

# MAAE 2300 – Fluid Mechanics 1

## Laboratory #2: Experiment 2

Jet Pump

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## **Summary**

The objective of this experiment was to apply the basic fluid mechanics techniques learnt in the course and analyze the performance of a jet pump under certain assumptions and compare that with the measured output of the device. The experiment was carried out to analyze two different flows, one being an open-throttle system referred to as Flow 1 and the other being a closed-throttle system referred to as Flow 2.

Jet Pump is a device in which a primary stream of flow is injected with a certain pressure and velocity into a tube with a cross-sectional area greater than that of the primary tube. The flow entering through the bell mouth inlet is the secondary flow taken from atmosphere surrounding and has relatively smaller velocity than that of the primary flow. The primary and the secondary flow along the mixing tube achieve a steady and stabilized velocity and static pressure profile. The purpose of this experiment was to compare the measured and predicted velocity and static pressure profiles based on the assumptions made and analysis carried out.

For open throttle flow, the measured velocity and static pressure at the outlet plane was found out to be  $20\text{ m/s}$  and  $101661.6\text{ Pa}(a)$  whereas the actual velocity and static pressure was found out to be  $22.51\text{ m/s}$  and  $101565.3\text{ Pa}(a)$ . The percentage error for velocity and pressure were found out to be  $12.55\%$  and  $28.57\%$  respectively. For closed throttle flow, the measured velocity and static pressure at the outlet plane was found out to be  $11.2\text{ m/s}$  and  $102619\text{ Pa}(a)$  whereas the actual velocity and static pressure was found out to be  $9.77\text{ m/s}$  and  $102031\text{ Pa}(a)$ . The percentage error for velocity and pressure were found out to be  $12.76\%$  and  $45.44\%$  respectively.

## **Nomenclature**

$\rho_{\text{air}}$ :	Density of air ( $1.20\text{ kg/m}^3$ )
$\rho_{\text{water}}$ :	Density of water ( $1000\text{ kg/m}^3$ )
$g$ :	Acceleration due to gravity ( $9.81\text{ m/s}^2$ )
$P_{\text{atm}}$ :	Atmospheric pressure ( $101325\text{ Pa}$ )
$C_q$ :	Dynamic pressure coefficient ( $0.93$ )
$C_p$ :	Static pressure coefficient ( $-0.045$ )
$Q$ :	Volume Flow Rate ( $\text{m}^3/\text{s}$ )
$h_{\text{manometer}}$ :	Water Column height of manometer (m)
$V_P$ :	Velocity of primary airflow ( $\text{m/s}$ )

$V_s$ :	Velocity of secondary airflow (m/s)
$V_{outlet}$ :	Velocity of air at outlet (m/s)
$V_{radius}$ :	Velocity at the tube's radius (m <sup>3</sup> /s)
$A_p$ :	Cross-sectional area of primary outlet (m <sup>2</sup> )
$A_s$ :	Cross-sectional area of secondary inlet (m <sup>2</sup> )
$A_{out}$ :	Cross-sectional area of mixing tube (m <sup>2</sup> )
$\Delta h$ :	Change in manometer height (m)
$P_p$ :	Primary static pressure (Pa)
$P_s$ :	Secondary static pressure (Pa)
$P_{out}$ :	Outlet static pressure (Pa)
$P_{C1}$ :	Primary supply pressure (Pa)
$P_{C2}$ :	Primary nozzle pressure (Pa)

## **Flow Analysis**

Please refer to appendix 4- Data Sheet for the complete set of data values recorded during the experiment.

### 1. Velocity and Static Pressure at the mixing tube outlet

The velocity and the static pressure at the mixing tube outlet plane involved the calculation of the primary inlet velocity and static pressure, secondary inlet velocity and static pressure for which the flow analysis is shown below.

