MHF4U-B

1

Exponents and Exponential Functions

Introduction

Mathematics has been around from the early beginnings of society. Shepherds in ancient times needed to count in order to keep track of tribal members, possessions, and sheep. As the idea of borrowing developed, people needed to invent negative numbers to handle the idea of loss and debt. As society developed and became settled, methods for calculating areas and surveying land became necessary to resolve disputes between landowners. As permanent homes were built, geometry developed so that people could design and build houses that would stand. Sharing resources led to the development of a way to handle ratios. The discovery of electricity required equations in order to be able to use it. The more sophisticated society became, the more mathematics was needed. As society progressed, it needed to be able to predict future trends and events, and so had to develop more powerful ways to model relationships and phenomena.

Many situations and relationships can be modelled (graphically or algebraically) using straight lines and parabolas. For example:

- The distance travelled by a vehicle moving at a constant speed can be modelled by a first-degree equation. It appears on a graph as a straight line.
- The path of a baseball as it is thrown from the outfield to home plate or a football as it leaves the kicker's foot and flies through the air are examples of parabolic paths. Parabolic paths can be modelled by a quadratic equation and are easily represented on a graph.
- Naturally occurring phenomena such as ocean tides, phases of the moon, and seasons of the year are repetitive and cyclic by nature. They can be graphed over a long period and modelled by trigonometric equations. These equations can then be used to predict future events or trends.

Many situations that involve growth can be modelled using exponential functions. The earth's population is growing exponentially. Scientists claim the amount of CO_2 (carbon dioxide) in the atmosphere is growing exponentially. You hope that the value of your investment portfolio will grow exponentially more quickly than inflation. The decay of radioactive material can be modelled using exponential functions. This behaviour, measured by half-life, is used to determine the age of relics and ancient bones.

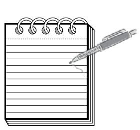
In this lesson, you will examine the features of exponential functions and curves after a brief review of the laws for exponents. You will then apply these rules and relationships to solve problems that need exponential functions to determine solutions.

Estimated Hours for Completing This Lesson	
Part A: Exponents	2.25
Part B: Graphs of Exponential Functions	1
Part C: Applications of Exponential Functions	1
Key Questions	0.75

What You Will Learn

After completing this lesson, you will be able to

- evaluate expressions that have exponents
- simplify expressions using the laws of exponents
- simplify complex expressions involving exponents
- graph basic exponential functions
- apply laws of exponents to solving problems from science and economics



Watch for this symbol:

Throughout this course, important concepts, definitions, and skills will be indicated with this symbol. You should keep a notebook with these Key Ideas for use when you are completing assignments and studying for tests.

Part A: Exponents

Powers and Exponents

When you see an expression such as 2^3, it has a very specific meaning and terminology:

- The number 2 is called the base.
- The number 3 is called the exponent.
- The entire expression 2^3 is called a power.
- 2^3 means to multiply 2 by itself three times; that is, $2^3 = 2 \times 2 \times 2$, which in turn has a value of 8.

Examples

1. The following are written as the product of numbers:

   a) $3^4 = 3 \times 3 \times 3 \times 3$

   b) $(-4)^7 = (-4)(-4)(-4)(-4)(-4)(-4)(-4)$

   c) $-5^2 = -5 \times 5$

(**Note:** In part c, the minus sign is written only once, as the exponent is only on the number 5, whereas in part b the exponent is on the entire quantity in the bracket.)

2. The following are written as powers:

   a) $2 \times 2 \times 2 \times 2 = 2^4$

   b) $(-3)(-3)(-3)(-3) = (-3)^4$

   c) $-3 \times 3 \times 3 \times 3 = -3^4$

(**Note:** In part c, the minus sign is written once in front of the power, whereas in part b, the minus sign is contained inside the bracket with an exponent on it.)

