

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	205	All
Examination	Date	Pages
Final	April 2013	2
Instructors:	Course Examiners	
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Special	Only Sharp EL 531 or Casio FX 300 MS	
Instructions:	calculators are allowed	

1 [10]. (a) Sketch a graph of the function

$$f(x) = \begin{cases} \sqrt{9-x^2} & 0 \leq x \leq 3 \\ |x+4| - 1 & -5 \leq x < 0 \end{cases}$$

on the interval $-5 \leq x \leq 3$ and find the definite integral $\int_{-5}^3 f(x) dx$
in terms of area (do not antidifferentiate).

(b) Use the Fundamental Theorem of Calculus to calculate the derivative of

$$F(x) = \int_{-x^2}^0 \cos(t^2) dt \quad ,$$

and determine whether F is increasing or decreasing at $x = 1$.

2 [15]. Find the following indefinite integrals:

$$(a) \int \frac{\sin x}{1 + \cos^2(x)} dx \quad (b) \int \frac{x}{x^2 + 3x - 4} dx \quad (c) \int \frac{(x^{3/2} - 1)^2}{\sqrt{x}} dx$$

3 [16]. Evaluate the following definite integrals (give the exact answers):

$$(a) \int_e^{e^4} \frac{dx}{x\sqrt{\ln x}} \quad (b) \int_4^9 \frac{\ln x}{\sqrt{x}} dx \quad (c) \int_0^{\pi/4} \sec^2(\theta) \tan^2(\theta) d\theta$$

4 [8]. Evaluate the given improper integral or show that it diverges:

$$(a) \int_0^{\infty} x e^{-x^2} dx \quad (b) \int_3^4 \frac{dx}{(4-x)^2}$$

5. [18] (a) Sketch the curves $x = y^2 - 4y$ and $x = 2y - y^2$ and find the area enclosed by the two curves. (HINT: find first the points of intersection of the curves.)
- (b) Sketch the curves defined by $y = x^2$ and $y = 2x$ and find the volume of the solid of revolution of the region bounded by these curves rotated about the axis $y = -1$.
- (c) Find the average value of the function $f(x) = \ln x$ on the interval $[1, e]$.
- 6 [8]. Find the limit of the sequence $\{a_n\}$ as $n \rightarrow \infty$ or prove that it does not exist:

(a) $a_n = \sqrt{\frac{n+1}{9n+1}}$ (b) $a_n = \frac{(-1)^{n+1}n}{(\sqrt{n}+100)^2}$

- 7 [15]. Determine whether the series is divergent or convergent, and if convergent, then whether absolutely or conditionally convergent:

(a) $\sum_{n=1}^{\infty} (-1)^{n+1} n e^{-n}$ (b) $\sum_{n=1}^{\infty} (-1)^{n+1} \cos\left(\frac{\pi}{n}\right)$ (c) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^{10}}{10^n}$

- 8 [5]. Find the radius and the interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(x-4)^n}{n 2^{n+1}}$$

- 9 [5]. (a) Derive the Maclaurin series of $f(x) = x^3 \sin 2x$
(HINT: start with the series for $\sin(z)$ and then let $z = 2x$).

Bonus question [5]. Suppose $\sum_{n=1}^{\infty} a_n$ is a convergent series. Prove that the series $\sum_{n=1}^{\infty} a_n^2$ also converges if $a_n \geq 0$ for all n . Also show that if the condition $a_n \geq 0$ is removed the series $\sum_{n=1}^{\infty} a_n^2$ may become divergent.