#### Stagnation and Static Pressure due to Static taps and Pitot tube

For the two open and closed throttle flows, the static pressure was calculated by applying the hydrostatic pressure equation on different manometer readings.

$$P_{atm} + \rho_{water} gh_{atm} - \rho_{water} gh_{static,i} = P_{static}$$

Here  $h_{atm}$  represents the height in the manometer column that was open to the atmosphere and  $h_{static,i}$  represents the height, also called static head in this case, achieved in the manometer column connected to the  $i^{th}$  static tap in the mixing tube. The manometer columns measuring the static pressures were connected from static tap 1 up to static tap 17. Static taps 18, 19 and 20 were connected to

manometer columns open to atmosphere. Rearranging the above equation introduced equation 1.

$$P_{static} = P_{atm} + \rho_{water} g \Delta h_{static} \quad [1]$$

Where  $\Delta h_{static} = h_{atm} - h_{static,i}$  is also known as the static head for the corresponding static tap.

Similarly, the stagnation pressure was measured due to the change in heights in manometer columns connected to the pitot tubes. By applying hydrostatic pressure equation at these columns, the stagnation pressure was formulated as

$$P_{stagnation} = P_{atm} + \rho_{water} g \Delta h_{stagnation} \quad [2]$$

Where  $\Delta h_{stagnation} = h_{atm} - h_{stagnation,i}$  is also known as the stagnation head for the corresponding pitot tube. Over here,  $h_{atm}$  is the height in water column open to atmosphere and  $h_{stagnation,i}$  is the height in the water column connected to the  $i^{th}$  pitot tube. Readings were recorded from pitot tubes 1 to 19.

The static and stagnation pressure values calculated for flow 1- open throttle and flow 2- closed throttle are been included in Table 1 and Table 2 respectively in Appendix 4- Data Sheets.

### Primary Velocity and Static Pressure

The primary velocity  $v_p$ , pressure upstream of the nozzle  $P_{C1}$  and the pressure at the nozzle outlet  $P_{C2}$  were found to have a relation with the dynamic pressure coefficient  $C_q$  and constant coefficient  $q$  as follows

$$C_q = \frac{q}{P_{C1} - P_{C2}}, \text{ and}$$

$$q = \frac{1}{2} \rho_{air} v_p^2$$

The above two equations were taken from the MAAE 2300 Course Manual- Experiment 2. Using these equations  $v_p$  was isolated to give equation 3.

$$v_p = \sqrt{\frac{2C_q (P_{C1} - P_{C2})}{\rho_{air}}} \quad [3]$$

Similarly, the coefficient of static pressure was defined by equation 4.

$$C_P = \frac{P_P - P_{C2}}{P_{C1} - P_{C2}} \quad [4]$$

Using equation 4, the primary pressure entering the mixing tube was formulated as

$$P_p = P_{C2} + C_p(P_{C1} - P_{C2}) \quad [5]$$

The value for the dynamic pressure coefficient and the static pressure coefficient were given during the experiment and are recorded in the data sheet included in Appendix 4. They required pressure upstream  $P_{C1}$  and pressure outlet of the nozzle  $P_{C2}$  which was again calculated using the hydrostatic pressure equations as defined by equations 6 and 7 respectively.

$$P_{C1} = P_{atm} + \rho_{water} g \Delta h_{U-tube} \quad [6]$$

$$P_{C2} = P_{atm} + \rho_{water} g \Delta h_{staticTap1} \quad [7]$$

Equations 3 and 5 gave the primary state of the fluid entering the mixing tube.

### Secondary Velocity and Static Pressure

The secondary static pressure of the fluid entering through the bell-mouth inlet was measured due to change in the static head from the second static tap. It was calculated by implementing hydrostatic pressure equation for water column connected to static tap 2.

$$P_s = P_{atm} + \rho_{water} g \Delta h_{staticTap2} \quad [8]$$

Since the pressure of the fluid entering through the secondary inlet wasn't constant and varying significantly, the velocity of the fluid entering through the bell-mouth inlet was calculated by applying Bernoulli's equation. To apply Bernoulli's equation, 1-D steady state friction less flow with no energy transfer in any form was assumed and a streamline was chosen in relation from a point far away from the bell-mouth inlet to a point just at the entrance of the mixing tube. This streamline was chosen so that the velocity at the first point could be assumed to be zero and the pressure to be atmospheric pressure.

$$P_{atm} + \rho g h_1 + \frac{1}{2} \rho v_1^2 = P_s + \rho g h_2 + \frac{1}{2} \rho v_s^2$$

Since the streamline chosen was at same elevation,  $h_1 = h_2$  and  $v_1 = 0$ , giving

$$v_s = \sqrt{\frac{2(P_{atm} - P_s)}{\rho_{air}}} \quad [9]$$

Where  $P_s$  in equation 9 was taken from equation 8.