3. Evaluate each of the following powers:

   a) 6^3 b) $(-5)^4$ c) -5^4 d) $(-3)^5$ e) $\left(-\frac{2}{3}\right)^3$

Solutions

3. a) $6^3 = 6 \times 6 \times 6 = 216$

b) $(-5)^4 = (-5)(-5)(-5)(-5) = 625$

c) $-5^4 = -5 \times 5 \times 5 \times 5 = -625$

d) $(-3)^5 = (-3)(-3)(-3)(-3)(-3) = -243$

Note: As a shortcut here, you could recognize that a negative number is to be multiplied an odd number of times, so the result will be negative: $(-3)^5 = -3^5 = -243$

e) $\left(-\frac{2}{3}\right)^3 = -\left(\frac{2}{3}\right)^3 = -\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = -\frac{8}{27}$

Laws of Exponents

Multiplication

Recall that 2^3 means $2 \times 2 \times 2$ and 2^4 means $2 \times 2 \times 2 \times 2$.

Then $2^3 \times 2^4 = (2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2) = 2^7$ since there are seven 2s.

Law of Exponents for Multiplication (LEM)

When multiplying powers with equal bases, add the exponents:

$a^m \times a^n = a^{m+n}$

Examples

a) $(3^7)(3^{11}) = 3^{7+11} = 3^{18}$

b) $(-7)^5(-7)^6 = (-7)^{5+6} = (-7)^{11}$

c) $-5^4 \times 5 = -(5^{4+1}) = -5^5$

d) $6^3 \times 6^5 \times 6^2 = 6^{3+5+2} = 6^{10}$

Division

Since $2^3 \times 2^4 = 2^7$, it follows that $2^7 \div 2^4 = 2^3$, which is 2^{7-4}

OR

$$\frac{2^7}{2^4} = \frac{2\times2\times2\times2\times2\times2\times2}{2\times2\times2\times2} = \frac{128}{16} = 8 = 2^3$$

You could also approach this by dividing each of the 2s in the denominator with one of the 2s in the numerator, leaving three 2s in the numerator.

Law of Exponents for Division (LED)

When dividing powers with equal bases, subtract the exponents:

$a^m \div a^n = a^{m-n}$

Examples

a) $3^{11} \div 3^5 = 3^{11-5} = 3^6$

b) $(-7)^8 \div (-7)^7 = (-7)^{8-7} = (-7)^1 = -7$

c) $5^9 \div 5 = 5^{9-1} = 5^8$

Powers of Powers

2^3 means $2 \times 2 \times 2$. You can infer that $(2^5)^3$ means $(2^5)(2^5)(2^5)$.

So, $(2^5)^3 = (2^5)(2^5)(2^5) = 2^{5+5+5} = 2^{15}$ or $2^{5\times3}$. (LEM was used here.)

Similarly, $(3^7)^4 = (3^7)(3^7)(3^7)(3^7) = 3^{7+7+7+7} = 3^{28}$ or $3^{7\times4}$.

Law of Exponents for Powers (LEP)

When raising a power to an exponent (a power of a power), multiply the inner exponent by the outer exponent:

$(a^m)^n = a^{mn}$

Examples

a) $(3^{11})^5 = 3^{11\times5} = 3^{55}$

b) $(y^3)^2 = y^{3\times2} = y^6$

c) $(5^3)^2(5^4)^3 = 5^{3\times2} \times 5^{4\times3} = 5^6 \times 5^{12} = 5^{18}$

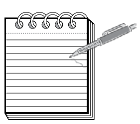
d) 8^5 as a power of 2 is $(2^3)^5 = 2^{15}$

e) 81^7 as a power of 3 is $(3^4)^7 = 3^{28}$

Powers of Products

$(xy)^3$ means $(xy)(xy)(xy)$, which in turn means x^3y^3.

Verify for yourself that $(2 \times 5)^3 = 2^3 \times 5^3$.



Law of Exponents for Powers of Products (LEPP)

When raising a product to a power, place the exponent of the product on each of the factors:

$(ab)^m = a^m b^m$

Examples

a) $(xy)^5 = x^5y^5$

b) $(3abc)^4 = 3^4a^4b^4c^4 = 81a^4b^4c^4$

c) $(-2x^2y^3)^5 = (-2)^5(x^2)^5(y^3)^5 = (-2)^5x^{2\times5}y^{3\times5} = -32x^{10}y^{15}$

Powers of Quotients

$\left(\frac{x}{y}\right)^3$ means $\left(\frac{x}{y}\right)\left(\frac{x}{y}\right)\left(\frac{x}{y}\right)$, which in turn means $\frac{x^3}{y^3}$.

