Midterm 2: Review

Chapter 9:

Section 9.3

- Use integral tables to solve more difficult integrals
  - Manipulate integral into correct form
  - Integral tables do not need to be memorized
- Introduced numerical tools
  - Maple and Wolfram|Alpha
  - Not being tested

Section 9.4 : Numerical Integration

- Rectangle Rule:

$$R_n = \Delta x \left[f(x_0) + f(x_1) + \ldots + f(x_{n-1}) + f(x_n) \right]$$

- Trapezoidal Rule

$$T_n = \frac{\Delta x}{2} \left[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-1}) + f(x_n) \right]$$

- Simpson's Rule

$$S_n = \frac{\Delta x}{3} \left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_2) + \ldots + 4f(x_{n-1}) + f(x_n) \right]$$

→ n is even

Chapter 10:

Section 10.1: Functions of Several Variables

- Functions of several variables
- Domain of functions with several variables
- Level curves:

$$f(x, y, z) = C$$

- Sketch domain of functions
- Read and draw contour plots

Section 10.2: Partial Derivatives

- First Partial Derivatives

$$f_x = \frac{\partial f}{\partial x} \quad \text{and} \quad f_y = \frac{\partial f}{\partial y}$$

- Second Partial Derivatives:

$$f_{xx} = \frac{\partial^2 f}{\partial x^2}, \quad f_{yy} = \frac{\partial^2 f}{\partial y^2}, \quad f_{xy} = \frac{\partial^2 f}{\partial y \partial x}, \quad f_{yx} = \frac{\partial^2 f}{\partial x \partial y}$$

- Definition of Partial Derivatives

$$\frac{\partial f}{\partial x}(a,b) = \lim_{h \to 0} \frac{f(a+h, b) - f(a,b)}{h}$$

$$\frac{\partial f}{\partial y}(a,b) = \lim_{h \to 0} \frac{f(a, b+h) - f(a,b)}{h}$$

- Marginal Productivity Application

Section 10.3: Find Max/Min

- Find critical points:

$$f_x = 0 \quad \text{and} \quad f_y = 0$$

- Classify critical points as a local max, min, or saddle point.
- Second Derivative Test
  1. Find $D = f_{xx} f_{yy} - (f_{xy})^2$
  2. Classify critical point
     - If $D > 0$ and $f_{xx} > 0 \Rightarrow$ local min
     - If $D > 0$ and $f_{yy} < 0 \Rightarrow$ local max
     - If $D < 0 \Rightarrow$ Saddle Point
     - If $D = 0 \Rightarrow$ Not enough info

Section 10.4: Method of Least Squares

- Minimization problem (area of squares A)
- Found normal equations by looking for critical points $\left(\frac{\partial A}{\partial m} = \frac{\partial A}{\partial b} = 0\right)$
- Solved normal equations to find the best fit line of data.
- Find best fit for $y = ae^{ct}$ — using linear fit equations
- In exam, I will give A and normal equations (if needed)

Section 10.7: Double Integrals

- Double Integrals over rectangles:

$$\int_a^b \int_c^d f(x,y)\, dy\, dx$$

- Fubini's Theorem:

$$\int_a^b \int_c^d f(x,y)\, dy\, dx = \int_c^d \int_a^b f(x,y)\, dx\, dy$$

if $f(x,y)$ is continuous

- Riemann Sum Interpretation

$$\lim_{m\to\infty} \sum_{j=1}^{m} \left[\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i, y_j) \Delta x \right] \Delta y$$

- Separable Integrals

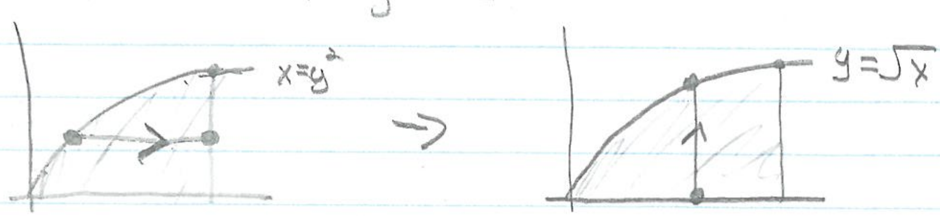
$$\int_c^d \int_a^b f(x) g(y)\, dx\, dy = \left(\int_a^b f(x)\, dx \right) \left(\int_c^d g(y)\, dy \right)$$

- Integrals over Blob domains

  - $\displaystyle\int_c^d \int_{g(y)}^{h(y)} f(x,y)\, dx\, dy \leftarrow x$ first

  - $\displaystyle\int_a^b \int_{g(x)}^{h(x)} f(x,y)\, dy\, dx \leftarrow y$ first

- Domain of Integration



Section 10.8: Applications of Double Integral

- Finding Volume
  - Might not be given plot
  - Example: Assignment 6: Question 2
- Average Value
- Population Density