Equations 8 and 9 gave the secondary state of the fluid entering the mixing tube.

### Velocity and Static Pressure at the mixing tube outlet plane (Pitot tube rake)

To measure the predicted velocity and the pressure at the outlet plane of the mixing tube, a control volume was chosen from the very entrance of the primary fluid

(static tap 3, in particular) to the plane just before the pitot tube rake (up to static tap 17, in particular). This control volume (shown in Figure 1) was chosen because of the assumption made that there was no friction acting on the fluid and hence no formation boundary layers to oppose the flow of the fluid. The primary and secondary state at one plane of control volume was known from above calculations and the pressure at the other plane was to be calculated.

In addition to steady state flow, incompressible flow was assumed which meant that the density of the fluid ( $\rho_{air}$ ) remained constant over time and did not change. This allowed applying the continuity equation by equating the inlet volume flow rate to the outlet volume flow rate.

$$A_P v_P + A_S v_S = A_{outlet} v_{outlet}$$

Where  $A$  is the cross-sectional area given by  $A = \frac{\pi d^2}{4}$

Using the above equation, the velocity at outlet plane was formulated as

$$v_{outlet} = \frac{A_P v_P + A_S v_S}{A_{outlet}} \quad [10]$$

To calculate the pressure at the outlet plane, linear momentum equation was deployed on the above mentioned control volume.

$$\sum F_x = \sum \dot{m}_{outlet} v_{outlet} - \sum \dot{m}_{inlet} v_{inlet}$$

$$P_P A_P + P_S A_S - P_{outlet} A_{outlet} = \rho_{air} A_{out} v_{out}^2 - \rho_{air} A_P v_P^2 - \rho_{air} A_S v_S^2$$

Isolating for  $P_{outlet}$ , equation 11 was introduced to calculate the pressure at the outlet plane of the control volume shown in Figure 1.

$$P_{outlet} = \frac{P_P A_P + P_S A_S - \rho_{air} (A_{out} v_{out}^2 - A_P v_P^2 - A_S v_S^2)}{A_{outlet}} \quad [11]$$

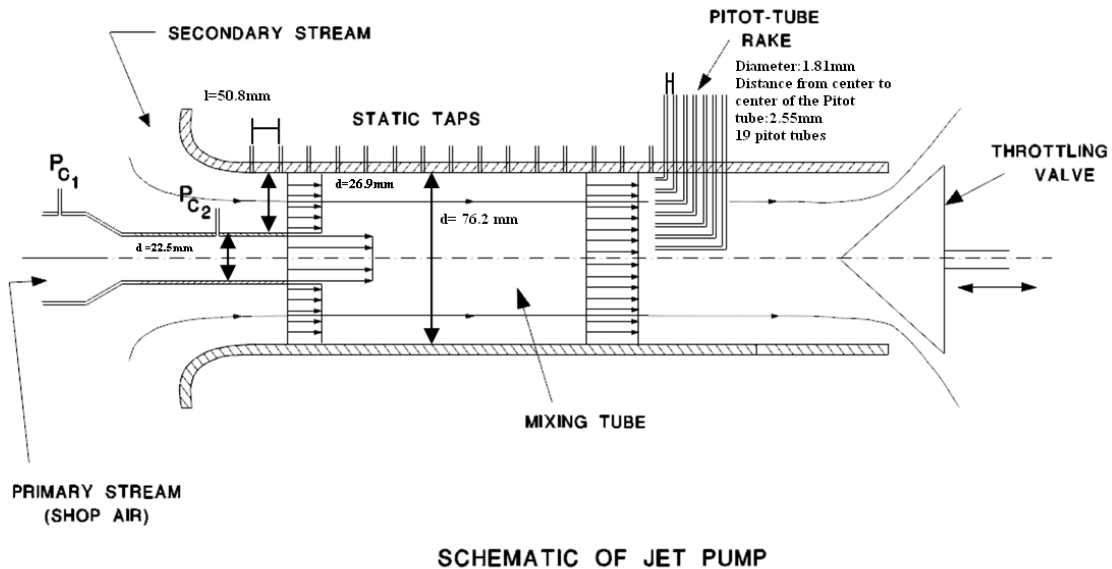
## 2. Outlet Velocity as a function of radius

