CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Section**
Mathematics	205	CA
Examination	**Date**	**Pages**
Final	August 2011	2
Instructor:		**Course Examiners**
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Special Instructions:	Only Sharp EL 531 or Casio FX-300MS calculators are allowed	

MARKS

[8] **1.** **(a)** Sketch a graph of the function $f(x) = 4 - x^2$, write the formula in Σ-notation for its right Riemann sum $R(n)$ on the interval [0,2] with partitioning on n subintervals of equal length, and calculate the area enclosed by the graph of f and $x-$ axis on that interval as the limit of $R(n)$ at $n \to \infty$.

NOTE: you may need the formula $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

(b) Use the Fundamental Theorem of Calculus to calculate the derivative of the function $F(x) = \int_{x^2}^{1} \arctan(t)\,\sqrt{1+t}\,\mathrm{d}t$ at $x = 1$

[20] **2.** Calculate the following indefinite integrals:

(**a**) $\displaystyle\int \frac{(1+\sqrt{x})^2}{x}\,\mathrm{d}x$

(**b**) $\displaystyle\int x^2\,e^{-2x}\,\mathrm{d}x$

(**c**) $\displaystyle\int \frac{x^2-1}{x^2-4}\,\mathrm{d}x$

(**d**) $\displaystyle\int \frac{\cos^3 x}{\sin^3 x}\,\mathrm{d}x$

[12] **3.** Evaluate the following definite integrals (give the exact answers):

(**a**) $\displaystyle\int_0^2 \frac{\arctan(\frac{x}{2})}{x^2+4}\,\mathrm{d}x$

(**b**) $\displaystyle\int_0^3 \ln(1+x)\,\mathrm{d}x$

[10] **4.** Evaluate the given improper integral or show that it diverges:

$$\textbf{(a)} \quad \int_e^\infty \frac{\mathrm{d}x}{x \ln^2 x} \qquad \textbf{(b)} \quad \int_0^{\pi/2} \tan(x)\, \mathrm{d}x$$

[16] **5.** **(a)** Sketch the curves $y = \sqrt{2x}$ and $y = x$ and find the area enclosed.

(b) Sketch the region bounded by $f(x) = \sec(x)$ and the lines $y = 0$, $x = 0$ and $x = \frac{\pi}{4}$, and find the volume of the solid of revolution of this region about the x-axis.

(c) Find the average value of the function $f(x) = \sqrt{25 - x^2}$ on the interval [-5,5].

[6] **6.** Find the limit of the sequence $\{a_n\}$ when $n \to \infty$ or prove that it does not exist:

$$\textbf{(a)} \quad a_n = \frac{(2^n + 3)^2}{3^n} \qquad \textbf{(b)} \quad a_n = \frac{\sqrt{9 + 2n + n^4} - 3n^2}{10 + 3n^2}$$

[12] **7.** Determine whether the series is divergent or convergent, and if convergent, then absolutely or conditionally :

$$\textbf{(a)} \quad \sum_{n=1}^\infty \frac{(-1)^n}{\sqrt{1 + n^2}} \qquad \textbf{(b)} \quad \sum_{n=0}^\infty \frac{(-2)^n}{e^n + 1} \qquad \textbf{(c)} \quad \sum_{n=2}^\infty \frac{1}{n \ln n}$$

[10] **8.** Find the radius of convergence and the interval of convergence of the series

$$\textbf{(a)} \quad \sum_1^\infty \frac{2^n}{n+1}(x+2)^n \qquad \textbf{(b)} \quad \sum_{n=1}^\infty \frac{(4x-1)^{2n}}{n^2}$$

[6] **9.** **(a)** Find the radius of convergence of the power series $F(x) = \sum_{n=0}^\infty 9^n\, x^{2n+1}$.

(b) Express the function $F(x)$ as an elementary function (i.e. sum the series within its conversion radius).

[5] **Bonus Question.** Calculate the definite integral

$$\int_0^\pi \sin t\, \sin^9(\cos t)\, \mathrm{d}t$$