Verify for yourself that $\left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3}$.

Law of Exponents for Powers of Quotients (LEPQ)

When raising a quotient to a power, place the exponent of the quotient on the numerator and the denominator:

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Examples

a) $\left(\frac{x}{y}\right)^5 = \frac{x^5}{y^5}$

b) $\left(\frac{-2}{3}\right)^3 = \frac{(-2)^3}{3^3} = \frac{-8}{27}$

c) $\left(\frac{x^2}{y^3}\right)^5 = \frac{\left(x^2\right)^5}{\left(y^3\right)^5} = \frac{x^{10}}{y^{15}}$

d) $\left(\frac{2x^3y^5}{3z^4}\right)^2 = \frac{2^2\left(x^3\right)^2\left(y^5\right)^2}{3^2\left(z^4\right)^2} = \frac{2^2x^6y^{10}}{3^2z^8} = \frac{4x^6y^{10}}{9z^8}$

e) $\left(\frac{2}{5}\right)^3 = \left(\frac{2}{5}\right)\left(\frac{2}{5}\right)\left(\frac{2}{5}\right) = \frac{2\times2\times2}{5\times5\times5} = \frac{2^3}{5^3}$

Zero and Negative Exponents

Zero Exponents

Recall that any number divided by itself equals 1. That is, $7 \div 7 = 1$, $23 \div 23 = 1$, $0.1 \div 0.1 = 1$, $(-73) \div (-73) = 1$.

In the same way, you can determine that $2^3 \div 2^3 = 1$ $(8 \div 8 = 1)$.

Using LED, you can determine that $2^3 \div 2^3 = 2^{3-3} = 2^0$.

Therefore, $2^3 \div 2^3$ gives two different answers: $2^3 \div 2^3 = 1$ and $2^3 \div 2^3 = 2^0$.

One question cannot give two different answers, so you must conclude that $2^0 = 1$.

Similarly, $x^a \div x^a = 1$ and $x^a \div x^a = x^0$.

You must then conclude that a number raised to the exponent 0 equals 1:

Rule: $x^0 = 1$

(This works for all numbers but 0, as 0^0 is undefined.)

Negative Exponents

Try dividing 2^3 by 2^7.

Using LED: $2^3 \div 2^7 = 2^{3-7} = 2^{-4}$

Using numerical equivalences: $2^3 \div 2^7 = \frac{2^3}{2^7} = \frac{8}{128} = \frac{1}{16} = \frac{1}{2^4}$

You then conclude that $2^{-4} = \frac{1}{2^4}$.

Rule: $x^{-a} = \frac{1}{x^a}$ and $x^a = \frac{1}{x^{-a}}$

Examples

Evaluate each of the following:

a) 7^0

b) $(-3)^{-3}$

c) $\frac{1}{7^{-2}}$

d) $3^0 + 3^{-2}$

e) $\frac{2^{-3} + 3^{-2}}{3^{-1} - 2^{-2}}$

Solutions

a) $7^0 = 1$

b) $(-3)^{-3} = \dfrac{1}{(-3)^3} = \dfrac{1}{-27} = -\dfrac{1}{27}$

c) $\dfrac{1}{7^{-2}} = 7^2 = 49$

d) $3^0 + 3^{-2} = 1 + \dfrac{1}{3^2} = 1 + \dfrac{1}{9} = \dfrac{10}{9}$

e) $\dfrac{2^{-3} + 3^{-2}}{3^{-1} - 2^{-2}} = \dfrac{\dfrac{1}{2^3} + \dfrac{1}{3^2}}{\dfrac{1}{3^1} - \dfrac{1}{2^2}} = \dfrac{\dfrac{1}{8} + \dfrac{1}{9}}{\dfrac{1}{3} - \dfrac{1}{4}} = \dfrac{\dfrac{9}{72} + \dfrac{8}{72}}{\dfrac{4}{12} - \dfrac{3}{12}} = \dfrac{\dfrac{17}{72}}{\dfrac{1}{12}} = \dfrac{17}{72} \times \dfrac{12}{1} = \dfrac{17}{6}$

Rational Exponents

What is meant by $8^{\frac{1}{3}}$?