The measured velocity along the radius of the tube was calculated using the dynamic pressure as follows.

$$v_{radiusMeasured} = \sqrt{\frac{2(P_{stagnation} - P_{static})}{\rho_{air}}} \quad [12]$$

The predicted velocity along the radius of the tube was calculated based on the control volume analysis done to find the velocity and the static pressure at the outlet of the plane i.e. at the pitot tube rack.

## Experimental Setup and Procedure



**Figure 1: Schematic Diagram of a cross-sectional view of Jet Pump [modified from 1].**

The procedure followed in this experiment was entirely based on the procedure outlined in MAAE 2300 course manual 'Experiment 2: Jet Pump'. No major deviations were encountered, however, human and experimental errors are anticipated.

Assumptions were made to carry out this experiment which are clearly mentioned in Flow Analysis. The flow was assumed to be steady state 1-D flow having no friction and no transfer of heat energy to be able to apply Bernoulli's Equation.

## Results and Discussion

The primary pressures at the upstream and the downstream of the nozzle as well as pressure of the secondary flow was calculated using Bernoulli's equation. To apply Bernoulli's equation, 1-D steady state frictionless flow was assumed with no transfer of heat in any form such as sound energy or energy due to friction. Bernoulli's equation contain the hydrostatic pressure equation within itself and since the fluid in the manometers was not moving, Bernoulli's equation was reduced to hydrostatic pressure equation to calculate corresponding pressures.

The absolute pressure at the upstream nozzle was calculated from equation 6 and was found out to be  $109526.2 \text{ Pa}$  for the open throttle system and  $109035.7 \text{ Pa}$  for the closed throttle system. The absolute pressure at the downstream nozzle was calculated from equation 7 and was found out to be  $101795.9 \text{ Pa}$  for the open throttle system and  $101560.5 \text{ Pa}$  for the closed throttle system. Using the values of the pressure upstream and

downstream of the nozzle, and the coefficient of dynamic pressure, the primary velocity of the flow was calculated using equation 3. The primary velocity for the open throttle system was found out to be 109.5 m/s whereas for closed throttle, it was found out to be 107.6 m/s. The coefficient of static pressure with the pressures upstream and downstream of the nozzle gave the primary pressure of the fluid entering the mixing tube by equation 5. For the open throttle system, this absolute primary pressure was found out to be 1101448 Pa and for closed throttle system the absolute pressure was 101224.1 Pa. Please refer to Appendix 2 for all the sample calculations done for the analysis.

The secondary velocity was then calculated by applying Bernoulli's equation from a point outside the bell-mouth inlet where the relative velocity could be assumed to zero and pressure to be atmospheric pressure to a point just at the bell-mouth inlet. This introduced equation 9 from which the secondary velocity of the flow could be calculated. This equation involved static pressure which was measured from the total head change due to static taps given by equation 8. Using these equations, the secondary velocity and absolute pressure for open throttle system was found out to be 21.4 m/s and 101050.3 Pa respectively. The same values for closed throttle system was found out to be 4.04 m/s and 101334.8 m/s.

