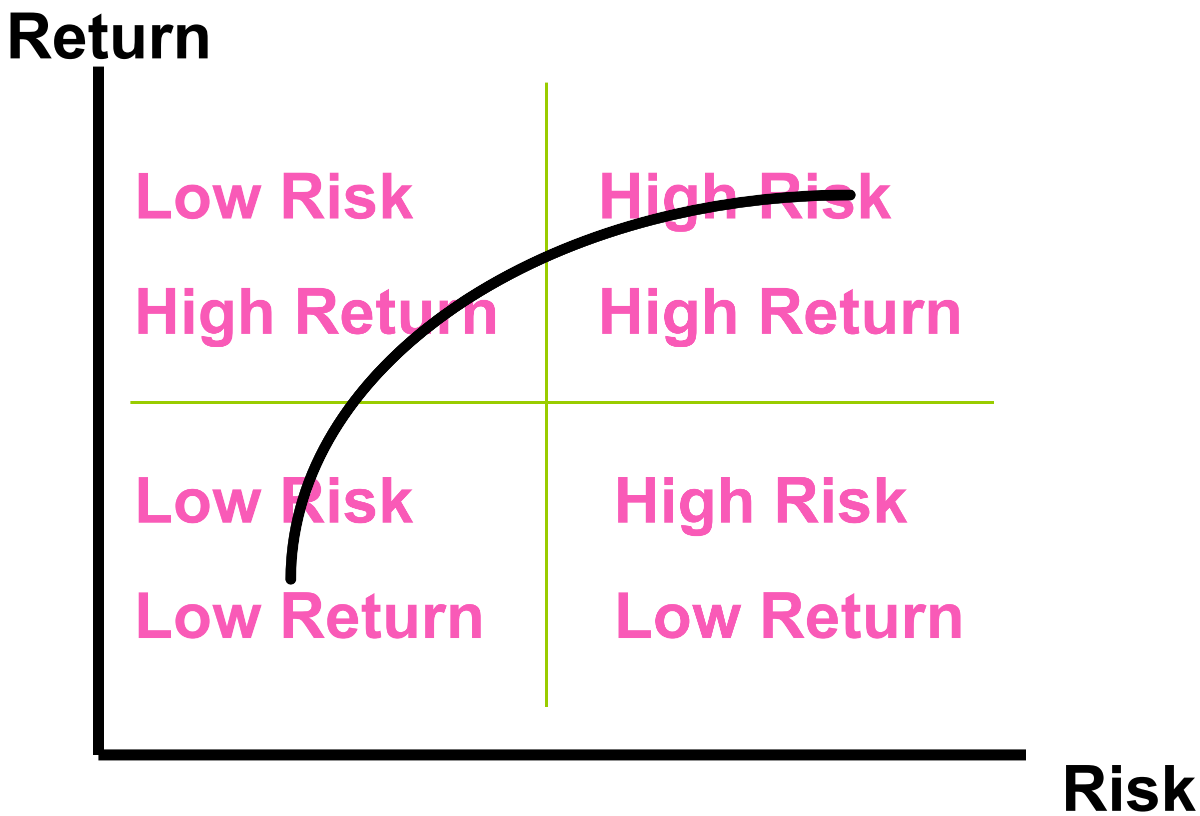
Efficient Frontier



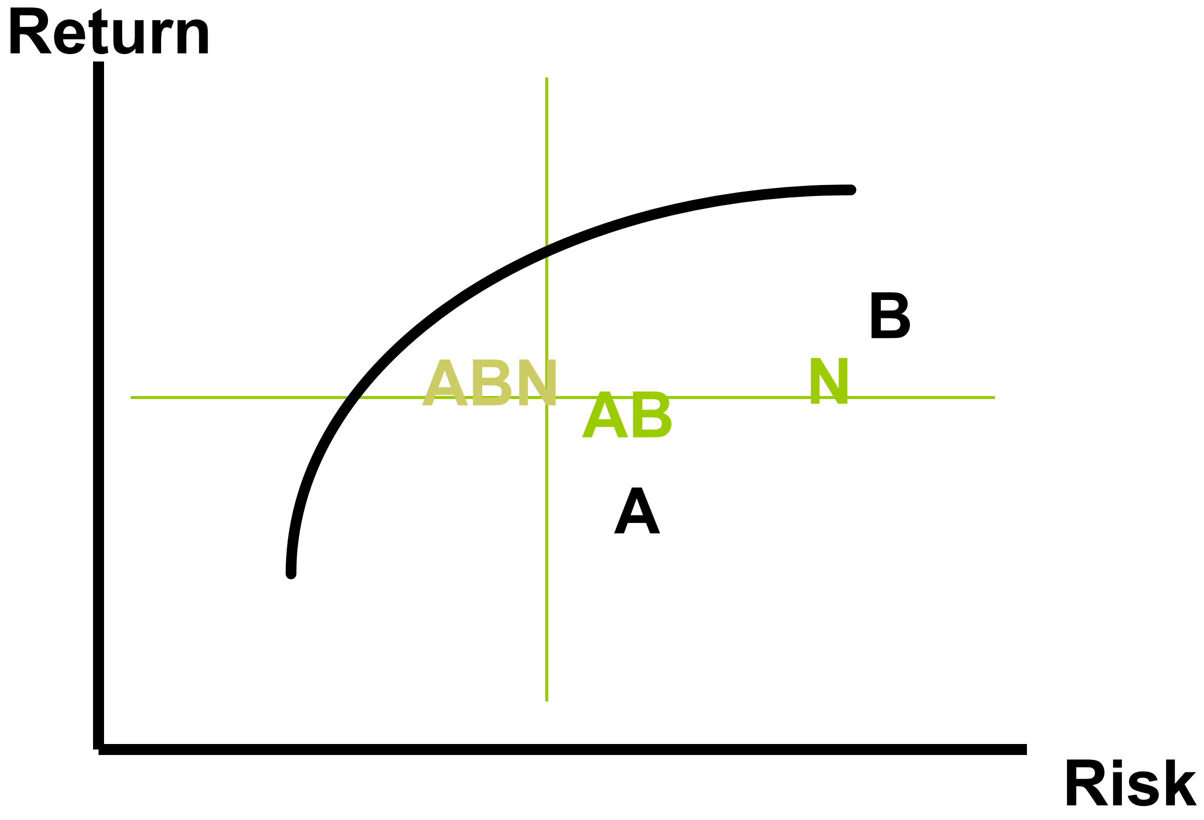