Recall the rules for multiplying exponents to rationalize the following statement: $8^{\frac{1}{3}} \times 8^{\frac{1}{3}} \times 8^{\frac{1}{3}} = 8^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = 8^{\frac{3}{3}} = 8^1 = 8$

In other words, this is a number that multiplied by itself three times gives 8.

Another number that multiplies by itself three times to give 8 is 2.

So $8^{\frac{1}{3}} \times 8^{\frac{1}{3}} \times 8^{\frac{1}{3}} = 8$ and $2 \times 2 \times 2 = 8$.

You can conclude that $8^{\frac{1}{3}}$ = 2. **Note:** $2 = \sqrt[3]{8}$

Similarly, $25^{\frac{1}{2}} \times 25^{\frac{1}{2}} = 25^{\frac{1}{2}+\frac{1}{2}} = 25^{\frac{2}{2}} = 25^1 = 25$ and $5 \times 5 = 25$.

You can conclude that $25^{\frac{1}{2}} = 5$. **Note:** $5 = \sqrt{25}$

Rule: $x^{\frac{1}{n}} = \sqrt[n]{x}$

In other words, what number multiplied by itself n times results in x?

(Many calculators have a button labelled either $x^{\frac{1}{n}}$ or $\sqrt[n]{x}$, while others have a key with a "^" or exponentiation symbol. While these are valuable, they cannot always be used, and it is often preferable to have a fractional answer rather than a decimal result.)

Examples

Evaluate each of the following:

a) $81^{\frac{1}{4}}$

b) $(-27)^{\frac{1}{3}}$

c) $49^{-\frac{1}{2}}$

d) $16^{\frac{3}{4}}$

e) $125^{-\frac{2}{3}}$

Solutions

a) $81^{\frac{1}{4}} = 3$

b) $(-27)^{\frac{1}{3}} = -3$

c) $49^{-\frac{1}{2}} = \frac{1}{49^{\frac{1}{2}}} = \frac{1}{7}$

d) $16^{\frac{3}{4}} = \left(16^{\frac{1}{4}}\right)^3 = (2)^3 = 8$

e) $125^{-\frac{2}{3}} = \left(125^{\frac{1}{3}}\right)^{-2} = (5)^{-2} = \frac{1}{5^2} = \frac{1}{25}$

Support Questions
(do not send in for evaluation)

Be sure to try the Support Questions on your own before looking at the suggested answers provided. Click on each "Suggested answer" button to check your work.

1. Simplify each of the following:

   a) $(x^2)(x^3)$ b) $x^{19} \times x^9$ c) $(2x^2y^3)(-3x^3y^5)$

   d) $x^{19} \div x^9$ e) $x^4 \div x$ f) $\dfrac{-27x^{12}y^{15}}{9x^6y^7}$

   g) $(2^3)^4$ h) $(3x^4)^3$ i) $\dfrac{\left(2x^3y^7\right)^5}{\left(2x^2y^5\right)^7}$

