Interest rates

Types of rates

https://www.youtube.com/watch?v=sx6ttA-kmos
https://www.youtube.com/watch?v=-dV4OFW6BbY

- **Treasury rates**: Rates on instruments issued by a government in its own currency
- **LIBOR and LIBID rates**:
- LIBOR (London Interbank Offer Rate) is the rate of interest at which a bank is prepared to deposit money with another bank. (The second bank must typically have a AA rating)
- LIBOR is compiled once a day by the British Bankers Association on all major currencies for maturities up to 12 months
- LIBID (London Interbank Bid Rate) is the rate which a AA bank is prepared to pay on deposits from anther bank
- **Eurocurrency market**: Interbank market

Example

- **LIBOR rates**: large international banks actively trade with each other 1-month, 3-month, 6-month, and 12-month deposits dominated in all of the world’s major currencies. E.g., Citibank might quote a bid rate of 6.25% and an offer rate of 6.375% to other banks for 6-month deposits in Australian dollars. It is prepared to pay 6.25% per year on 6-month deposits from other bank; advance deposits to another bank at and 6.375% per year.

The risk-free rate

- The short-term risk-free rate traditionally used by derivatives practitioners is LIBOR
- The Treasury rate is considered to be artificially low for a number of reasons

Measuring interest rates

- The compounding frequency used for an interest rate is the unit of measurement
- Impact of compounding:

When we compound m times per year at rate R an amount A grows to $A(1+r/m)^m$ in one year

Compounding frequency	*Value of $100 in one year at 10%*
Annual (m=1)	110.00
Semiannual (m=2)	110.25
Quarterly (m=4)	110.38
Monthly (m=12)	110.47
Weekly (m=52)	110.51
Daily (m=365)	110.52

Continuous compounding

- In the limit as we compound more and more frequently we obtain continuously compounded interest rates
- \$100 grows to \$100e^{rT} when invested at a continuously compounded rate *r* for time *T*
- \$100 received at time *T* discounts to \$100e^{-rT} at time zero when the continuously compounded discount rate is *r*

Conversion formulas

Define

r_c : continuously compounded rate

r_m: same rate with compounding *m* times per year

$$r_c = m \ln\left(1 + \frac{r_m}{m}\right)$$

$$r_m = m\left(e^{r_c/m} - 1\right)$$

Examples

- 10% with semiannual compounding is equivalent to $2\ln(1.05)=9.758\%$ with continuous compounding
- 8% with continuous compounding is equivalent to $4(e^{0.08/4}-1)=8.08\%$ with quarterly compounding
- Rates used in option pricing are nearly always expressed with continuous compounding

Zero Rates (zero-coupon rates)

A zero rate (or spot rate), for maturity *T*, is the rate of interest earned on an investment that provides a payoff only at time *T*.

Suppose the 5-year Treasury zero rate with continuous compounding is quoted as 5% per annum. This means that $100, if invested at the risk-free rate for 5 years, would grow to

$$100e^{0.05\times5} = 128.40$$

Example

Maturity (years)	Zero Rate (% cont comp)
0.5	5.0
1.0	5.8
1.5	6.4
2.0	6.8

Bond Pricing

- To calculate the cash price of a bond we discount each cash flow at the appropriate zero rate
- In our example, the theoretical price of a two-year bond providing a 6% coupon semiannually is

$$3e^{-0.05\times 0.5} + 3e^{-0.058\times 1.0} + 3e^{-0.064\times 1.5}$$
$$+ 103e^{-0.068\times 2.0} = 98.39$$

Bond Yield

- The bond yield is **the discount rate** that makes the present value of the cash flows on the bond equal to the market price of the bond
- Suppose that the market price of the bond in our example equals its theoretical price of 98.39
- The bond yield is given by solving

$$3e^{-y\times 0.5}+3e^{-y\times 1.0}+3e^{-y\times 1.5}+103e^{-y\times 2.0}=98.39$$

to get y=0.0676 or 6.76%.

Par Yield

- The par yield for a certain maturity is **the coupon rate** that causes the bond price to equal its face value.
- In our example we solve

$$\frac{c}{2}e^{-0.05\times 0.5}+\frac{c}{2}e^{-0.058\times 1.0}+\frac{c}{2}e^{-0.064\times 1.5}$$
$$+\left(100+\frac{c}{2}\right)e^{-0.068\times 2.0}=100$$

to get $c = 6.87$ (with semiannual compounding)

Par Yield continued

In general if m is the number of coupon payments per year, d is the present value of $1 received at maturity and A is the present value of an annuity of $1 on each coupon date

$$c = \frac{(100 - 100d)m}{A}$$

Sample Data for Determining the Zero Curve

Bond Principal (dollars)	Time to Maturity (years)	Annual Coupon (dollars)	Bond Price (dollars)
100	0.25	0	97.5
100	0.50	0	94.9
100	1.00	0	90.0
100	1.50	8	96.0
100	2.00	12	101.6

The Bootstrapping the Zero Curve

- An amount 2.5 can be earned on 97.5 during 3 months.
- The 3-month rate is 4 times 2.5/97.5 or 10.256% with quarterly compounding
- This is 10.127% with continuous compounding
- Similarly the 6 month and 1 year rates are 10.469% and 10.536% with continuous compounding