Efficient Frontier



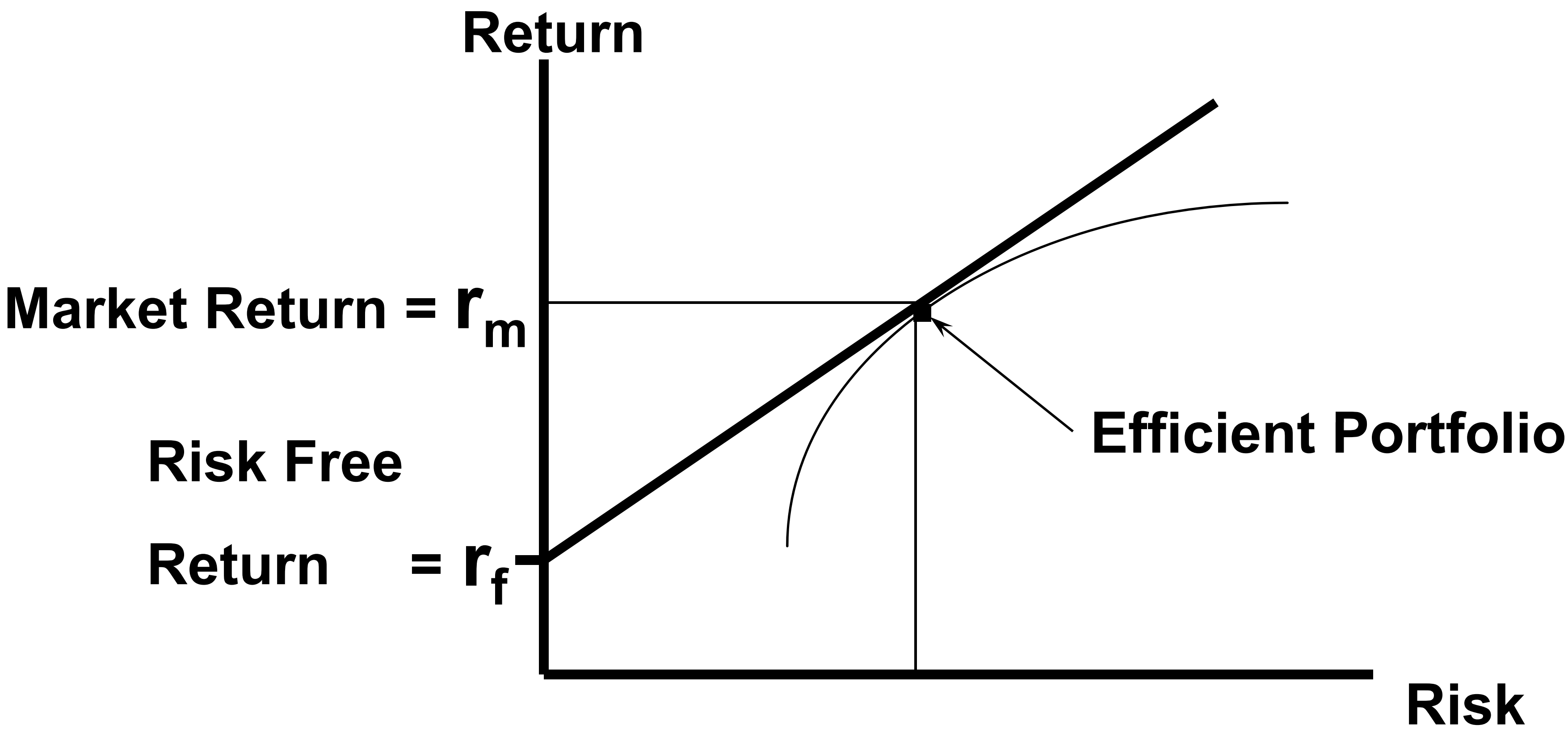
Efficient Frontier



Efficient Frontier