To measure the velocity and static pressure at the outlet of the mixing tube, a control volume as shown in Figure 1 was chosen. This control volume was chosen because the dynamic state at one plane of the CV was known and the state at the other CV was to be calculated. Continuity equations with linear momentum equations were used to find a relation for the outlet pressure and velocity. Equation for the outlet velocity and equation 11 for the outlet pressure were derived to calculate the actual values. Please refer to the Flow Analysis section for the derivation of these equations. For the open throttle system, the velocity was found out to be 20 m/s and the absolute pressure was found out to be 101661.6 Pa. Similarly for the closed throttle system, the velocity was found out to be 11.2 m/s and the absolute pressure was found out to be 102619.8 Pa.

Equation 1 was then again used to measure the static pressure due to static taps present at an interval of 2 inches. Stagnation pressure was calculated by using equation 2 due to the stagnation head change due to pitot tubes. Please refer to Appendix 1 for the tabulated data of static and stagnation pressures for the two flows. To analyze this data, static pressure variation due to the static taps for both the flows was plotted on the graph. The predicted values of pressure were also plotted on the same graph.

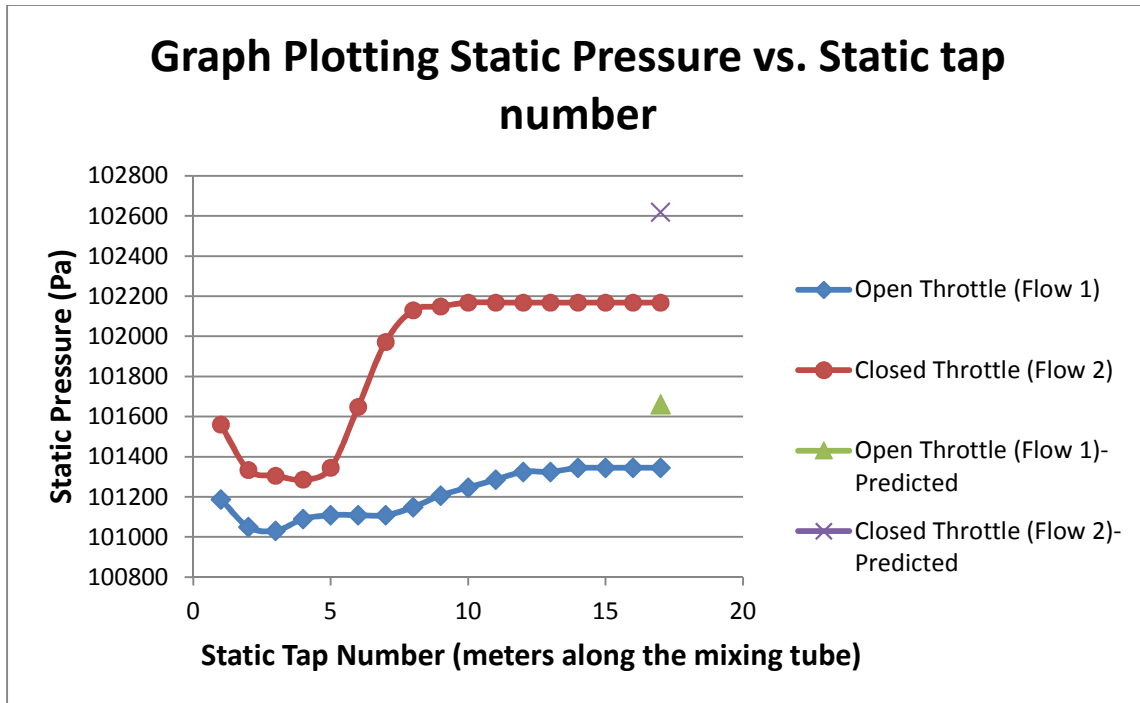
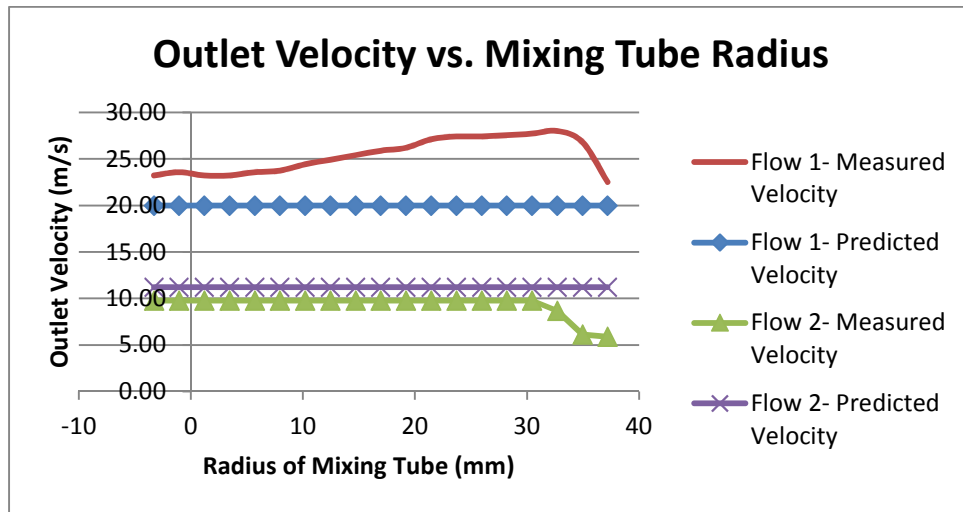


