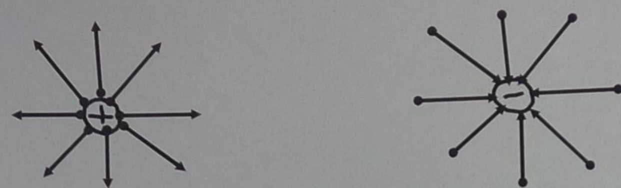
Chapter 23: Electric Fields

Charged particles create electric fields around them.



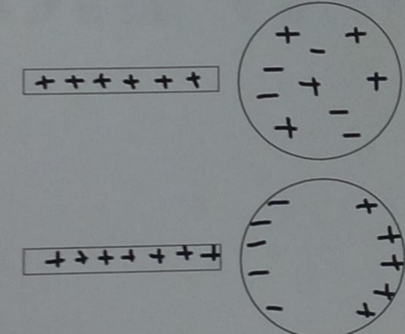
like charges repel , unlike charges attract

Charging by induction (without touching)

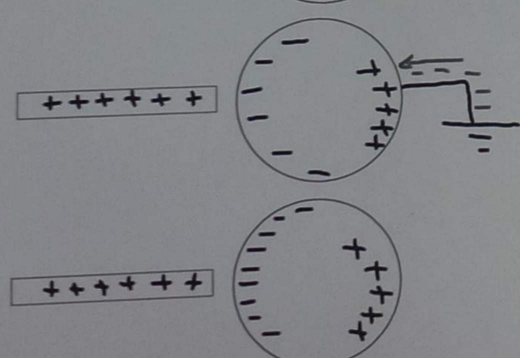
How to charge an object without touching it?

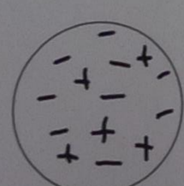
▭ positively charged

O electrically neutral 5+ 5−



connect O to "ground" → is a pool of electrons. e^- can either come from ground or go to ground.





→ negatively charged (4^-)

Coulomb's Law

The force between 2 particles:

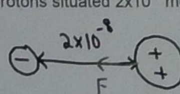
$$\boxed{\vec{F} = \frac{|kq_1q_2|}{r^2}\hat{r}}$$

$k = \frac{1}{4\pi\epsilon_0} = 9\times10^9$, ϵ_0: permitivity of free space
8.854×10^{-12}

q = charge [coulombs, C]

r = distance.

Example: What is the force exerted by an electron on a nucleus containing 2 protons situated 2×10^{-8} meters away?

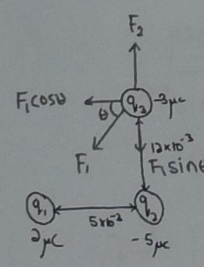


charge on an electron = e
charge on a proton = e

$$\vec{F} = \frac{k|q_1q_2|}{r^2} = \frac{9\times10^9(1.602\times10^{-19})(2\times1.602\times10^{-19})}{(2\times10^{-8})^2}$$

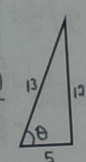
$= 1.155\times10^{-12}$ N (towards the electron)

Ex 2: Calculate $\vec{F}$ on q_3:



$F_1 = \frac{kq_1q_2}{r^2} = \frac{9\times10^9(2\times10^{-6})(3\times10^{-6})}{(13\times10^{-3})^2}$

$= 319.5$ N



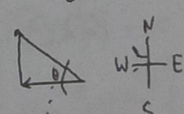
$F_2 = \frac{9\times10^9(5\times10^{-6})(3\times10^{-6})}{(12\times10^{-3})^2}$

$= 937.5$ N

$\vec{F}_T = \vec{F}_1 + \vec{F}_2 = -F_1\cos\theta\,\hat{\imath} - F_1\sin\theta\,\hat{\jmath} + F_2\hat{\jmath}$

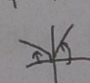
$= -319.5\left(\frac{5}{13}\right)\hat{\imath} - 319.5\left(\frac{12}{13}\right)\hat{\jmath} + 937.5\hat{\jmath}$

$$\boxed{\vec{F}_T = -122.9\hat{\imath} + 642.6\hat{\jmath}}$$



magnitude of $\vec{F}_T$ $\|\vec{F}\| = \sqrt{(-122.9)^2 + (642.6)^2} = 654.24$ N

direction of $\vec{F}_T$ $\theta = \tan^{-1}\left(\frac{642.6}{-122.9}\right) = 79.17^\circ$ N of W



