Discounted cash flow analysis

Introduction to valuation

- Valuation is the process of estimating what something is worth.
- Valuations are needed for many reasons such as investment analysis, capital budgeting, financial reporting, etc.
- Items that are usually valued are a financial asset or liability.
- Valuations can be done on assets (for example, investments in marketable securities such as stocks, options, business enterprises
- Valuation can be done on liabilities (e.g., bonds issued by a company).

Example

Suppose that you come across a vacant lot that you can buy for $50,000 and you are seeking an advice from your real estate advisor. Your real estate advisor thinks there will be a shortage of office space a year from now and that an office building will fetch $420,000. The total cost of buying the land and constructing the building would be $370,000. Would you buy the land now?

The time value of money

- Introduction to Present Value
- Foundations of the Net Present Value Rule

Present and future value

Future Value

Amount to which an investment will grow after earning interest

Present Value

Value today of a future cash flow.

Discount factors and rates

Discount Rate

Interest rate used to compute present values of future cash flows.

Discount Factor

Present value of a $1 future payment.

Future values

Future Value of $100 = FV

$$FV = \$100 \times (1+r)^t$$

Future values

$$FV = \$100 \times (1+r)^t$$

<u>Example - FV</u>

What is the future value of $100 if interest is compounded annually at a rate of 6% for five years?

$$FV = \$100 \times (1+.06)^5 = \$133.82$$

Future Values

$$FV = \$100 \times (1 + r)^t$$

Example - FV

What is the future value of $400,000 if interest is compounded annually at a rate of 5% for one year?

$$FV = \$400{,}000 \times (1 + .05)^1 = \$420{,}000$$

Present value

$$\text{Present Value} = \text{PV}$$

$$\text{PV} = \text{discount factor} \times C_1$$

Present Value

Discount Factor = DF = PV of $1

$$DF = \frac{1}{(1+r)^t}$$

Discount Factors can be used to compute the present value of any cash flow.

Valuing an office building

Step 1: Forecast cash flows

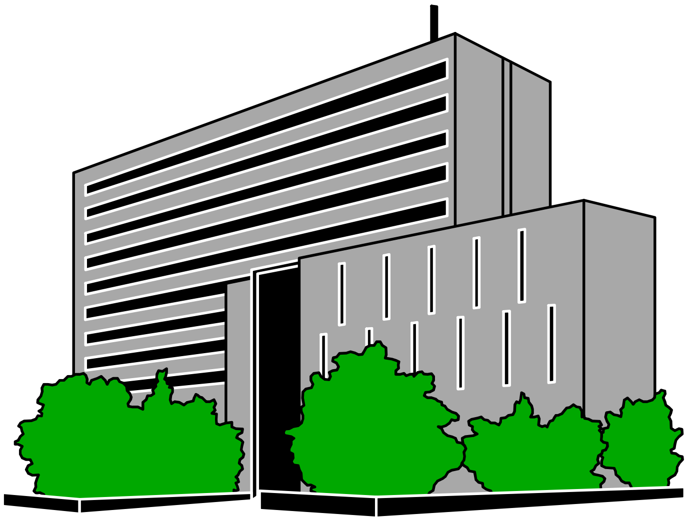
Cost of building = C_0 = $370,000

Sale price in Year 1 = C_1 = $420,000

Step 2: Estimate opportunity cost of capital

If equally risky investments in the capital market offer a return of 5%, then

Cost of capital = r = 5%



Valuing an office building

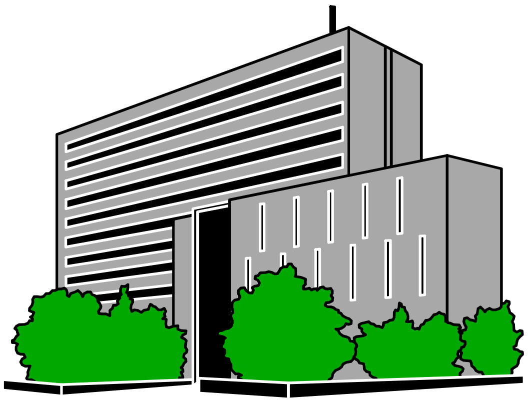
Step 3: Discount future cash flows

$$PV = \frac{C_1}{(1+r)} = \frac{420{,}000}{(1+.05)} = 400{,}000$$

Step 4: Go ahead if PV of payoff exceeds investment

$$NPV = 400{,}000 - 370{,}000$$
$$= 30{,}000$$

NPV = PV - required investment



Net Present Value

$$\mathrm{NPV} = \mathrm{C}_0 + \frac{C_1}{1+r}$$

Risk and Present Value

- Higher risk projects require a higher rate of return
- A higher required rate of return causes a lower PV

$$\text{PV of } C_1 = \$420{,}000 \text{ at } 5\%$$

$$PV = \frac{420{,}000}{1+.05} = 400{,}000$$

Risk and Present Value

$$\text{PV of } C_1 = \$420{,}000 \text{ at } 12\%$$

$$PV = \frac{420{,}000}{1+.12} = 375{,}000$$

$$\text{PV of } C_1 = \$420{,}000 \text{ at } 5\%$$

$$PV = \frac{420{,}000}{1+.05} = 400{,}000$$

Risk and Net Present Value

$$NPV = PV - \text{required investment}$$

$$NPV = 375{,}000 - 370{,}000$$
$$= \$5{,}000$$

Rate of Return Rule

- Accept investments that offer rate of return in excess of their opportunity cost of capital

Example

In the project listed below, the foregone investment opportunity is 12%. Should we do the project?

$$\text{Return} = \frac{\text{profit}}{\text{investment}} = \frac{420{,}000 - 370{,}000}{370{,}000} = .135 \text{ or } 13.5\%$$

Net Present Value Rule

- Accept investments that have positive net present value

Example

Suppose we can invest $50 today and receive $60 in one year. Should we accept the project given a 10% expected return?

$$NPV = -50 + \frac{60}{1.10} = \$4.55$$

Opportunity Cost of Capital

Example

You may invest $100,000 today. Depending on the state of the economy, you may get one of three possible cash payoffs:

