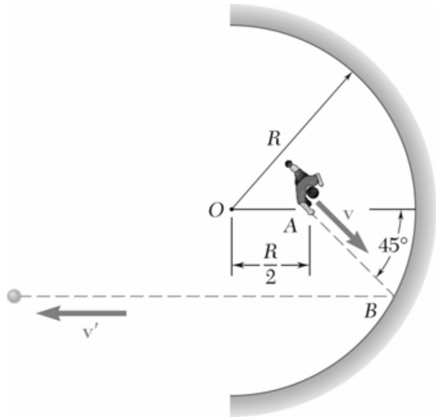
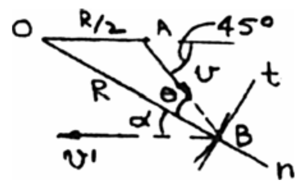


PROBLEM 13.169

A boy located at Point A halfway between the center O of a semicircular wall and the wall itself throws a ball at the wall in a direction forming an angle of 45° with OA. Knowing that after hitting the wall the ball rebounds in a direction parallel to OA, determine the coefficient of restitution between the ball and the wall.

SOLUTION

Law of sines:



$$\frac{\sin\theta}{\frac{R}{2}} = \frac{\sin 135°}{R}$$

$$\theta = 20.705°$$

$$\alpha = 45° - 20.705°$$

$$= 24.295°$$

Conservation of momentum for ball in t-direction:

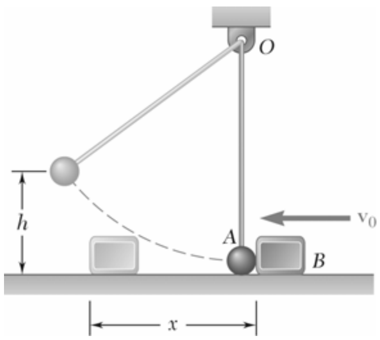
$$-v\sin\theta = -v'\sin\alpha$$

Coefficient of restitution in n: $\qquad v(\cos\theta)e = v'\cos\alpha$

Dividing, $\qquad \dfrac{\tan\theta}{e} = \tan\alpha$

$$e = \frac{\tan 20.705°}{\tan 24.295°}$$

$e = 0.837$ ◀

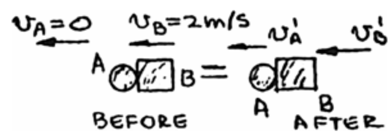


PROBLEM 13.175

A 1-kg block B is moving with a velocity $\mathbf{v}_0$ of magnitude $v_0 = 2$ m/s as it hits the 0.5-kg sphere A, which is at rest and hanging from a cord attached at O. Knowing that $\mu_k = 0.6$ between the block and the horizontal surface and $e = 0.8$ between the block and the sphere, determine after impact (*a*) the maximum height h reached by the sphere, (*b*) the distance x traveled by the block.

SOLUTION

Velocities just after impact



Total momentum in the horizontal direction is conserved:

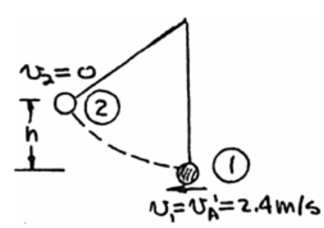
$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$$
$$0 + (1\text{ kg})(2\text{ m/s}) = (0.5\text{ kg})(v'_A) + (1\text{ kg})(v'_B)$$
$$4 = v'_A + 2v'_B \qquad (1)$$

Relative velocities:

$$(v_A - v_B)e = (v'_B - v'_A)$$
$$(0-2)(0.8) = v'_B - v'_A$$
$$-1.6 = v'_B - v'_A \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously:

$$v'_B = 0.8\text{ m/s}$$
$$v'_A = 2.4\text{ m/s}$$



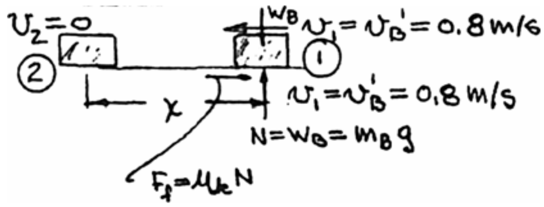
(*a*) Conservation of energy:

$$T_1 = \frac{1}{2} m_A v_1^2 \quad V_1 = 0$$
$$T_1 = \frac{1}{2} m_A (2.4\text{ m/s})^2 = 2.88\, m_A$$
$$T_2 = 0$$
$$V_2 = m_A g h$$
$$T_1 + V_1 = T_2 + V_2 \qquad 2.88\cancel{m}_A + 0 = 0 + \cancel{m}_A(9.81)h$$

$h = 0.294$ m ◀

PROBLEM 13.175 (Continued)

(*b*) Work and energy:



