

### Logical connectives (operators)

Name Compound proposition	Truth value	Corresponding English expressions
Negation $\neg p$	true <i>if and only if</i> $p$ is false	“not $p$ ” “it is not the case that $p$ ”
Conjunction $(p \wedge q)$	true <i>if and only if</i> both $p$ and $q$ are true	“ $p$ and $q$ ” “ $p$ but $q$ ”
Disjunction $(p \vee q)$	false <i>if and only if</i> both $p$ and $q$ are false	“ $p$ or $q$ (or both)” “ $p$ unless $q$ ”
Exclusive or $(p \oplus q)$	true <i>if and only if</i> exactly one of $p$ and $q$ is true	“either $p$ or $q$ ” “ $p$ or $q$ but not both”
Biconditional $(p \leftrightarrow q)$	true <i>if and only if</i> $p$ and $q$ have the same truth value	“ $p$ if and only if $q$ ” “ $p$ is necessary and sufficient for $q$ ” “if $p$ , then $q$ , and conversely”
Implication $(p \rightarrow q)$	false <i>if and only if</i> $p$ is true and $q$ is false	“ $p$ implies $q$ ” “if $p$ , then $q$ ” “ $p$ only if $q$ ” “ $p$ is sufficient for $q$ ” “ $q$ follows from $p$ ” “ $q$ if $p$ ” “ $q$ when(ever) $p$ ” “ $q$ is necessary for $p$ ” “ $q$ unless not $p$ ” “ $q$ or else not $p$ ”

Note: We will not be using *exclusive or* very frequently, since  $p \oplus q \equiv \neg(p \leftrightarrow q)$ .

**Precedence of logical operators:** } for simplicity, we omit outer  
 1)  $\neg$ ; 2)  $\wedge$ ; 3)  $\vee$ ; 4)  $\rightarrow$ ; 5)  $\leftrightarrow$  } parentheses.

**More about the implication:**

$$p \rightarrow q$$

(hypothesis *or* premise)                      (conclusion *or* consequence)

**Propositions related to the implication  $p \rightarrow q$ :**

- The **converse** of  $p \rightarrow q$ :  $q \rightarrow p$
- The **contrapositive** of  $p \rightarrow q$ :  $\neg q \rightarrow \neg p$
- The **inverse** of  $p \rightarrow q$ :  $\neg p \rightarrow \neg q$

Equivalences: (see handout on **Boolean Equivalences**)

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

$$q \rightarrow p \equiv \neg p \rightarrow \neg q$$

$$p \rightarrow q \not\equiv q \rightarrow p$$