Economy	Slump	Normal	Boom
Payoff	80,000	110,000	140,000

$$\text{Expected payoff} = C_1 = \frac{80{,}000 + 110{,}000 + 140{,}000}{3} = \$110{,}000$$

Opportunity Cost of Capital

Example - continued

The stock is trading for $95.65. Next year's price, given a normal economy, is forecast at $110

The stocks expected payoff leads to an expected return.

$$\text{Expected return} = \frac{\text{expected profit}}{\textit{investment}} = \frac{110 - 95.65}{95.65} = .15 \text{ or } 15\%$$

Opportunity Cost of Capital

Example - continued

Discounting the expected payoff at the expected return leads to the PV of the project

$$PV = \frac{110{,}000}{1.15} = \$95{,}650$$

NPV requires the subtraction of the initial investment

$$NPV = 95{,}650 - 100{,}000 = \$ - 4{,}350$$

Opportunity Cost of Capital

Example - continued

Notice that you come to the same conclusion if you compare the expected project return with the cost of capital.

$$\text{Expected return} = \frac{\text{expected profit}}{\textit{investment}} = \frac{110{,}000 - 100{,}000}{100{,}000} = .10 \text{ or } 10\%$$

Present Values

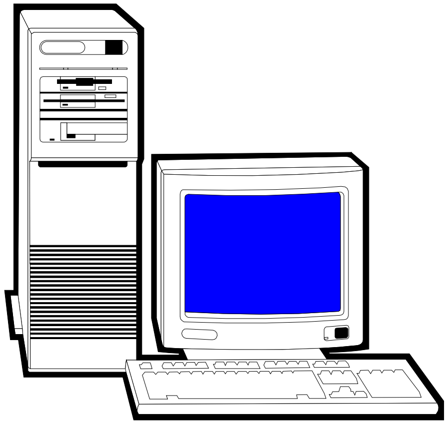
$$PV = DF \times C_t = \frac{C_t}{(1+r)^t}$$

- Replacing "1" with "*t*" allows the formula to be used for cash flows that exist at any point in time

Present Values

Example

You just bought a new computer for $3,000. The payment terms are 2 years same as cash. If you can earn 8% on your money, how much money should you set aside today in order to make the payment when due in two years?



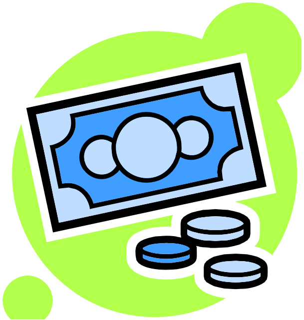
$$PV = \frac{3000}{(1.08)^2} = \$2,572$$

Present Values

Example

You will receive $200 risk free in two years. If the annual rate of interest on a two year treasury note is 7.7%, what is the present value of the $200?

$$PV = \frac{200}{(1.077)^2} = \$172.42$$



Present Values

- PVs can be added together to evaluate multiple cash flows.

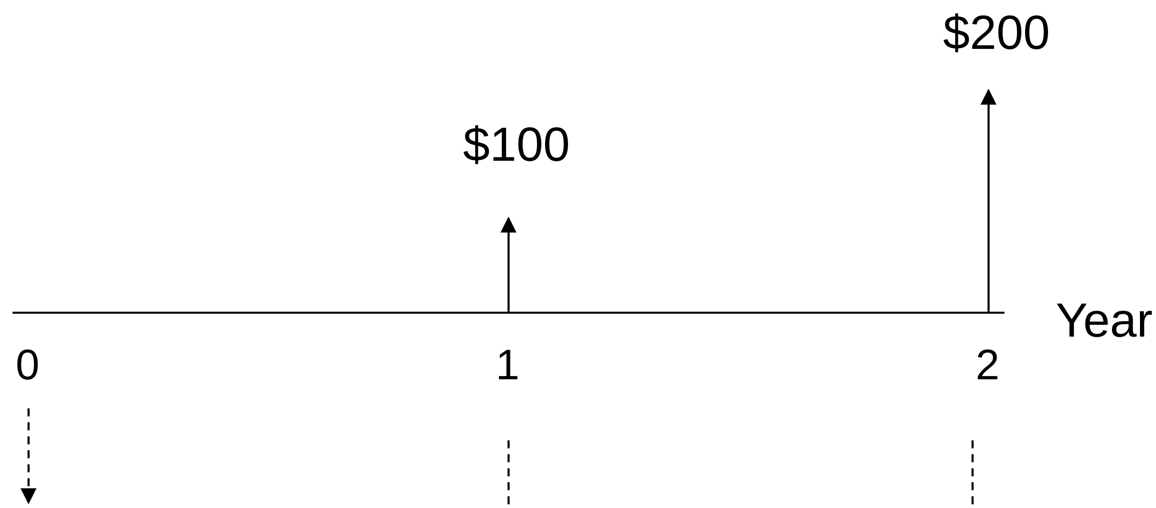
$$PV = \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \ldots .$$

Present Values

- PVs can be added together to evaluate multiple cash flows.

$$PV = \frac{100}{(1+.07)^1} + \frac{200}{(1+.077)^2} = 265.88$$

Present Values



Present Value

<u>Year 0</u>

$100/1.07$ = $93.46

$200/1.077^2$ = $172.42

Total = $265.88

Present Values

- Given two dollars, one received a year from now and the other two years from now, the value of each is commonly called the Discount Factor. Assume r_1 = 20% and r_2 = 7%.