Electric Field

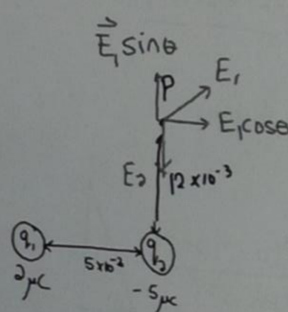
The electric Field at a point is given by

$$\boxed{\vec{E} = \frac{kq}{r^2}\hat{r}}$$

Note that the electric force & field are related by:

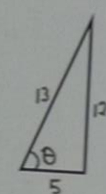
$$\boxed{\vec{F} = q\vec{E}}$$

<u>Ex 2</u>: Calculate $\vec{E}$ at P



$$E_1 = \frac{kq_1}{r^2} = \frac{9\times10^9(2\times10^{-6})}{(13\times10^{-3})^2}$$

$$= 1.065\times10^8$$



$$E_2 = \frac{9\times10^9(5\times10^{-6})}{(12\times10^{-3})^2}$$

$$= 3.125\times10^8$$

$$\vec{E}_T = \vec{E}_1 + \vec{E}_2 = E_1\cos\theta\,\hat{i} + E_1\sin\theta\,\hat{j} - E_2\hat{j}$$

The faster way to get $\vec{E}$ is to use the previous result:

$$\vec{F} = -122.9\hat{i} + 642.6\hat{j} \quad \text{for } q_3 = -3\times10^{-6}$$

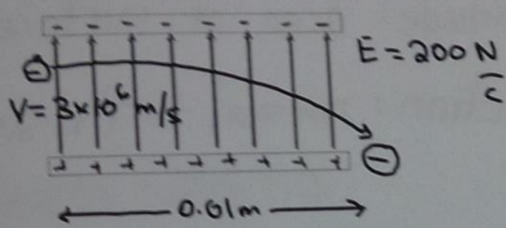
$$\vec{F} = q\vec{E}$$

$$\vec{E} = \frac{\vec{F}}{q} = \frac{-122.9\hat{i} + 642.6\hat{j}}{-3\times10^{-6}}$$

$$\boxed{\vec{E} = 4.1\times10^{+7}\hat{i} - 2.14\times10^{7}\hat{j} \quad N/C}$$

Motion in $\vec{E}$ fields.

An electron enters the regoin of a uniform electric field as shown with v=3x10^6 m/s and E= 200N/C. The horizontal length of the plates is l=0.100m. (A)Find the acceleration of the electron while it is in the electric field



(a) $F = qE$

$= (1.602\times10^{-19})(200)$

$= 3.204\times10^{-17}\ N$

$F = ma$

$\frac{F}{m} = a$

$3.52\times10^{13}\ \frac{m}{s^2} = \frac{3.204\times10^{-17}}{9.11\times10^{-31}} = a$

Ⓑ How long does it take the electron to exit? And with what velocity does it exit?

	x	y
V_i =	3×10^6	0
V_F =	3×10^6	
a =	0	3.52×10^{13}
d =	0.01	
t =	?	$\boxed{3\times10^{-9}\ s}$

$d = V_i t + \frac{1}{2}at^2$

$0.01 = 3\times10^6 t + 0$

$t = 3\times10^{-9}$

$V_F = V_i + at$

$V_F = 0 + (3.52\times10^{13})(3\times10^{-9})$

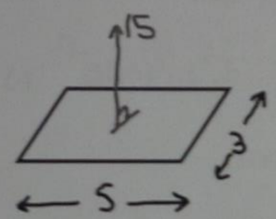
$V_F = 1.17\times10^5\ \frac{m}{s}$

Exits:

$V = 3\times10^6\,\hat{i} - 1.17\times10^5\,\hat{j}$

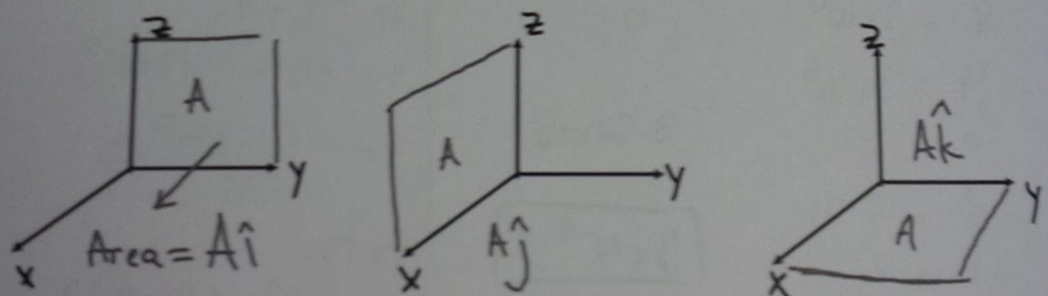
Chapter 24: Gauss' Law.