   j) $\left(\dfrac{2x^2}{3y^3}\right)^6 \div \left(\dfrac{2x}{3y^2}\right)^4$

   Suggested answer

2. Evaluate each of the following:

   a) 3^{-2} b) $7^0 + (-9)^0$ c) $3^{-1} + 2^{-3}$

   d) $16^{\frac{3}{4}}$ e) $(-125)^{\frac{5}{3}}$ f) $8^{-\frac{2}{3}}$

   Suggested answer

3. Express each of the following with a single base:

   a) $(2^3)^2(4^2)^4(8^3)^5$ b) $(-125)^3 \div (25)^4$

   Suggested answer

Part B: Graphs of Exponential Functions

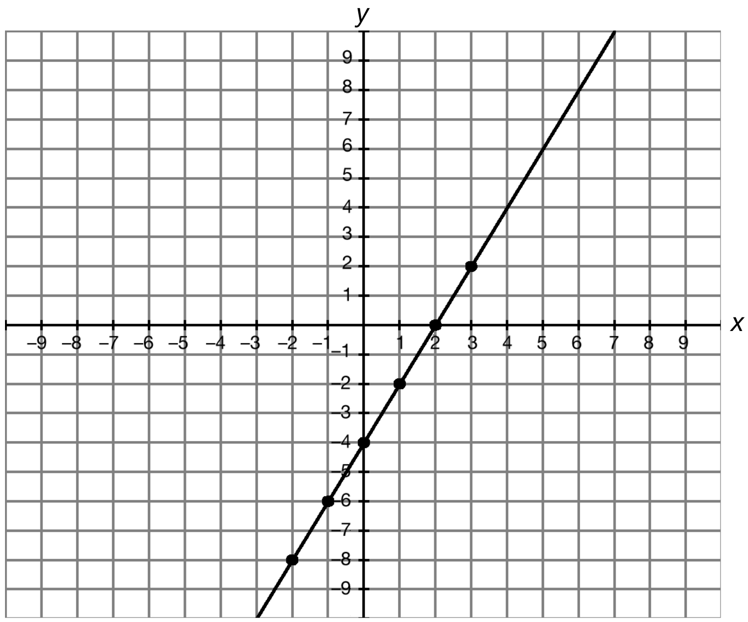
In earlier courses, you first learned to graph functions by substituting values for x into an equation that then gave you some y-values. This combination of x and y values together became defined as ordered pairs or points, which you then graphed. The points you graphed were then joined by a line or curve that passed through all the points.

Example 1

To graph the equation $y = 2x - 4$, you first constructed a table of values using selected values of x to find values for y, and then determined the set of points.

x	*y*	point
–2	2(–2) – 4 = –4 – 4 = –8	(–2,–8)
–1	2(–1) – 4 = –2 – 4 = –6	(–1,–6)
0	2(0) – 4 = 0 – 4 = –4	(0,–4)
1	2(1) – 4 = 2 – 4 = –2	(1,–2)
2	2(2) – 4 = 4 – 4 = 0	(2,0)
3	2(3) – 4 = 6 – 4 = 2	(3,2)

The points were then plotted on a grid and joined with a line.

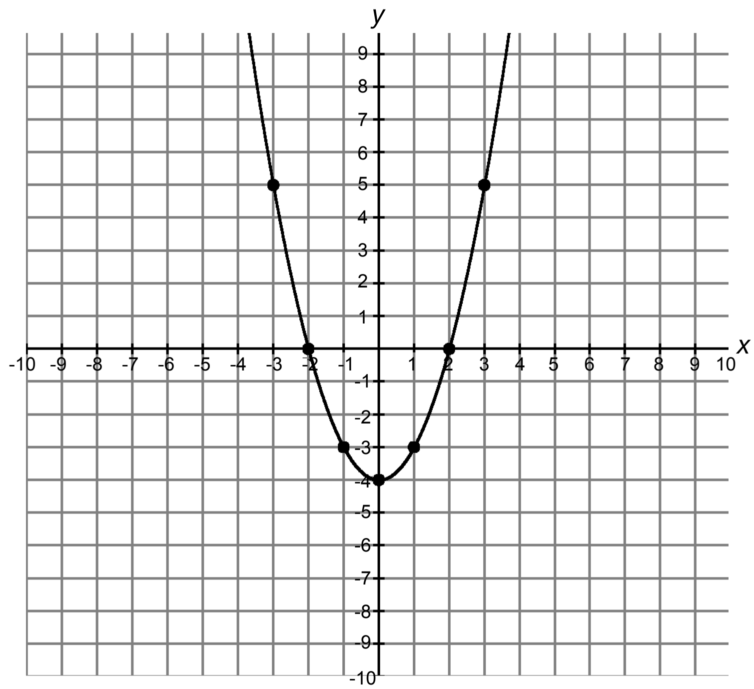


Example 2

To graph the equation $y = x^2 - 4$, you first constructed a table of values using selected values of x to find values for y, and then determined the set of points.

x	*y*	point
–3	$(-3)^2 - 4 = 9 - 4 = 5$	(–3,5)
–2	$(-2)^2 - 4 = 4 - 4 = 0$	(–2,0)
–1	$(-1)^2 - 4 = 1 - 4 = -3$	(–1,–3)
0	$(0)^2 - 4 = 0 - 4 = -4$	(0,–4)
1	$(1)^2 - 4 = 1 - 4 = -3$	(1,–3)
2	$(2)^2 - 4 = 4 - 4 = 0$	(2,0)
3	$(3)^2 - 4 = 9 - 4 = 5$	(3,5)

The points were then plotted on a grid and joined with a curve.