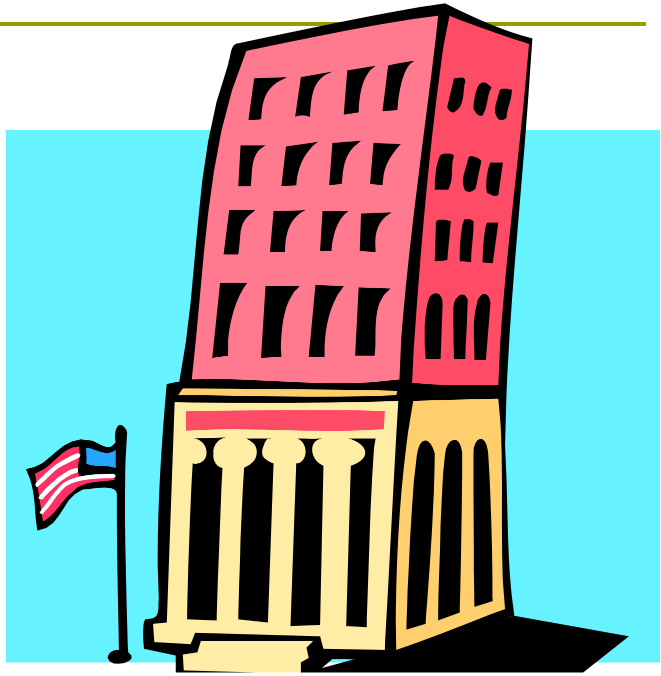
$$DF_1 = \frac{1.00}{(1+.20)^1} = .83$$

$$DF_2 = \frac{1.00}{(1+.07)^2} = .87$$

Present Values

Example

Assume that the cash flows from the construction and sale of an office building is as follows. Given a 5% required rate of return, create a present value worksheet and show the net present value.



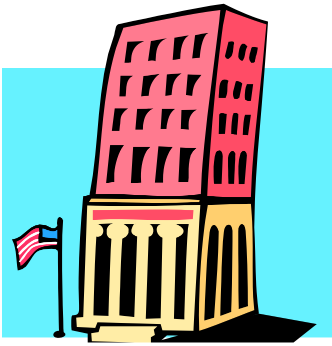
Year 0	Year 1	Year 2
−170,000	−100,000	+320,000

Present Values

Example - continued

Assume that the cash flows from the construction and sale of an office building is as follows. Given a 5% required rate of return, create a present value worksheet and show the net present value.

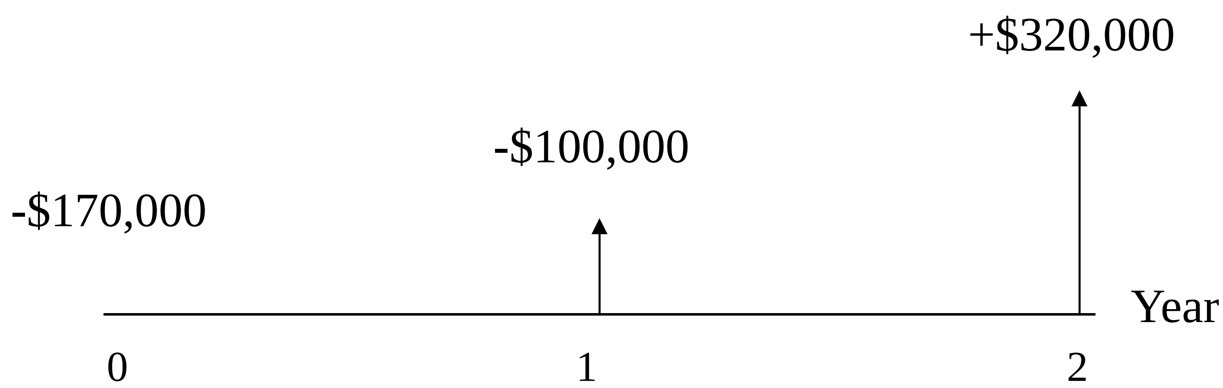
Period	Discount Factor	Cash Flow	Present Value
0	1.0	$-170{,}000$	$-170{,}000$
1	$\frac{1}{1.05} = .952$	$-100{,}000$	$-95{,}238$
2	$\frac{1}{(1.05)^2} = .907$	$+320{,}000$	$+290{,}249$
		$NPV = Total =$	$25,011



Present Values

Example - continued

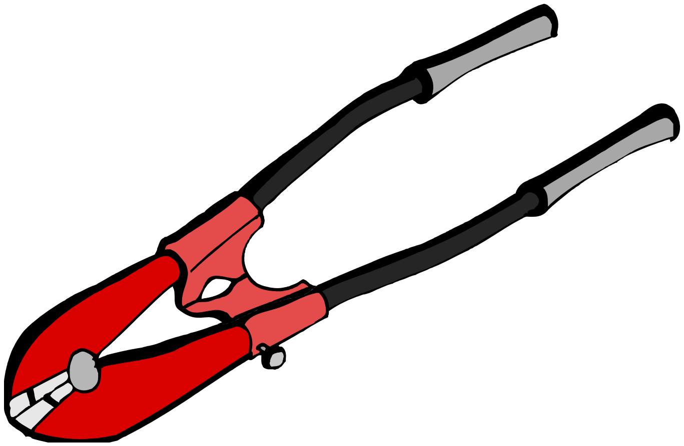
Assume that the cash flows from the construction and sale of an office building is as follows. Given a 5% required rate of return, create a present value worksheet and show the net present value.



Present Value	
Year 0	
-170,000	= -$170,000
-100,000/1.05	= $95,238
$320,000/1.05^2$	= $290,249
Total = NPV	= $25,011

Short Cuts

- Sometimes there are shortcuts that make it very easy to calculate the present value of an asset that pays off in different periods. These tools allow us to cut through the calculations quickly.



Short Cuts

Perpetuity - Financial concept in which a cash flow is theoretically received forever.

$$\text{Return} = \frac{\text{cash flow}}{\text{present value}}$$

$$r = \frac{C}{PV} \Rightarrow PV = \frac{C}{r}$$

Present Values

Example

What is the present value of $1 billion every year, for all eternity, if you estimate the perpetual discount rate to be 10%??

$$PV = \frac{\$1\,\text{bil}}{0.10} = \$10\ \text{billion}$$

Short Cuts

Annuity - An asset that pays a fixed sum each year for a specified number of years.