Example

Find all second partial derivatives of

$$f(x,y) = e^{xy} \cdot \cos(x)$$

$$f_x = \frac{\partial}{\partial x}\left[e^{xy} \cdot \cos(x)\right]$$

$$= -\sin(x)e^{xy} + ye^{xy}\cos(x) = e^{xy}\left[y\cos(x) - \sin(x)\right]$$

$$f_y = \frac{\partial}{\partial y}\left[e^{xy} \cdot \cos(x)\right] = xe^{xy}\cos(x)$$

$$f_{xx} = \frac{\partial}{\partial x}\left[e^{xy}(y\cos(x) - \sin(x))\right]$$

$$= ye^{xy}\left(y\cos(x) - \sin(x)\right) + e^{xy}\left(-y\sin(x) - \cos(x)\right)$$

$$= y^2e^{xy}\cos(x) - 2ye^{xy}\sin(x) - e^{xy}\cos(x)$$

$$f_{yy} = x^2e^{xy}\cos(x)$$

$$f_{xy} = \frac{\partial}{\partial x}\left[e^{xy}(y\cos(x) - \sin(x))\right]$$

$$= xe^{xy}\left(y\cos(x) - \sin(x)\right) + e^{xy}\cos(x)$$

$$f_{yx} = \frac{\partial}{\partial x}\left[xe^{xy}\cos(x)\right]$$

$$= e^{xy}\cos(x) + xye^{xy}\cos(x) - xe^{xy}\sin(x)$$

Find the level curve for the function

$$f(x,y) = \frac{x^2 + 3y}{4 - y^2}$$

when $f(x,y)=1$.

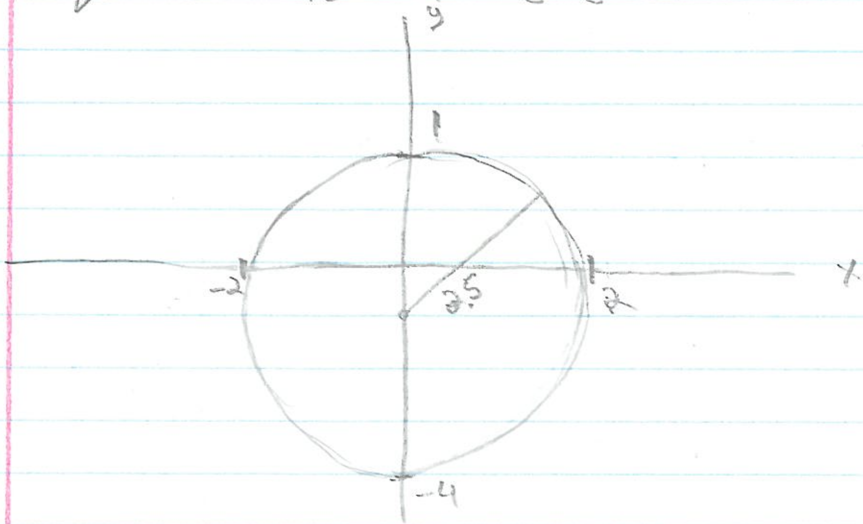
$$\frac{x^2 + 3y}{4-y^2} = 1$$

$\Rightarrow x^2 + 3y = 4 - y^2$

$\Rightarrow x^2 + y^2 + 3y = 4$

$\Rightarrow x^2 + y(y+3) = 4$

Equation is a circle



Classify the critical points for the function
$$f(x,y) = x^3 - 12xy + 8y^3$$

$f_x = 3x^2 - 12y$, $f_y = -12x + 24y^2$

Critical Points

$$\begin{cases} 3x^2 - 12y = 0 \\ -12x + 24y^2 = 0 \end{cases} \Rightarrow \begin{matrix} x^2 = 4y & (1) \\ x = 2y^2 & (2) \end{matrix}$$

Plug (2) into (1)

$$(2y^2)^2 = 4y \Rightarrow 4y^4 = 4y \Rightarrow y^3 = 1 \Rightarrow y = 1$$

Therefore, $(2,1)$ is a critical point. $\boxed{\text{Plug } y=1 \text{ into } (2)}$

We also know that $(0,0)$ is a critical point.

Second Derivatives:

$f_{xx} = 6x$, $f_{yy} = 48y$, $f_{xy} = -12$

Therefore,

At $(2,1)$

$$D = 48 \cdot 6xy - (-12)^2$$
$$= 48 \cdot 6 \cdot 2 \cdot 1 - (-12)^2$$
$$= 432$$

At $(0,0)$ $D = 0 - (-12)^2$

Since $D > 0$ and $f_{xx} > 0$, we know $(2,1)$ is a local min

Since $D < 0$, we know $(0,0)$ is a saddle point