Eventually, you were able to graph the function right from the equations without calculating and then plotting individual points. You learned the features of the class of curves.

Example 1 is an example of a linear function, which you can graph after understanding that the graph of a line depends on the slope and y-intercept.

Example 2 is an example of a quadratic function, which you can graph once you know how to find the vertex, direction of opening, and stretch factor.

In this section, you will examine the curve of the exponential function by looking first at some examples of basic exponential functions.

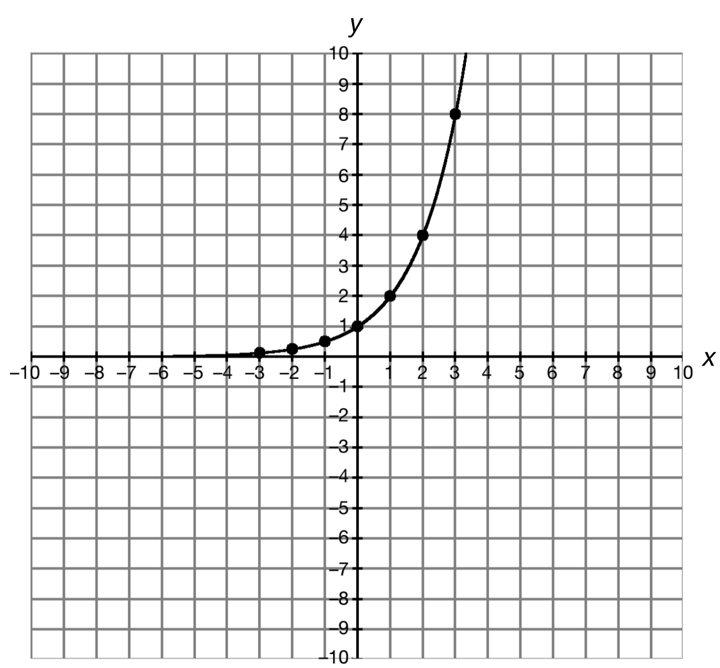
1. You will plot individual points.
2. You will join them with a smooth curve.
3. You will then look at some of the common features of the curves.
4. You will make predictions on what a curve should look like.

Example 3

To graph the equation $y = 2^x$, first construct a table of values using selected values of x to find values for y and then determine the set of points.

x	*y*	point
–3	$2^{-3} = \frac{1}{8}$	$(-3, \frac{1}{8})$
–2	$2^{-2} = \frac{1}{4}$	$(-2, \frac{1}{4})$
–1	$2^{-1} = \frac{1}{2}$	$(-1, \frac{1}{2})$
0	$2^0 = 1$	(0,1)
1	$2^1 = 2$	(1,2)
2	$2^2 = 4$	(2,4)
3	$2^3 = 8$	(3,8)

The points are then plotted on a grid and joined with a curve.



Do the following Support Questions carefully. Summarize your observations in your notes so you may refer to them at a later date.

Support Questions
(do not send in for evaluation)

4. Graph the following on the same axes:

   a) $y = 2^x$ b) $y = 3^x$ c) $y = 4^x$ d) $y = 5^x$

   List all the features the graphs have in common.

   What effect does changing the base have on the function?

   Predict what would happen to the graph if the base were the number 1.

   Suggested answer

5. Graph the following on the same axes:

   a) $y = \left(\frac{1}{2}\right)^x$ b) $y = \left(\frac{1}{3}\right)^x$ c) $y = \left(\frac{1}{4}\right)^x$

   d) $y = \left(\frac{1}{5}\right)^x$

   List all the features the graphs have in common.

   How do each of these compare to the graphs in Support Question 4?

   Express each equation in another form.