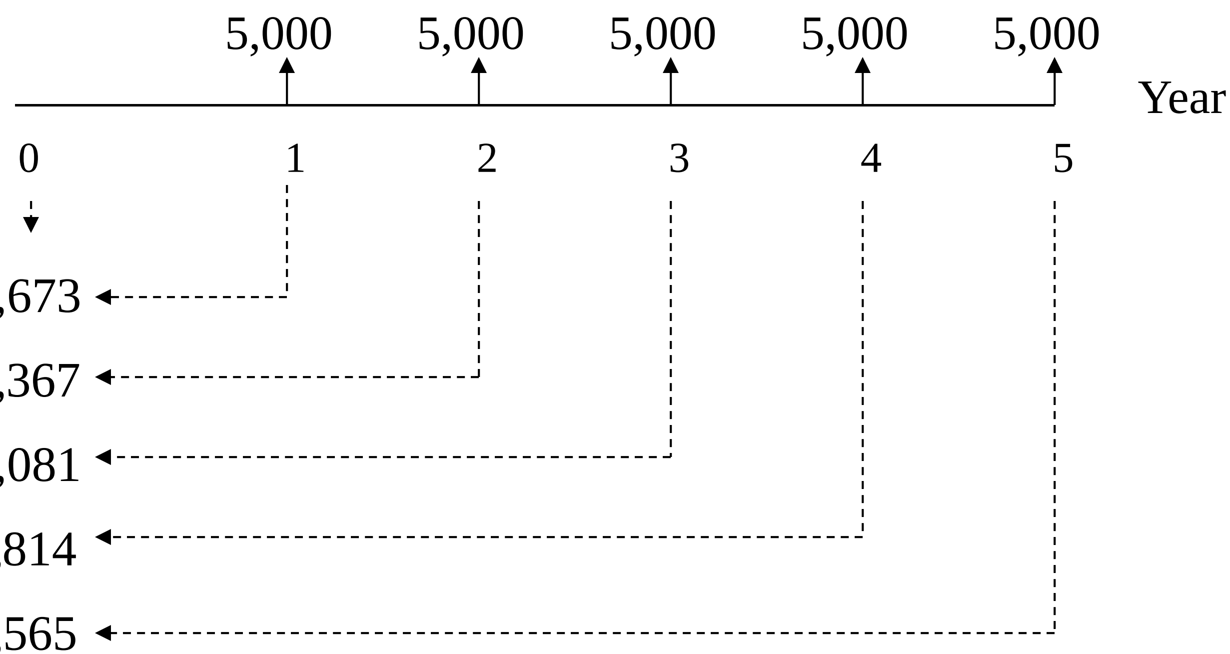
Asset	Year of Payment (1 … 2…..t … t + 1)	Present Value
Perpetuity (first payment in year 1)	1 → t + 1 and beyond	$\frac{C}{r}$
Perpetuity (first payment in year t + 1)	t + 1 and beyond	$\left(\frac{C}{r}\right)\frac{1}{(1+r)^t}$
Annuity from year 1 to year t	1 → t	$\left(\frac{C}{r}\right)-\left(\frac{C}{r}\right)\left(\frac{1}{(1+r)^t}\right)$

Present Values

Example

Tiburon Autos offers you "easy payments" of $5,000 per year, at the end of each year for 5 years. If interest rates are 7%, per year, what is the cost of the car?

Present Value at year 0



$$5{,}000/1.07 = 4{,}673$$

$$5{,}000/(1.07)^2 = 4{,}367$$

$$5{,}000/(1.07)^3 = 4{,}081$$

$$5{,}000/(1.07)^4 = 3{,}814$$

$$5{,}000/(1.07)^5 = 3{,}565$$

$$\text{Total NPV} = 20{,}501$$

Short Cuts

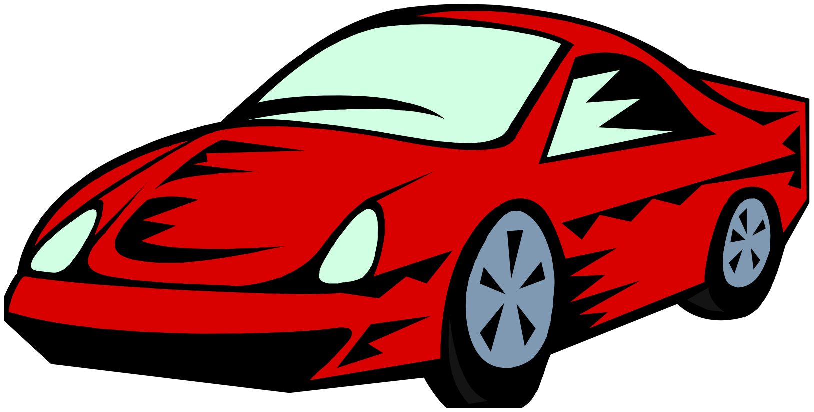
<u>Annuity</u> - An asset that pays a fixed sum each year for a specified number of years.

$$\text{PV of annuity} = C \times \left[\frac{1}{r} - \frac{1}{r(1+r)^t} \right]$$

Annuity Short Cut

Example

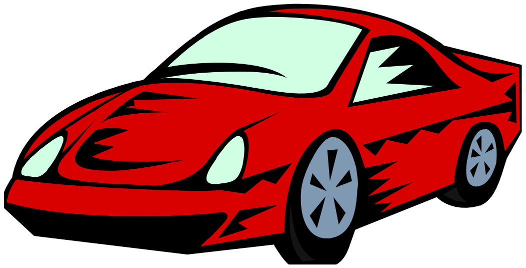
You agree to lease a car for 4 years at $300 per month. You are not required to pay any money up front or at the end of your agreement. If your opportunity cost of capital is 0.5% per month, what is the cost of the lease?