Area Vector: magnitude: area of surface
direction: normal to the surface



$$\text{Electric Flux} = \Phi_E = \vec{E} \cdot \vec{A} = \int E \cdot dA = \frac{q_{inclosed}}{\epsilon_0} \left[\frac{Nm^2}{C}\right]$$
$$= |E||A| \cos\theta$$

5. A uniform electric field $a\hat{i} + b\hat{j}$ intersects a surface of area A. What is the flux through this area if the surface lies (a) in the yz plane? (b) in the xz plane? (c) in the xy plane?



$\phi - E \cdot A$
$= (a\hat{i} + b\hat{j}) \cdot (A\hat{i})$
$= aA$

$\phi = (a\hat{i} + b\hat{j})(A\hat{j})$
$= bA$

$\phi = 0$

Charge Densities:

λ σ ρ charge density = $\dfrac{\text{TOTAL CHARGE, } Q}{\text{a) TOTAL LENGTH } \lambda \quad \text{b) TOTAL AREA } \sigma \quad \text{c) TOTAL VOLUME } \rho}$

a) TOTAL LENGTH λ

b) TOTAL AREA σ

c) TOTAL VOLUME ρ

$\left.\begin{array}{l} \lambda\,(\text{length}) = \\ \sigma\,(\text{area}) = \\ \rho\,(\text{Volume}) = \end{array}\right\} Q$

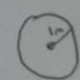
10 m

5m

$Q_T = 15c$

$A_T = 50$

$\sigma = \frac{15}{50} \frac{C}{m^2}$



$= \pi r^2$

$= \pi$

$\frac{15}{50}\pi =$

$$\vec{E} = \frac{kq}{r^2}$$

$$\Delta V = \frac{kq}{r}$$

ELECTRIC POTENTIAL
OR
POTENTIAL DIFFERENCE
OR
VOLTAGE

$$\boxed{F = qE}$$

$$\boxed{U = q\Delta V}$$

$$\vec{F} = \left| \frac{kq_1q_2}{r_2} \right|$$

$$U = \frac{kq_1q_2}{r}$$

POTENTIAL ENERGY

If E is uniform V = -Ed

$$\Delta V = -\int E \cdot ds$$

Chapter 26: Capacitance & dielectrics.

A capacitor is a circuit element that stores energy

Capacitance is defined by: $\longrightarrow C = \frac{Q}{V}$

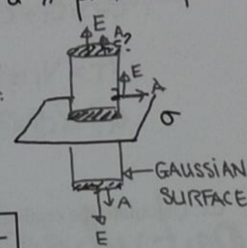
Where C: Capacitance [Farads F], Q: Charge [Couloumbs, C] and V: Potential difference [Volts, v]

Even though C = Q/V the capacitance doesn't depend on Q or V. It depends on the geometry of the configuration (distance, area, length, radius etc)

Calculating Capacitance:

Ex1: Calculate the capacitance of a // plate cap. from scratch.

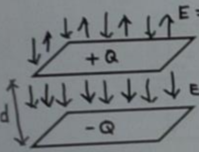
1. Electric Field Due to an ∞ plane:



$$\int E \cdot dA = \frac{q_{in}}{\epsilon_0} : \int_1 E \cdot dA + \int_2 E\,dA + \int_3 E\,dA = \frac{q_{in}}{\epsilon_0}$$

$$2EA = \frac{\sigma A}{\epsilon_0} \rightarrow \boxed{E = \frac{\sigma}{2\epsilon_0}}$$

2. Electric Field between the plates of a capacitor



$E = \frac{\sigma}{2\epsilon_0}$ $E = \frac{\sigma}{2\epsilon_0}$ $E_{TOTAL} = 0$

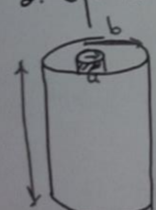
$E = \frac{\sigma}{2\epsilon_0}$ $E = \frac{\sigma}{2\epsilon_0}$ $E_{TOTAL} = \frac{\sigma}{\epsilon_0}$ UNIFORM $V = Ed$