The Bootstrap Method continued

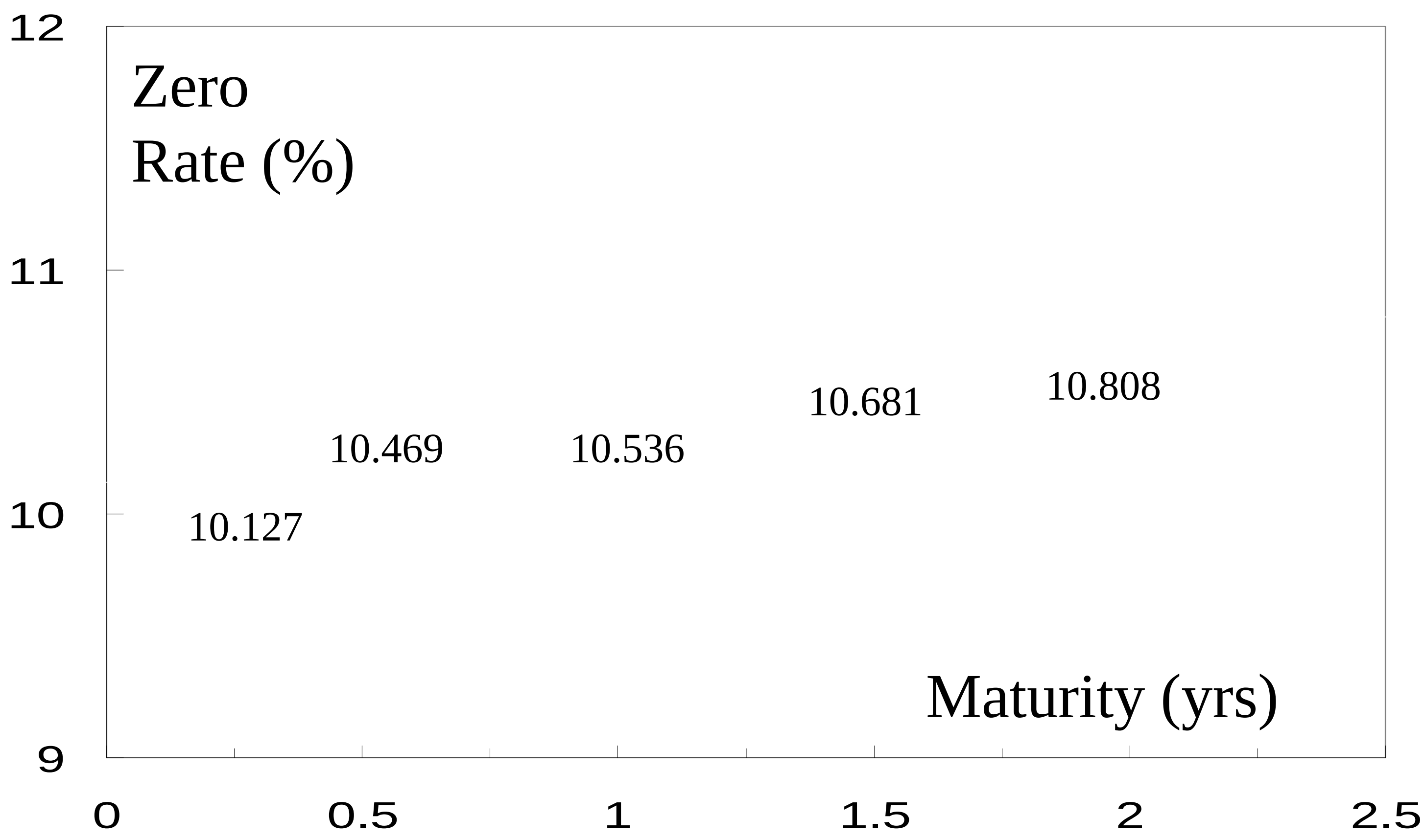
- To calculate the 1.5 year rate we solve

$$4e^{-0.10469\times 0.5}+4e^{-0.10536\times 1.0}+104e^{-R\times 1.5}=96$$

to get R = 0.10681 or 10.681%

- Similarly the two-year rate is 10.808%

Zero Curve Calculated from the Data (Figure 5.1, page 98)



Forward Rates

The forward rate is the future zero rate implied by today's term structure of interest rates

Calculation of Forward Rates

Year (n)	Zero Rate for an n-year Investment (% per annum)	Forward Rate for nth Year (% per annum)
1	10.0	
2	10.5	11.0
3	10.8	11.4
4	11.0	11.6
5	11.1	11.5

Formula for Forward Rates

- Suppose that the zero rates for maturities T_1 and T_2 are R_1 and R_2 with both rates continuously compounded.
- The forward rate for the period between times T_1 and T_2 is

$$\frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$$

Instantaneous Forward Rate

- The instantaneous forward rate for a maturity T is the forward rate that applies for a very short time period starting at T. It is

$$R + T\frac{\partial R}{\partial T}$$

where R is the T-year rate

Upward vs Downward Sloping Yield Curve

- For an upward sloping yield curve:
Fwd Rate > Zero Rate > Par Yield

- For a downward sloping yield curve
Par Yield > Zero Rate > Fwd Rate

Forward Rate Agreement

- A forward rate agreement (FRA) is an agreement that a certain rate will apply to a certain principal during a certain future time period