Annuity Short Cut

Example - continued

You agree to lease a car for 4 years at $300 per month. You are not required to pay any money up front or at the end of your agreement. If your opportunity cost of capital is 0.5% per month, what is the cost of the lease?



$$\text{Lease Cost} = 300 \times \left[\frac{1}{.005} - \frac{1}{.005(1+.005)^{48}} \right]$$

$$Cost = \$12{,}774.10$$

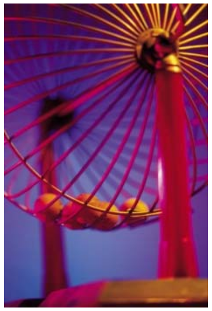
Annuity Short Cut

Example

The state lottery advertises a jackpot prize of $295.7 million, paid in 25 installments over 25 years of $11.828 million per year, at the end of each year. If interest rates are 5.9% what is the true value of the lottery prize?

$$\text{Lottery Value} = 11.828 \times \left[\frac{1}{.059} - \frac{1}{.059(1+.059)^{25}} \right]$$

$$\textit{Value} = \$152,600,000$$



FV Annuity Short Cut

Future Value of an Annuity – The future value of an asset that pays a fixed sum each year for a specified number of years.

$$\text{FV of annuity} = C \times \left[\frac{(1+r)^t - 1}{r} \right]$$

Annuity Short Cut

Example

What is the future value of $20,000 paid at the end of each of the following 5 years, assuming your investment returns 8% per year?

$$FV = 20{,}000 \times \left[\frac{(1+.08)^5 - 1}{.08} \right]$$

$$= \$117{,}332$$

Perpetuities

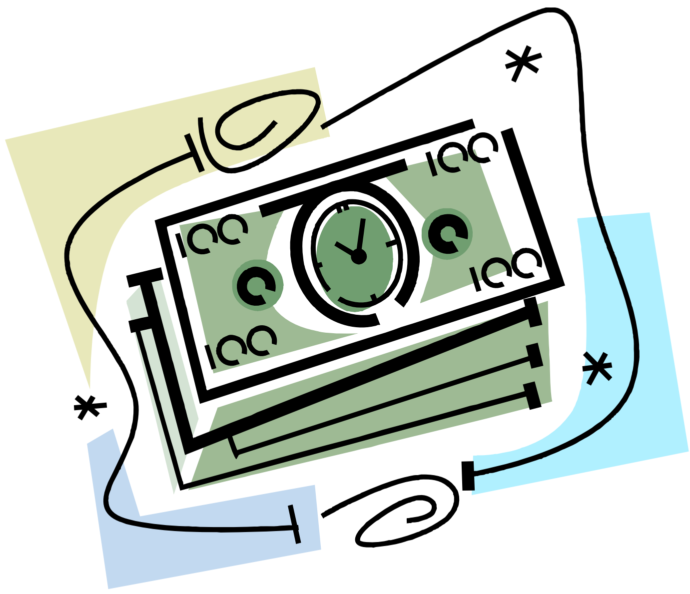
A three-year stream of cash flows that grows at the rate g is equal to the difference between two growing perpetuities.

	Cash flow						
Year:	1	2	3	4	5	6 . . .	Present value
1. Growing perpetuity A	\$1	$\$1 \times (1+g)$	$\$1 \times (1+g)^2$	$\$1 \times (1+g)^3$	$\$1 \times (1+g)^4$	$\$1 \times (1+g)^5 \ldots$	$\frac{1}{r-g}$
2. Growing perpetuity B				$\$1 \times (1+g)^3$	$\$1 \times (1+g)^4$	$\$1 \times (1+g)^5 \ldots$	$\frac{1}{(r-g)(1+r)^3}$
3. Growing 3-year annuity (1 − 2)	\$1	$\$1 \times (1+g)$	$\$1 \times (1+g)^2$				$\frac{1}{r-g} - \frac{1}{(r-g)(1+r)^3}$

Constant Growth Perpetuity

$$PV_0 = \frac{C_1}{r - g}$$

g = the annual growth rate of the cash flow



Constant Growth Perpetuity

NOTE: This formula can be used to value a perpetuity at any point in time.

$$PV_0 = \frac{C_1}{r - g}$$

$$PV_t = \frac{C_{t+1}}{r - g}$$

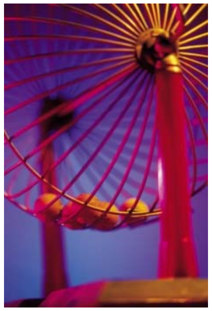
Constant Growth Perpetuity

Example

What is the present value of $1 billion paid at the end of every year in perpetuity, assuming a rate of return of 10% and a constant growth rate of 4%?