Security Market Line



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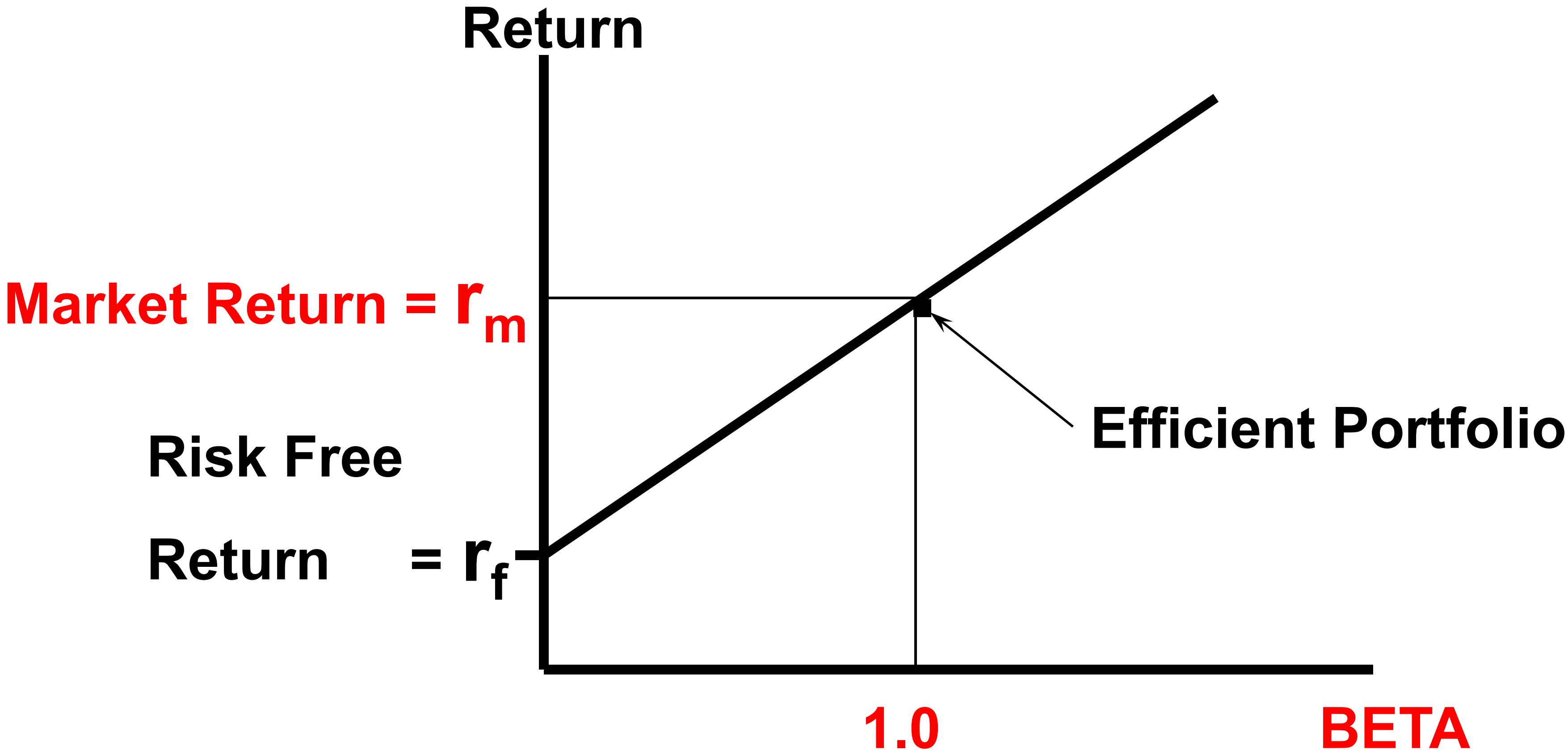

