CONCORDIA UNIVERSITY
Faculty of Engineering and Computer Science
Department of Mechanical and Industrial Engineering

ENGR 391 - NUMERICAL METHODS IN ENGINEERING

PROBLEM SET # 1

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Objective:
- Review some basic mathematical concepts
- Be aware of round-off errors
- Apply the Taylor series to estimate truncation errors

1. Problem **1.18, 1.19, 1.20, 1.21, 1.22** in the textbook

2. Problem **2.2, 2.8, 2.9, 2.22** in the textbook

3. Given the following function:

$$f(x) = x^3 - 6.1\,x^2 + 3.2\,x + 1.5$$

   a) Use a calculator to obtain the exact solution with value determined to be at least six digits for $x = 4.71$
   b) Evaluate the same function using three-digit arithmetic with 1) chopping and 2) rounding. Compute the relative errors for the three-digit methods.
   c) Redo the calculation in part b) by first rewriting the above equation in a nested manner.

4. Let $f(x) = \dfrac{e^x - e^{-x}}{x}$
   a) Find $\lim_{x \to 0} f(x)$
   b) Use three digit rounding arithmetic to evaluate $f(0.1)$
   c) Replace each trigonometric function with its third Maclaurin polynomial and repeat part b)
   d) The actual value is $f(0.1) = 2.003335000$. Find the relative error for the values obtained in part b) and c).

5. Given the following equation:

$$f(x) = \frac{8.00}{\sqrt{1.50 + x^2} - x}$$

   a) Use three-digit rounding arithmetic to evaluate $f(2.50)$
   b) Redo the same three-digit rounding arithmetic calculation by first re-formulating the equation using rationalization
   c) Calculate the percentage relative errors in part a) and b) to the true result $f(2.50) = 28.18070497$

6. Given $f(x)=\dfrac{x\cdot\cos x-\sin x}{x-\sin x}$

a) Find $\underset{x\to 0}{Limit}\, f(x)$

b) Use four digit rounding arithmetic to evaluate $f(0.1)$

c) Replace each trigonometric function with its third Maclaurin polynomial and repeat part b)

d) The actual value is $f(0.1) = -1.99899998$. Find the relative error for the values obtained in part b) and c).

7. Find the 2nd order Taylor polynomial $P_2(x)$ for the function $f(x)=\ln(2+x)$ about $x_o= -1$.

a) Use $P_2(-0.5)$ to approximate $f(-0.5)$.

b) Find a bound for the error $|f(x)-P_2(x)|$ in using $P_2(x)$ to approximate $f(x)$ on the interval [-1, 0].

c) Approximate $\int_{-1}^{0} f(x)dx$ using $\int_{-1}^{0} P_2(x)dx$.

8. Find the second Taylor polynomial $P_2(x)$ for the function $f(x) = e^{x}\cos(x)$ about $x_o= 0$.

a) Use $P_2(0.5)$ to approximate $f(0.5)$.

b) Find a bound for the error $|f(x)-P_2(x)|$ in using $P_2(x)$ to approximate $f(x)$ on the interval [0, 1].

c) Approximate $\int_{0}^{1} f(x)dx$ using $\int_{0}^{1} P_2(x)dx$.

9. Find the second-order Taylor polynomial $P_2(x)$ for $f(x)=\sqrt{x+1}$ about $x_o= 0$.

a) Find an upper bound for the error $|f(x)-P_2(x)|$ in using $P_2(x)$ to approximate $f(x)$ on the interval [0, 1].

b) Approximate $\int_{0}^{1} f(x)dx$ using $\int_{0}^{1} P_2(x)dx$

c) Find an upper bound for the error in using $\int_{0}^{1} P_2(x)dx$ to approximate $\int_{0}^{1} f(x)dx$

10. Find the third-order Taylor polynomial $P_3(x)$ for the function $f(x)=\ln(1+x)$ about $x_o= 0$.

a) Use $P_3(0.5)$ to approximate $f(0.5)$.

b) Find a bound for the error $|f(x)-P_3(x)|$ in using $P_3(x)$ to approximate $f(x)$ on the interval [-1/2, 1/2].

c) Approximate $\int_{-0.5}^{0.5} f(x)dx$ using $\int_{-0.5}^{0.5} P_3(x)dx$. Find an upper bound for the error in using $\int_{-0.5}^{0.5} |R_3(x)|\,dx$

Text Book: *Numerical Methods for Engineers and Scientists: An Introduction with Applications Using Matlab.* A. Gilat and V. Subramaniam, John Wiley & Sons, Inc. **3rd Edition.**