$$PV_0 = \frac{1}{.10 - .04}$$

$$= \$16.667 \text{ billion}$$

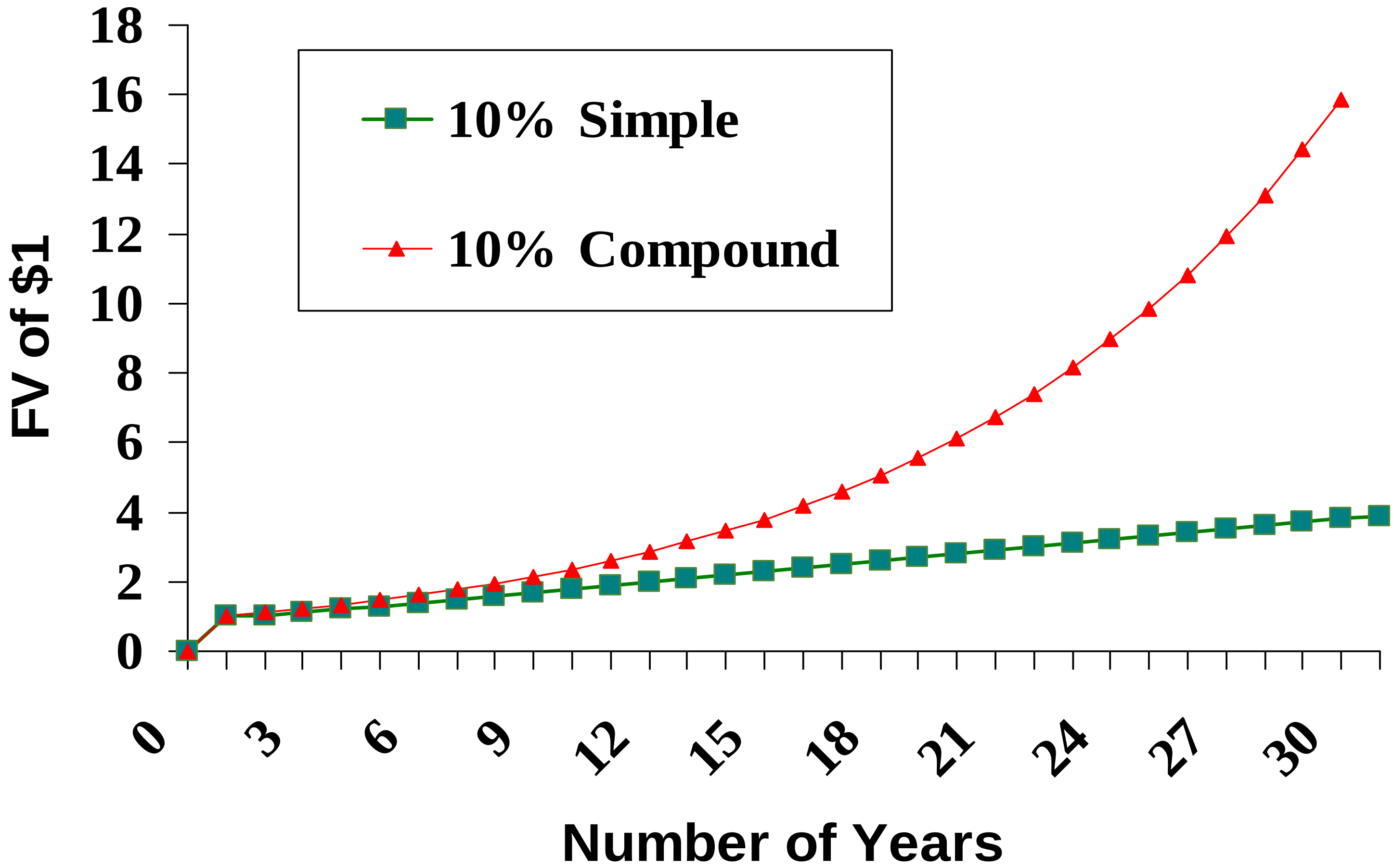


Simple and Compound Interest

The value of a $100 investment earning 10% annually.

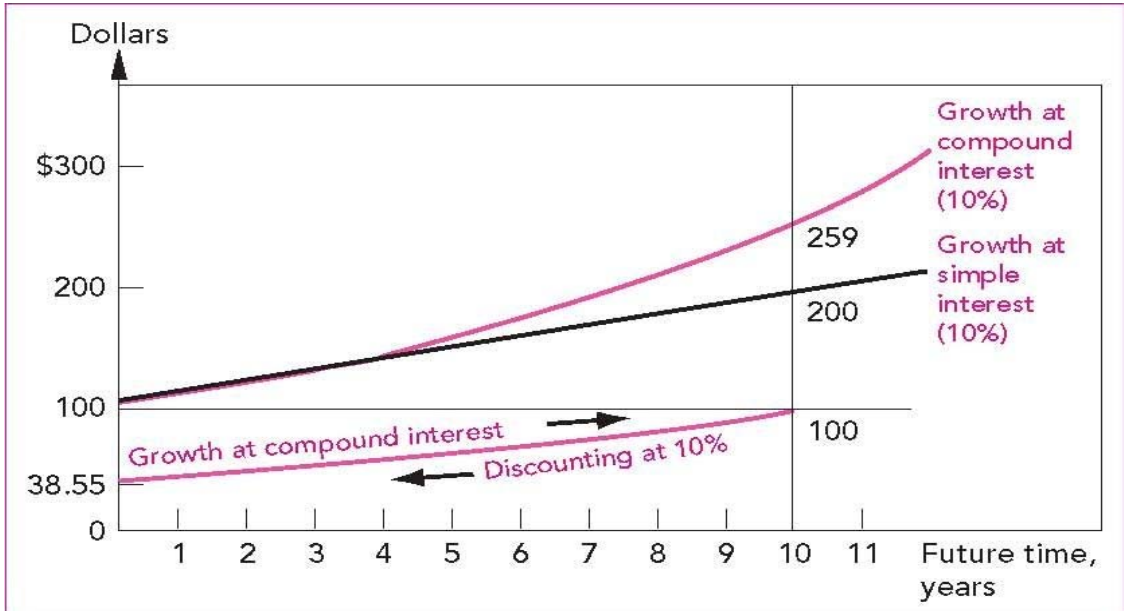
	Simple Interest					Compound Interest				
Year	Starting Balance	+	Interest	=	Ending Balance	Starting Balance	+	Ending Interest	=	Balance
1	$100	+	10	=	$110	$100	+	10	=	$110
2	110	+	10	=	120	110	+	11	=	121
3	120	+	10	=	130	121	+	12.1	=	133.1
4	130	+	10	=	140	133.1	+	13.3	=	146.4
10	190	+	10	=	200	236	+	24	=	259
100	1,090	+	10	=	1,100	1,252,783	+	125,278	=	1,378,061
200	2,090	+	10	=	2,100	17,264,116,042	+	1,726,411,604	=	18,990,527,646
230	2,390	+	10	=	2,400	301,248,505,631	+	30,124,850,563	=	331,373,356,194