   Suggested answer

6. Graph the following on the same axes:

   a) $y = 2^x$ b) $y = 2^{-x}$ c) $y = -2^x$

   Suggested answer

7. Graph the following on the same set of axes:

   a) $y = 3^x$ b) $y = 3^{-x}$ c) $y = -3^x$

   Suggested answer

Part C: Applications of Exponential Functions

Many naturally occurring phenomena grow or decay exponentially:

- Bacteria divide at a constant rate, thereby consistently doubling their number.
- Half the amount of a radioactive element will change to a different isotope at a constant rate.
- Money invested in a compound interest savings account will grow at an exponential rate.

In this section, you will look at some examples of applications of exponential functions.

Example

A type of bacteria doubles in number every six hours. If you started with 2000 bacteria, how many would there be after three days?

The formula for exponential doubling is $N(t) = N_0 2^{\frac{t}{d}}$:

$N(t)$: the number at time t

N_0: the initial number or concentration at time 0

2: because the population is doubling

t: represents the elapsed time since measuring started (express this in the same units as d)

d: the doubling period

Solution

$N_0 = 2000$, $t = 72$ (3 days × 24 hours per day), $d = 6$

$$N(t) = N_0 2^{\frac{t}{d}}$$

$$N(72) = 2000 \times 2^{\frac{72}{6}}$$

$N(72) = 2000 \times 2^{12}$

$N(72) = 2000 \times 4096$

$N(72) = 8\ 192\ 000$

There would be 8 192 000 bacteria after three days.

Examples

Carbon-14 changes to nitrogen-14 with a half-life of 5730 years. (Half-life means one half of the remaining C-14 will change to N-14 after 5730 years.)

a) If you start with 16 g, how much will you have after 17 190 years?

b) How long would it take to decay to $\frac{1}{8}$ g if you start with 16 g?

c) A bone was found in which $\frac{31}{32}$ of the C-14 had decayed to N-14. How old was the bone?

The formula for half-life is $C(t) = C_0 2^{\frac{-t}{H}}$ or $C(t) = C_0 \left(\frac{1}{2}\right)^{\frac{t}{H}}$

(Explain to yourself why both are equivalent.)

$C(t)$: the number at time t

C_0: the initial number or concentration at time 0

2^{-1} or $\left(\frac{1}{2}\right)$: because the population is halving

t: represents the elapsed time since measuring started (express this in the same units as H)

H: the half-life

Solutions

a)
$$C(t) = C_0 2^{\frac{-t}{H}}$$
$$C(t) = 16 \times (2)^{\frac{-17190}{5730}}$$
$$C(t) = 16 \times (2)^{-3}$$
$$C(t) = 16\left(\frac{1}{8}\right)$$
$$C(t) = 2$$

2 g of C-14 would remain.

b)
$$C(t) = C_0 2^{\frac{-t}{H}}$$
$$C(t) = \frac{1}{8}, C_0 = 16, H = 5730$$
$$\frac{1}{8} = 16 \times 2^{\frac{-t}{5730}}$$
$$\frac{1}{128} = 2^{\frac{-t}{5730}}$$
$$2^{-7} = 2^{\frac{-t}{5730}}$$
$$-7 = \frac{-t}{5730} \quad \text{(Exponents are equal since bases are equal.)}$$
$$t = 40\,110$$

It would take 40 110 years to decay to $\frac{1}{8}$ g.

c) $\frac{31}{32}$ had decayed, therefore $\frac{1}{32}$ remained, so $C(t) = \frac{1}{32}C_0$.

$$C(t) = C_0 2^{\frac{-t}{H}}$$
$$\frac{1}{32}C_0 = C_0 2^{\frac{-t}{5730}}$$
$$\frac{1}{32} = 2^{\frac{-t}{5730}}$$
$$2^{-5} = 2^{\frac{-t}{5730}}$$
$$-5 = \frac{-t}{5730}$$
$$t = 28\,650$$

The bone is 28 650 years old.

Examples

Money is deposited into a compound interest savings account.

a) $3000 is invested at 7% per annum for nine years.

What will it be worth?

b) What interest rate is needed for $5000 to grow to $15 000 in 10 years?