$\Delta V = \frac{\sigma d}{\epsilon_0}$

$E = \frac{\sigma}{2\epsilon_0}$ $E = \frac{\sigma}{2\epsilon_0}$ $E_{TOTAL} = 0$

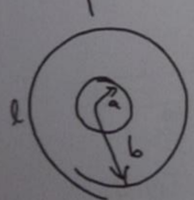
3. Capacitance: $C = \frac{Q}{\Delta V} = \frac{\sigma A_T}{\frac{\sigma d}{\epsilon_0}} = \frac{\epsilon_0 A}{d}$

2. Cylindrical Capacitor:



$$C = \frac{l}{2k\ln\left(\frac{b}{a}\right)}$$

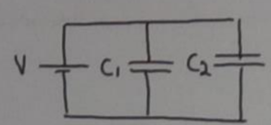
3. Spherical Capacitor:



$$C = \frac{ab}{k(b-a)}$$

Combination of Capacitors:

Parallel Circuit

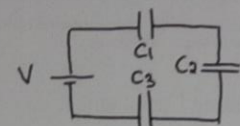


$C_T = C_1 + C_2$

$V_T = V_1 = V_2$

$Q_T = Q + Q_2$

Series Circuit



$C_T = \dfrac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$

$V_T = V_1 + V_2 + V_3$

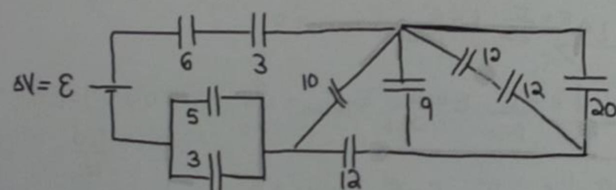
$Q_T = Q_1 = Q_2 = Q_3$

FOR A 2 CAPACITOR CIRCUIT

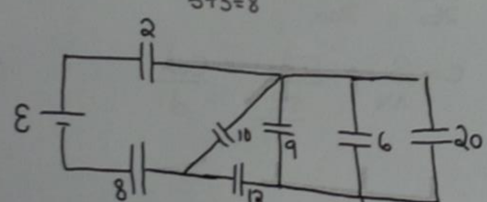
$C_T = \dfrac{C_1 C_2}{C_1 + C_2}$

Ex: Calculate the equivalent capacitance in the following circuit.

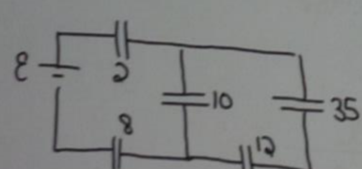
$\dfrac{6(3)}{6+3} = 2$ $\dfrac{12(12)}{12+12} = 6$



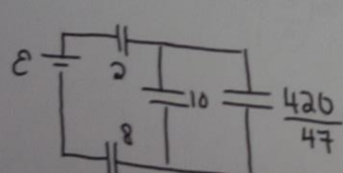
$5+3=8$



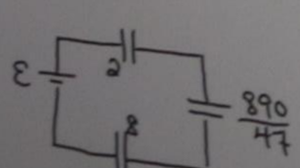
$9+6+20=35$



$\dfrac{35(12)}{35+12} = \dfrac{420}{47}$



$\dfrac{420}{47} + 10 = \dfrac{890}{47}$



$C_T = \dfrac{1}{\frac{1}{2} + \frac{1}{\frac{890}{47}} + \frac{1}{10}}$

$= 1.532$ Farad

Energy Stored in a capacitor:

$$\boxed{U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}}$$

Capacitors with dielectrics:

- A dielectric is a non-conducting material that you place inside a capacitor to: ↑ Capacitance

  ↑ Operating Voltage ↓V_c

- The capacitance increases by a factor

  K: dielectric constant of the material

$$\boxed{C = KC_0}$$

Chapter 27: Current and Resistance

- Current: $I = \frac{dQ}{dt} = \frac{\Delta Q}{\Delta t}$ [Amperes]

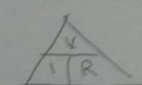
$$I = nqv_dA$$

n: density of charge carriers $\left(\frac{\text{electrons}}{m^3}\right)$
q: charge (coulombs)
v_d: drift velocity (m/s)
A: Cross Sectional Area (m^2 or cm^2)

- Current density: J [Amps/m^2]

$$J = \frac{I}{A} = nqv_d \qquad J = \sigma E$$

- Ohm's Law: $V = IR$



- Resistance: $R = \frac{\rho l}{A}$ [Ohms, Ω]