Compound Interest

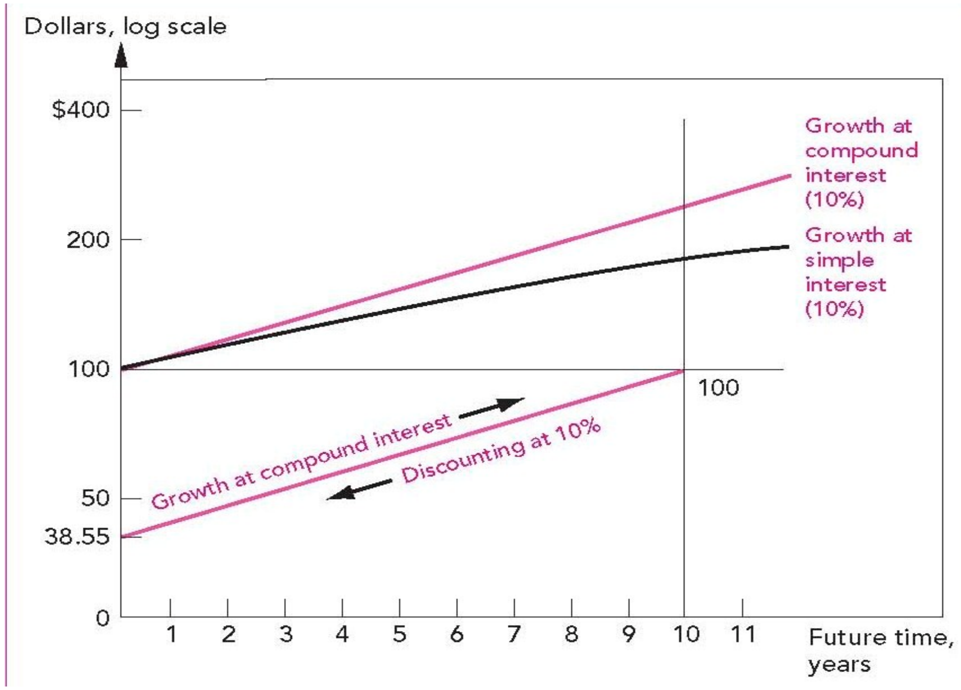


Compound Interest

Compound interest versus simple interest. The top two ascending lines show the growth of $100 invested at simple and compound interest. The longer the funds are invested, the greater the advantage with compound interest. The bottom line shows that $38.55 must be invested now to obtain $100 after 10 periods. Conversely, the present value of $100 to be received after 10 years is $38.55.



Compound Interest



The same story as the previous chart, except that the vertical scale is logarithmic. A constant compound rate of growth means a straight ascending line. This graph makes clear that the growth rate of funds invested at simple interest actually declines as time passes.

Topics Covered

- Company cost of capital
- Measuring the cost of equity
- Certainty equivalents

Company Cost of Capital

Category	Discount Rate
Speculative ventures	30%
New products	20%
Expansion of existing business	15% (Company COC)
Cost improvement, known technology	10%

- A firm's value can be stated as the sum of the value of its various assets

$$\text{Firm value} = PV(A) + PV(B)$$

Debt and COC

IMPORTANT

E, D, and V are all market values of Equity, Debt, and Total Firm Value

$$COC = r_{portfolio} = r_{assets}$$

$$r_{assets} = WACC = r_{debt} \frac{(D)}{(V)} + r_{equity} \frac{(E)}{(V)}$$

$$\text{After tax WACC} = (1-T_c) r_{debt} \frac{(D)}{(V)} + r_{equity} \frac{(E)}{(V)}$$

$$r_{equity} = r_f + \beta_{equity}(r_m - r_f)$$

Measuring Betas

Intel Computer

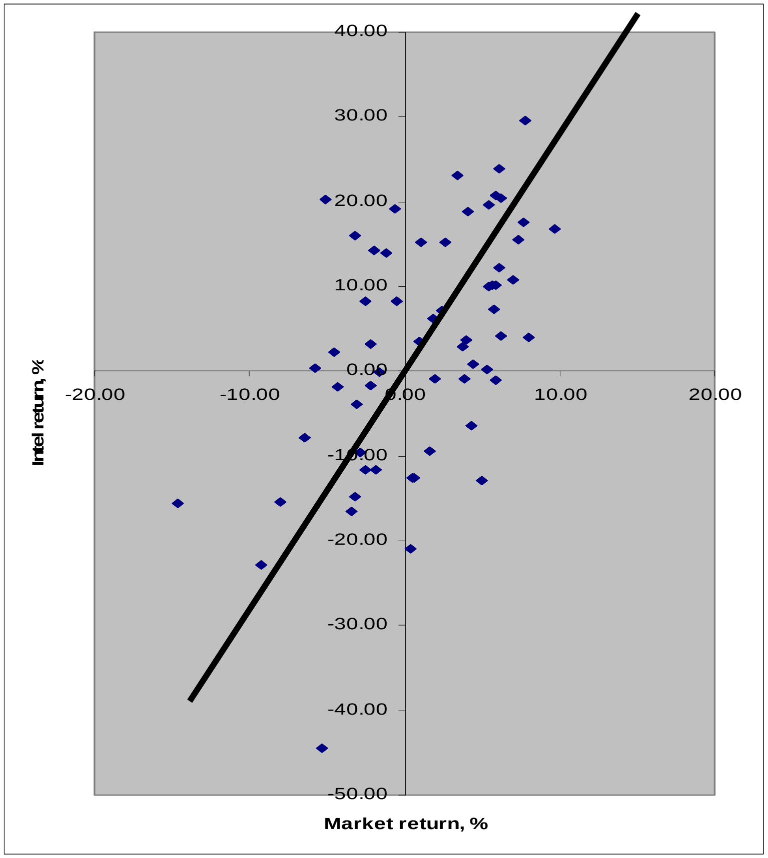
Price data: July 1996 – June 2001

$R^2 = .29$

Beta = 1.54

Slope determined from plotting the line of best fit.

