

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	205	All
Examination	Date	Pages
Final	December 2012	2
Instructors:		Course Examiners
A. Boyarsky, J. Brody U. Mgbemena, T. Hughes		A. Atoyan & H. Proppe
Special Instructions:	Only calculators approved by the Department are allowed. For full marks show all your work.	

[10] 1. (a) Sketch a graph of the function

$$f(x) = \begin{cases} -\sqrt{4-x^2} & \text{for } |x| \leq 2 \\ |x-3|-1 & \text{for } 2 < x \end{cases}$$

on the interval $-2 \leq x \leq 4$ and calculate the definite integral $\int_{-2}^4 f(x) dx$ in terms of signed area (*do not* antidifferentiate).

(b) Find the derivative $F'(x)$ of the function $F(x) = \int_{x^3}^1 \sqrt{1+t} \cos(\pi t) dt$, and use it to determine whether $F(x)$ is increasing or decreasing at $x = 1$.

[10] 2. Find the antiderivative $F(x)$ of the function $f(x)$ that satisfies the given condition:

$$(a) \quad f(x) = \frac{5^x}{5x+1}, \quad F(0) = 1. \quad (b) \quad f(x) = \frac{\sec^2 x}{(1+\tan x)^3}, \quad F\left(\frac{\pi}{4}\right) = 0.$$

[16] 3. Find the following indefinite integrals:

$$(a) \quad \int \frac{\ln x}{x^2} dx \quad (b) \quad \int \frac{x}{x^2 - 2x - 3} dx \quad (c) \quad \int (1 - e^x)^2 dx.$$

[11] 4. Evaluate the following definite integrals (give the exact answers):

$$(a) \quad \int_1^e \frac{1}{x(1+\ln^2 x)} dx \quad (b) \quad \int_0^{\pi/2} x^2 \cos(2x) dx$$

[8] 5. Evaluate the given improper integral or show that it diverges:

$$(a) \int_0^{\infty} \frac{x}{1+x^2} dx \quad (b) \int_0^4 \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}} dx$$

- [15] 6. (a) Sketch the curves $y = x(x^2 - 2)$ and $y = 2x$, and find the area enclosed by these curves.
(b) Find the volume of a solid obtained by rotating the region bounded by the curve $y = (2 - \sqrt{2x})$ and the lines $y = 0$ and $x = 0$ about the x-axis.
(c) Find the average value of $f(x) = \sin^2 x$ on the interval $[0, \pi]$.

[6] 7. Find the limit of the sequence $\{a_n\}$ at $n \rightarrow \infty$ or prove that it does not exist:

$$(a) a_n = \frac{n \cos^2(n)}{\sqrt{1+4n^3}} \quad (b) a_n = \frac{(2^n + 1)^2}{e^n}$$

[15] 8. Determine whether the series is divergent or convergent, and if convergent, then absolutely or conditionally :

$$(a) \sum_{n=1}^{\infty} \frac{(-1)^n n}{4+n^2} \quad (b) \sum_{n=0}^{\infty} (-1)^n e^{-n} 2^{n+3} \quad (c) \sum_{n=2}^{\infty} \frac{1}{n \ln^3(n)}$$

[9] 9. (a) Find the radius of convergence and the interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(x+1)^n}{n 3^n}$$

(b) Find the Maclaurin series for $f(x) = x \cos(x^2)$.

(Hint: start with the series for $\cos(t)$, then replace t with x^2 .)

[5] **Bonus Question.** It is known that on any given interval $[0, a]$ the average value of some continuous function $f(x)$ is equal to the square of the interval's length, i.e. a^2 . Is this information sufficient to find $f(x)$? Find the function f if it is, otherwise explain why it is insufficient.