ρ: resistivity of the material ($\Omega \cdot m$)
l: length (m)
A: Area (m^2)

- Conductance $G = \frac{1}{R} = \frac{A}{\rho l}$ (Siemens, S)

- Resistance & Temperature

$$R = R_0\left[1 + \alpha\left(T - T_0\right)\right]$$

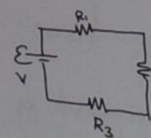
R_0 = Resistance @ T_0
α = temperature coefficient of resistivity

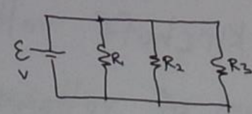
- Power P (W, watts)

$$\boxed{P = VI = \frac{V^2}{R} = I^2R}$$

Chapter 28: DC Circuits

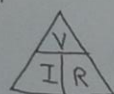
Series | Parallel





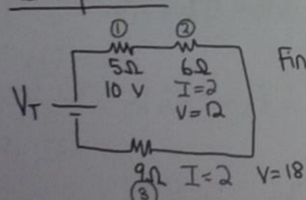
Series	Parallel
$V_T = V_1 + V_2 + V_3$	$V_T = V_1 = V_2 = V_3$
$I_T = I_1 = I_2 = I_3$	$I_T = I_1 + I_2 + I_3$
$R_T = R_1 + R_2 + R_3$	$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

At any given element V, I, R are related by ohm's law: $V = IR$



- V / $\mathcal{E}$ = Voltage or Potential difference
  Emf or electromotive force
  → like \$ given to electrons → must come back with \$0
- I Current, measured in Amperes, A.
  → like water flowing in pipes.
- R, resistance, measured in ohms, Ω
  → like speed bumps → the lower the R the ↑ the I.

Example 1:



Find V_T

$I_1 = \frac{V_1}{R_1} = \frac{10}{5} = 2A$ $\quad I_T = I_1 = I_2 = I_3 = 2$

$V_T = 18 + 12 + 10 = 40v$

Kirchoff's laws :

1 KVL: Kirchoff voltage law:

The sum of voltages around a loop = 0

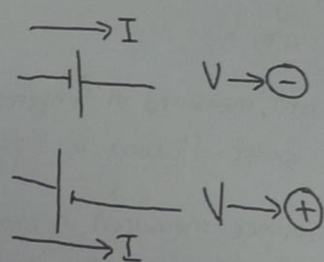
$$\sum_{loop} V_i = 0$$

2. KCL: Kichoff's current law:

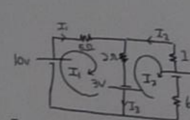
The sum of the net current entering a node = 0.

$$\sum_{node.} i = 0$$

Applying KVL & KCL: Mainly we apply KVL:



Example :



Find I_1, I_2 & I_3

KVL:

Loop 1: $-10 + 5I_1 + 2(I_1 - I_2) - 3 = 0$

$$7I_1 - 2I_2 = 13$$

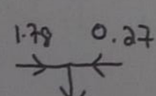
Loop 2: $1I_2 + 3 + 6I_2 + 3 + 2(I_2 - I_1) = 0$

$$-2I_1 + 9I_2 = -6$$

$I_1 = 1.78A \quad I_2 = -0.27$

$I_1 = 1.78$

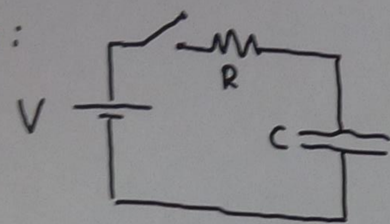
$I_2 = +0.27$



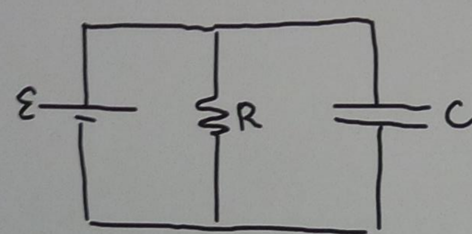
$I_3 = 1.78 + 0.27$

RC Circuits

Ex:



or

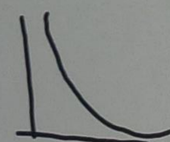


<u>Charging</u>: $q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right)$

$= Q\left(1 - e^{-t/RC}\right)$

$I(t) = \frac{\mathcal{E}}{R} e^{-t/RC}$

<u>Discharging</u>: $q(t) = Qe^{-t/RC}$



$I(t) = \frac{-Q}{RC} e^{-t/RC}$

$= -I_0 e^{-t/RC}$