

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	205	All
Examination	Date	Pages
Final	December 2009	2
Instructors:	Course Examiner	
All	A. Atoyan	
Special Instructions:		
▷ Materials allowed	Only approved calculators	
▷ Time allowed	3 hours	
▷	Marks for questions are indicated	

1 [10]. a. Sketch a graph of the function $f(x) = 2|1 - x| + x - 1$, and calculate the definite integral $\int_0^3 f(x) dx$.

b. Find the derivative of the function $F(x) = \int_1^{2x} \cos(t) \sqrt{1+t} dt$ and calculate its value $F'(0)$. (HINT: *do not try to integrate*)

2 [20]. Find the following indefinite integrals:

(a) $\int \frac{(2 - \sqrt{x})^2}{x} dx$

(b) $\int \frac{x + 3}{x^2 - 2x - 3} dt$

(c) $\int \sin^5(x) dx$

(d) $\int \sec^2(x) \sqrt{1 + \tan(x)} dx$

3 [10]. Evaluate the following definite integrals (give the exact answers):

(a) $\int_0^1 (x^2 - 2x + 1) e^x dx$

(b) $\int_0^{\pi/4} \frac{\cos(2t)}{1 + \sin^2(2t)} dt$

4 [15]. a. Find the area enclosed by the curves $y = x^3 - 3x$ and $y = x$.

b. Find the volume of a solid obtained by rotating the region bounded by the curve $y = 1 - \cos(x)$ and the line $y = 1$, on the interval $\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ about the x -axis.

c. Find the mean value of the function $f(x) = \frac{1}{\sqrt{4-x^2}}$ on the interval $[-1, 1]$.

5 [8]. Evaluate the given improper integral or show that it diverges:

$$(a) \int_0^1 \frac{dx}{x^{3/4}} \quad (b) \int_1^{\infty} \frac{dx}{x^{3/4}}$$

6 [10]. Find the limit of the sequence $\{a_n\}$ or prove that the limit does not exist:

$$(a) a_n = \frac{\sqrt{3+n+n^2}}{1+3n} \quad (b) a_n = \cos(\sqrt{n+1} - \sqrt{n})$$

7 [15]. Determine whether the series is divergent or convergent, and if convergent, then absolutely or conditionally :

$$(a) \sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{2+n}} \quad (b) \sum_{n=1}^{\infty} \frac{e^n}{2^{n+3}} \quad (c) \sum_{n=2}^{\infty} \frac{(-1)^n}{n^2+4n+3}$$

8 [7]. Find (a) the radius of convergence, and (b) the interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(4x+1)^n}{n^2-1}.$$

9 [5]. Find the radius of convergence of the series and, within this radius, the sum of the series as a function of x :

$$\sum_{n=0}^{\infty} \frac{(x-1)^{2n}}{4^n}$$

Bonus question [5]. Consider a function

$$F(x) = \int_1^{1-x^2} t f(t) dx$$

where f is a continuous positively defined for all real numbers. Find the set of critical points $\{x_i\}$ where the local extrema are located, and determine whether these points correspond to a local maximum or a local minimum of $F(x)$.