https://www.youtube.com/watch?v=etlv7qTQUSY



Measuring Betas

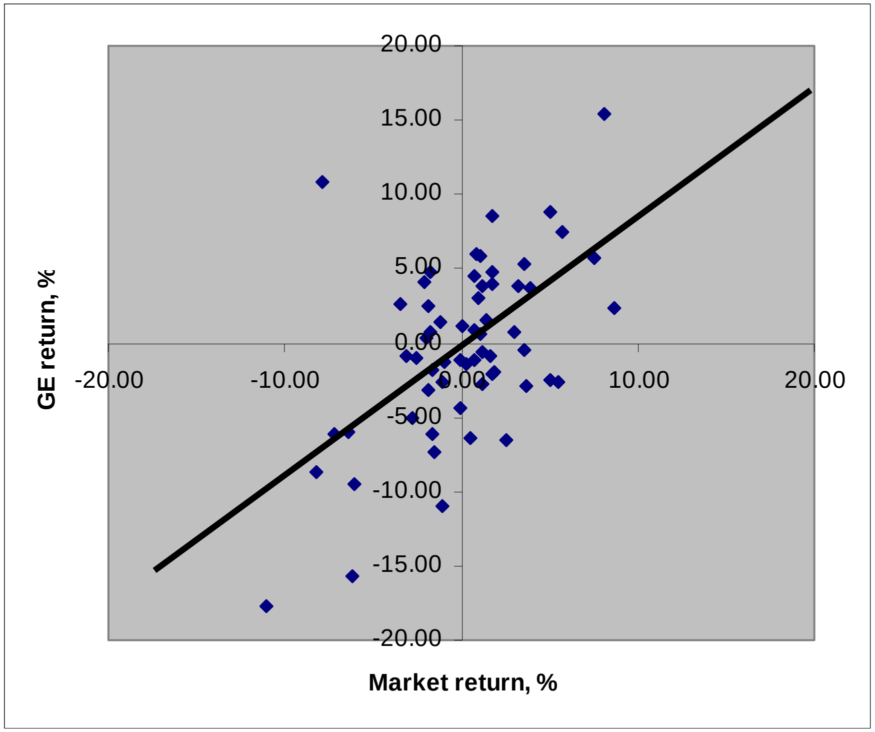
GE

Price data: July 2001 – June 2006

$R^2 = .17$

Beta = .83

Slope determined from plotting the line of best fit.



Company Cost of Capital

simple approach

Company Cost of Capital (COC) is based on the average beta of the assets

The average Beta of the assets is based on the % of funds in each asset

Example

1/3 New Ventures Beta=2.0

1/3 Expand existing business Beta=1.3

1/3 Plant efficiency Beta=0.6

AVG Beta of assets = 1.3

Debt and COC

Firm's market-value balance sheet

Asset value	100	Debt value (D)	30
		Equity value (E)	70
Asset value	100	Firm value (V)	100

- r_{debt} = 7.5% and r_{equity} = 15%
- Debt value is 30 and equity value is 70
- What is the expected return on the assets?
- (30/100)*7.5+(70/100)*15=12.75%

Capital Structure

Capital Structure = the mix of debt & equity within a company

Expand CAPM to include capital structure

$$R = r_f + \beta(r_m - r_f)$$

becomes

$$R_{equity} = r_f + \beta_{equity}(r_m - r_f)$$

Risk, DCF, and CEQ

$$PV = \frac{C_t}{(1+r)^t} = \frac{CEQ_t}{(1+r_f)^t}$$

Risk, DCF, and CEQ

Example

Project A is expected to produce CF = \$100 mil for each of three years. Given a risk free rate of 6%, a market premium of 8%, and beta of .75, what is the PV of the project?

Risk, DCF, and CEQ

Example

Project A is expected to produce CF = $100 mil for each of three years. Given a risk free rate of 6%, a market premium of 8%, and beta of .75, what is the PV of the project?

$$r = r_f + \beta(r_m - r_f)$$

$$= 6 + .75(8)$$

$$= 12\%$$

Risk, DCF, and CEQ

Example

Project A is expected to produce CF = $100 mil for each of three years. Given a risk free rate of 6%, a market premium of 8%, and beta of .75, what is the PV of the project?

$$r = r_f + B(r_m - r_f)$$
$$= 6 + .75(8)$$
$$= 12\%$$

Project A

Year	Cash Flow	PV @ 12%
1	100	89.3
2	100	79.7
3	100	71.2
	Total PV	240.2

Risk, DCF, and CEQ

Example

Project A is expected to produce CF = $100 mil for each of three years. Given a risk free rate of 6%, a market premium of 8%, and beta of .75, what is the PV of the project?.. Now let us assume that the cash flows change but RISKFREE. What is the new PV?

Project A

Year	Cash Flow	PV @ 12%
1	100	89.3
2	100	79.7
3	100	71.2
	Total PV	240.2

Project B

Year	Cash Flow	PV @ 6%
1	94.6	89.3
2	89.6	79.7
3	84.8	71.2
	Total PV	240.2

Risk, DCF, and CEQ

Example

Project A is expected to produce CF = $100 mil for each of three years. Given a risk free rate of 6%, a market premium of 8%, and beta of .75, what is the PV of the project?.. Now let us assume that the cash flows change, but RISKFREE. What is the new PV?

Project A		
Year	Cash Flow	PV @ 12%
1	100	89.3
2	100	79.7
3	100	71.2
	Total PV	240.2

Project B		
Year	Cash Flow	PV @ 6%
1	94.6	89.3
2	89.6	79.7
3	84.8	71.2
	Total PV	240.2

Since the 94.6 is risk free, we call it a ***Certainty Equivalent*** of the 100.

Risk, DCF, and CEQ

Example

Project A is expected to produce CF = $100 mil for each of three years. Given a risk free rate of 6%, a market premium of 8%, and beta of .75, what is the PV of the project? DEDUCTION FOR RISK

Year	Cash Flow	CEQ	Deduction for risk
1	100	94.6	5.4
2	100	89.6	10.4
3	100	84.8	15.2

Risk, DCF, and CEQ

Example

Project A is expected to produce CF = $100 mil for each of three years. Given a risk free rate of 6%, a market premium of 8%, and beta of .75, what is the PV of the project?.. Now let us assume that the cash flows change, but RISKFREE. What is the new PV?

The difference between the 100 and the certainty equivalent (94.6) is 5.7%…this % can be considered the annual premium on a risky cash flow

$$\frac{\text{Risky cash flow}}{1.057} = \text{certainty equivalent cash flow}$$

Risk, DCF, and CEQ

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$$\text{Year 1} = \frac{100}{1.057} = 94.6$$

$$\text{Year 2} = \frac{100}{1.057^2} = 89.6$$

$$\text{Year 3} = \frac{100}{1.057^3} = 84.8$$