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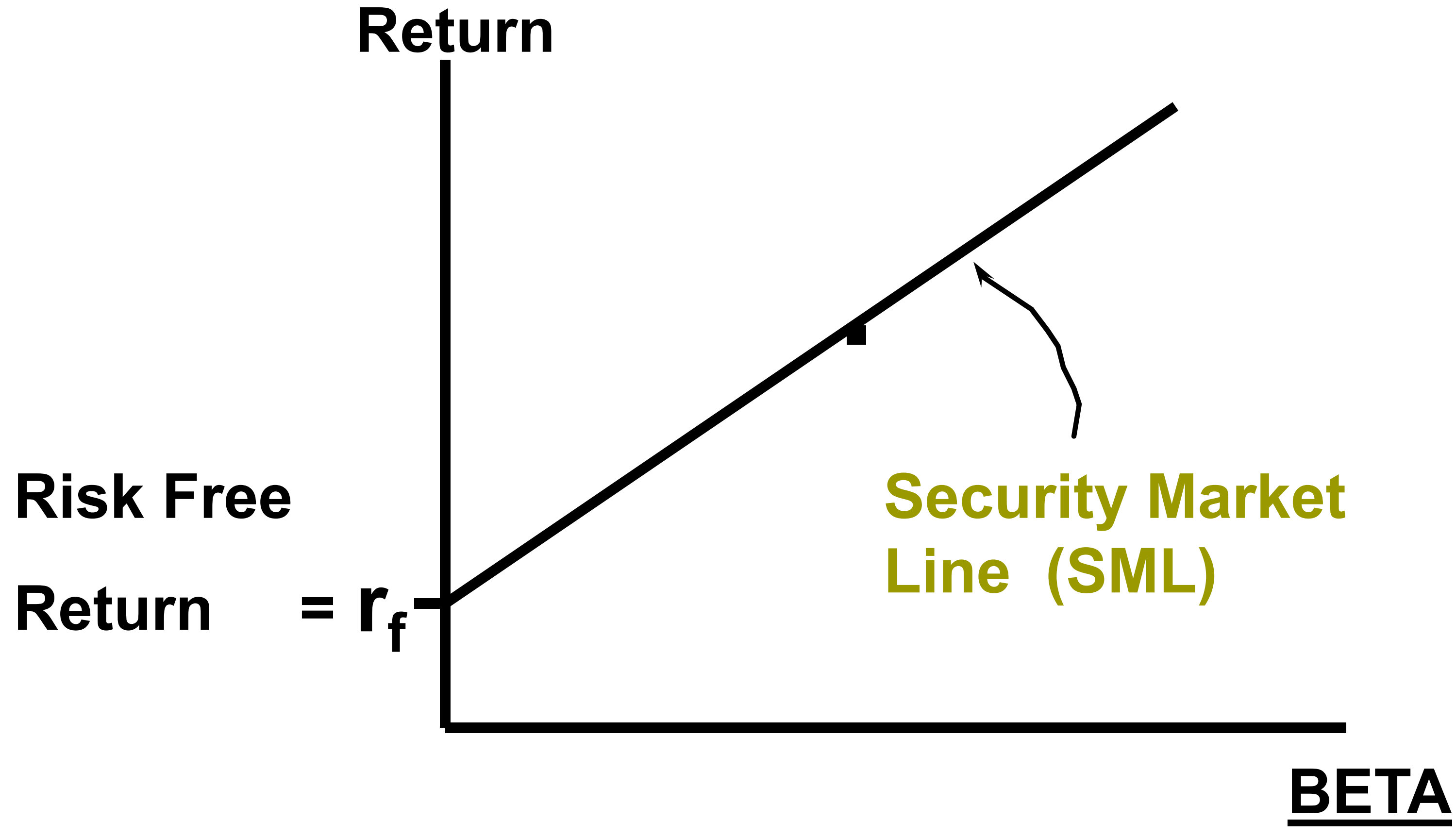

