CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Sections**
Mathematics	205	All
Examination	**Date**	**Pages**
Alternate	December 2010	2
Instructors:		**Course Examiners**
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Special Instructions:	Only Sharp EL 531 or Casio FX 300 MS calculators are allowed	

1 [10]. **a.** Sketch a graph of the function

$$f(x) = \begin{cases} 1+\sqrt{4-x^2} & |x| \le 2 \\ 3-x & x > 2 \end{cases}$$

on the interval $-2 \le x \le 4$ and find the definite integral $\int_{-2}^{4} f(x)\,\mathrm{d}x$ in terms of area (do not antidifferentiate).

b Find the derivative of the function $F(x) = \int_0^{\sqrt{x}} \sqrt{1+t^2}\,\sin(t^2)\,\mathrm{d}t$ and calculate its value $F'(1)$. (HINT: *do not try to integrate*)

2 [16]. Find the following indefinite integrals:

(**a**) $\displaystyle\int \frac{e^x}{4+e^{2x}}\,\mathrm{d}x$ (**b**) $\displaystyle\int \frac{(1-\sqrt{x})^2}{x^{3/2}}\mathrm{d}x$ (**c**) $\displaystyle\int \frac{1+x}{x^3-4x}\mathrm{d}x$

3 [18]. Evaluate the following definite integrals (give the exact answers):

(**a**) $\displaystyle\int_1^e \frac{\ln x^2}{x^2}\,\mathrm{d}x$ (**b**) $\displaystyle\int_0^{\pi/2} (1-\sin x\,)\,\cos^3(x)\,\mathrm{d}x$ (**c**) $\displaystyle\int_0^2 \sqrt{4-x^2}\,\mathrm{d}x$

4 [8]. Evaluate the given improper integral or show that it diverges:

(**a**) $\displaystyle\int_{-\infty}^{0} x^3 e^{-2x^2}\,\mathrm{d}x$ (**b**) $\displaystyle\int_0^1 \frac{\mathrm{d}x}{(1-x)^{5/4}}$

5 [15]. **a.** Make a schematic plot of the curve $y = \ln x$, the lines $y = 1 - x$, and $x = 3$, and find the exact value of the area enclosed.

b. Find the volume of a solid obtained by rotating the region bounded by the curve $\cos(\pi x)$ and the lines $y = -1$, $x = 0$ and $x = 1$ about the axis $y = -1$.

c. Find the mean value of $f(x) = \frac{\sec^2 x}{\sqrt{1+3\tan(x)}}$ on the interval $[0, \frac{\pi}{2}]$.

6 [7]. Find the limit of the sequence $\{a_n\}$ at $n \to \infty$ or prove that it does not exist:

(**a**) $a_n = \dfrac{(-1)^n n}{\sqrt{1+9n^2}}$ (**b**) $a_n = \dfrac{\ln(1+e^{2n})}{1+n^2}$

7 [15]. Determine whether the series is divergent or convergent, and if convergent, then absolutely or conditionally :

(**a**) $\displaystyle\sum_{n=0}^{\infty} \frac{n2^n}{(1+n)!}$ (**b**) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \ln(1+n)}{1+n}$ (**c**) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n + n^2}{(1+2n)^2}$

8 [5]. Find the radius of convergence of the series and, within this radius, the sum of the series as a function of x:

$$\sum_{n=0}^{\infty} \frac{(x^2+1)^n}{3^{n+1}}$$

9 [6]. Find **(a)** the radius of convergence, and **(b)** the interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(x+5)^n}{n2^n} .$$

Bonus question [5]. (version 1) What is the functional form (i.e. the sum) of the series in Question 9? (HINT: use differentiability of the power series within the convergence radius).

(version 2) Calculate the indefinite integral

$$\int 2\cos^2(x)\, e^{-x}\, \mathrm{d}x$$

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