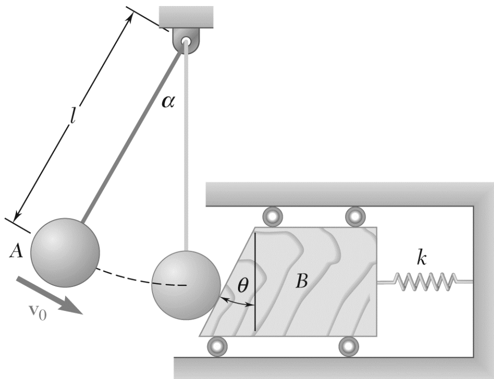
$$T_1 = \frac{1}{2} m_B v_1^2 = \frac{1}{2} m_B (0.8 \text{ m/s})^2 = 0.32 m_B \qquad T_2 = 0$$

$$U_{1-2} = -F_f x = -\mu_k N x = -\mu_x m_B g x = -(0.6)(m_B)(9.81)x$$

$$U_{1-2} = -5.886 m_B x$$

$$T_1 + U_{1-2} = T_2: \qquad 0.32 m_B - 5.886 m_B x = 0$$

$$x = 0.0544 \text{ m} \qquad\qquad x = 54.4 \text{ mm} \blacktriangleleft$$



PROBLEM 13.188

When the rope is at an angle of $\alpha = 30°$, the 0.5-kg sphere A has a speed $v_0 = 1.2$ m/s. The coefficient of restitution between A and the 0.9-kg wedge B is 0.7 and the length of rope $l = 0.8$ m. The spring constant has a value of 500 N/m and $\theta = 20°$. Determine (a) the velocity of A and B immediately after the impact, (b) the maximum deflection of the spring assuming A does not strike B again before this point.

SOLUTION

Masses: $m_A = 0.5 \text{ kg}, \quad m_B = 0.9 \text{ kg}$

Analysis of sphere A as it swings down.

Initial state: $\alpha = 30°, \quad h_0 = l(1-\cos\alpha) = (0.8)(1-\cos 30°) = 0.10718 \text{ m}$

$$V_0 = m_A g h_0 = (0.5)(9.81)(0.10718) = 0.52576 \text{ J}$$

$$T_0 = \frac{1}{2} m_A v_0^2 = \frac{1}{2}(0.5)(1.2)^2 = 0.36 \text{ J}$$

Just before impact: $\alpha = 0, \quad h_1 = 0, \quad V_1 = 0$

$$T_1 = \frac{1}{2} m_A v_A^2 = \frac{1}{2}(0.5) v_A^2 = 0.25\, v_A^2$$

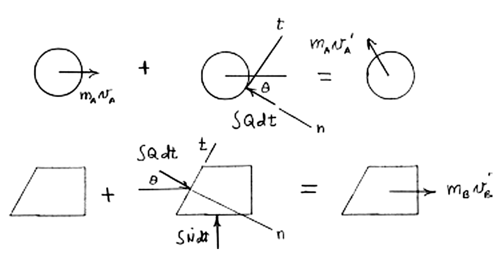
Conservation of energy: $T_0 + V_0 = T_1 + V_1$

$$0.36 + 0.52576 = 0.25\, v_A^2 + 0$$

$$v_A^2 = 3.54304 \text{ m}^2/\text{s}^2$$

$$\mathbf{v}_A = 1.8823 \text{ m/s} \longrightarrow \quad ◀$$

Analysis of the impact. Use conservation of momentum together with the coefficient of restitution $e = 0.7$.



Note that the rope does not apply an impulse since it becomes slack.

PROBLEM 13.188 (Continued)

Sphere A. Momentum in t-direction:

$$m_A v_A \sin\theta + 0 = m_A (v'_A)_t$$
$$(v'_A)_t = v_A \sin\theta = 1.8823 \sin 20° = 0.6438 \text{ m/s}$$
$$(\mathbf{v}_A)_t = 0.6438 \text{ m/s} \angle 70°$$

Both A and B. Momentum in x-direction:

$$m_A v_A + 0 = m_A (v'_A)_n \cos\theta + m_A (v'_A)_t \sin\theta + m_B v'_B$$
$$(0.5)(1.8823) = (0.5)(v_A)_n \cos 20° + (0.5)(0.6438) \sin 20° + 0.9 v'_B$$
$$(0.5)(v'_A)_n \cos 20° + 0.9 v'_B = 0.83105 \quad (1)$$

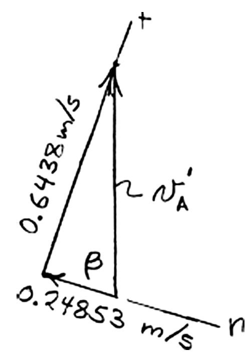
Coefficient of restitution:

$$(v'_B)_n - (v'_A)_n = e[(v_A)_n - (v_B)_n]$$
$$v'_B \cos\theta - (v'_A)_n = e[v_A \cos\theta - 0]$$
$$v'_B \cos 20° - (v'_A)_n = (0.7)(1.8823) \cos 20° \quad (2)$$

Solving Eqs. (1) and (2) simultaneously for $(v'_A)_n$ and v'_B,

$$(v'_A)_n = -0.24853 \text{ m/s}$$
$$v'_B = 1.053 \text{ m/s}$$

Resolve $\mathbf{v}_A$ into horizontal and vertical components.



$$\tan\beta = \frac{(v'_A)_t}{-(v'_A)_n} = \frac{0.6438}{0.24853}$$
$$\beta = 68.9° \qquad \beta + 20° = 88.9°$$
$$v'_A = \sqrt{(0.6438)^2 + (0.24853)^2} = 0.690 \text{ m/s}$$

$$\mathbf{v}'_A = 0.690 \text{ m/s} \measuredangle 88.9° \blacktriangleleft$$

$$\mathbf{v}'_B = 1.053 \text{ m/s} \rightarrow \blacktriangleleft$$