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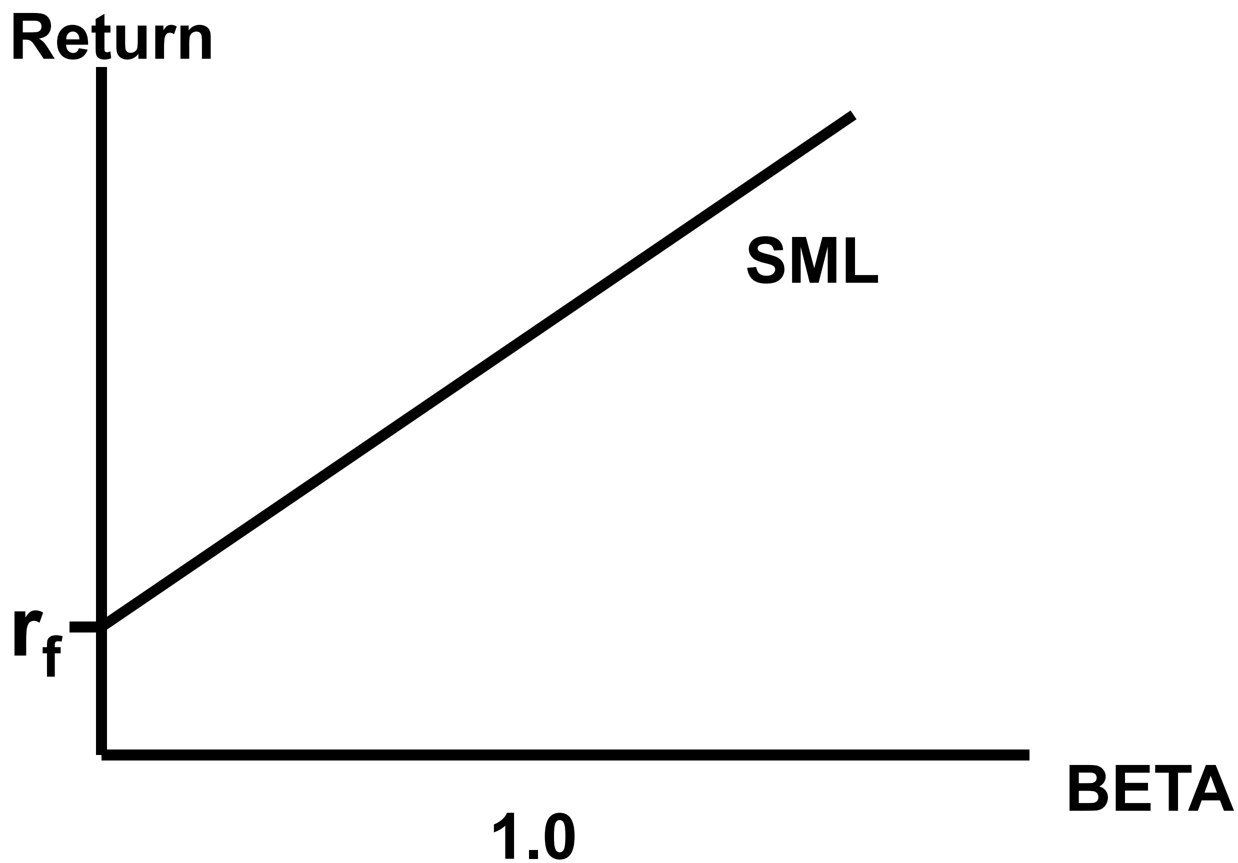