Solutions

The formula for compound interest is $A = P(1 + i)^n$:

A: the amount the account will grow to

P: the principal, or amount initially invested

i: the annual interest rate

n: the number of years for which the principal is invested

a) $P = 3000$, $i = 7\% = 0.07$, $t = 9$

$A = P(1 + i)^n$

$A = 3000(1 + 0.07)^9$

$A = 3000(1.07)^9$

$A = 3000 \times 1.838459$

$A = 5515.377$

The account will be worth \$5515.38.

b) $P = 5000$, $A = 15\ 000$, $n = 10$

$A = P(1 + i)^n$

$15\ 000 = 5000(1 + i)^{10}$

$$\frac{15\,000}{5000} = \frac{5000(1+i)^{10}}{5000}$$

$3 = (1 + i)^{10}$

$$(3)^{\frac{1}{10}} = \left((1+i)^{10}\right)^{\frac{1}{10}}$$

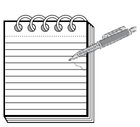
$$(3)^{\frac{1}{10}} = (1+i)^{\frac{10}{10}} = (1+i)^1$$

$$(3)^{\frac{1}{10}} = 1+i$$

$1.116123 = 1 + i$

$i = 0.116123$

The interest rate would have to be about 11.6%.



It will be important to remember the equations used in the above examples.

Support Questions
(do not send in for evaluation)

8. Carbon-14 has a half-life of 5730 years.

   a) If you start with 16 g, how much is left after

      i) 11 460 years?

      ii) 2865 years?

      iii) 57 300 years?

   b) How long does it take to decay to the point where there is

      i) 1 g?

      ii) 0.125 g?

   c) A bone is found in which $\frac{7}{8}$ of the original carbon-14 has decayed. How old is the bone?

Suggested answer

9. The formula for the amount earned at compound interest is $A = P(1 + i)^n$.

   a) What would \$3000 invested at 7% per annum for eight years grow to?

   b) What interest rate would cause \$2000 to grow to \$4000 after 12 years?

Suggested answer

10. Alberta has a population growth rate of 3.1% per year. Its population in 2007 was approximately 3.5 million.

   a) Write an expression that predicts Alberta's population. (Use 2007 as year 0.)

   b) Predict Alberta's population in i) 2017 and in ii) 2027 if the 2007 growth rate continues.

   c) If the growth rate has been consistent, what was Alberta's population in i) 2000 and in ii) 1997?

Suggested answer

Key Questions

Now work on your Key Questions in the online submission tool. You may continue to work at this task over several sessions, but be sure to save your work each time. When you have answered all the unit's Key Questions, submit your work to the ILC.

(26 marks)

1. Simplify each of the following. Express all answers with positive exponents if possible. **(5 marks)**

   a) $(2x^3)(-7x^5)$ b) $\dfrac{-45x^5y^7z^9}{9x^7y^7z^5}$ c) $(-5x^4)^3$

   d) $\dfrac{\left(3x^4y^5z^7\right)^5}{\left(-3x^3yz^4\right)^7}$ e) $\dfrac{\left(x^{a+b}\right)^{a-b}}{\left(x^{a-2b}\right)^{a+2b}}$

2. Evaluate each of the following. Leave your answers in fraction form with no decimals. **(4 marks)**

   a) 6^{-2} b) $25^{\frac{3}{2}}$ c) $-8^{\frac{5}{3}}$ d) $625^{-\frac{3}{4}}$

3. Express each of the following with a single base. **(6 marks: 3 marks each)**

   a) $9^3 \times 27^2 \times 81^3$ b) $\dfrac{5^7 \times 25^3}{125^4}$

4. Graph the following on the same set of axes. **(3 marks)**

   - $y = 5^x$
   - $y = -5^x$
   - $y = 5^{-x}$

5. State the equation of the exponential function that has the following features. **(2 marks)**

   - It has the x-axis as an asymptote.
   - It passes through $(0,-1)$, $(-1,-3)$, and $(-2,-9)$.

6. Beryllium-11 decomposes into boron-11 with a half-life of 13.8 seconds. How long will it take 2400 g of beryllium-11 to decompose into 75 g of beryllium-11? **(3 marks)**

7. A car depreciates at a rate of 30% per year. How much will a $20 000 car be worth after five years? **(3 marks)**

Now go on to Lesson 2. Send your answers to the Key Questions to ILC when you have completed Unit 1 (Lessons 1 to 5).