SML Equation = $r_f + \beta\,(r_m - r_f)$

Capital Asset Pricing Model

$$R = r_f + \beta (r_m - r_f)$$

CAPM

Arbitrage Pricing Theory

Alternative to CAPM

$$\text{Return} = \text{noise} + b_{factor1}(r_{factor1}) + b_{f2}(r_{f2}) + \ldots$$

$r_{factor1}$, $r_{factor2}$ ..... are macroeconomic factors
E.g., oil price, interest rate, etc

Risk from macroeconomic can not eliminated
Risk from noise (unique risk) can be eliminated by diversification

Arbitrage Pricing Theory

<u>Alternative to CAPM</u>

Return = noise + $b_{factor1}(r_{factor1}) + b_{f2}(r_{f2}) + \ldots$

Expected Risk

Premium = $r - r_f$

$= b_{factor1}(r_{factor1} - r_f) + b_{f2}(r_{f2} - r_f) + \ldots$

Arbitrage Pricing Theory

Alternative to CAPM

Expected Risk

Premium = $r - r_f$

$= b_{factor1}(r_{factor1} - r_f) + b_{f2}(r_{f2} - r_f) + \ldots$

- If all $b_i = 0$, then portfolio has zero sensitivity
- If the portfolio offered higher return, borrow at risk-free and invest on portfolio
- If the portfolio offered lower return, sell the portfolio and invest on Treasury bills

Arbitrage Pricing Theory

If only $b_1 > 0$,

Expected Risk Premium of Portfolio A

$$r_A - r_f = 2\, b_1(r_{factor1} - r_f)$$

Expected Risk Premium of Portfolio B

$$r_B - r_f = b_1(r_{factor1} - r_f)$$

$\frac{1}{2}(r_A) - \frac{1}{2}(r_f) = r_B - r_f$, i.e., you divide your money equally between treasury bills and portfolio A, then that portfolio has the same sensitivity as the portfolio B. Otherwise, there would be arbitrage.

Arbitrage Pricing Theory

Estimated risk premiums for taking on risk factors (1978-1990)

Factor	Estimated Risk Premium $(r_{\text{factor}} - r_f)$
Yield spread	5.10%
Interest rate	-.61
Exchange rate	-.59
Real GNP	.49
Inflation	-.83
Market	6.36