Figure 2: Graph plotting Stagnation pressure vs. static tap number for open and close throttle flows.

Figure 1 explains the behavior of the velocity profile of the fluid that contains both the primary and the secondary fluid. As evident from the graph for the closed throttle system, the velocity of the combined fluid first decreases up to static tap 5. This effect might have been caused since the secondary fluid entering the mixing tube with a relatively smaller velocity as that of the primary fluid might tend to slow down the combined velocity of the fluid. After static tap 5, the velocity of the fluid tending to combine increases significantly up to static tap 10 where after, the velocity profile becomes constant giving a constant static pressure of 102168.66 Pa. This proves that the assumption regarding the 1-dimensional flow of the fluid made in the beginning of the analysis was correct. This also proves that the length of the mixing tube was long enough for the two fluids to combine together and achieve a constant velocity profile, hence suggesting that the mixing process was complete by the end of the mixing tube.

The above graph also explains the open throttle system. The velocity profile follows the same trend, however, for the open throttle system, the static pressure along the mixing tube does not reach a constant value proving that the pressure was varying continuously. The reason for such a behavior might be the loss of energy since the mixing tube was well open to atmosphere and also due to the energy lost due to the friction acting on the walls of the tube thus generating boundary layers. These boundary layers would tend to grow bigger toward the centerline of the mixing tube introducing significant amount of turbulence in the fluid.

When comparing the actual velocity with the predicted velocity, an error of 12.55% for the first flow and an error of 12.76% for the second flow was encountered. This error is anticipated because the other assumptions made during the flow analysis were found to be not valid enough to support this velocity profile. A significant amount of energy lost due to friction force acting on the walls of the mixing tube as well as energy lost in the form of sound generated can be accounted as responsible for this error. Figure 3 shows the variation of velocity along the radius of the mixing tube that can be used to understand the error encountered above.



**Figure 3: Velocity variation of the fluid (air) along the radius of the mixing tube**

Figure 3 provides the velocity variation of the fluid measured along the radius of the mixing tube. The velocity was calculated by finding the dynamic pressure at each of these points along the radius of the tube. As it can be seen, for the open flow system, there is a significant change in the predicted velocity and the measured velocity profile. This is because the predicted velocity was calculated by making some assumptions mentioned above that were not found to be valid in this case. The measured velocity takes into account all the physical constraints that have been introduced into the system such as the friction due to the tube walls and the transfer of energy in any form such as sound. For the closed throttle system, the predicted and the measured velocity have a very minor deviation that suggest the application of Bernoulli's equation and assumptions made relating to it as explained above. The formation of boundary layers can also be explained since the viscous effects and viscosity of the fluid will tend to generate friction layers along the wall of the mixing tube.

Bonus analysis was done by comparing the inlet and the outlet flow rates for both the flows. Please refer Appendix 3- Bonus Analysis for sample calculations. For open-throttle system, the inlet and outlet volume flow rate were found out to be  $0.053\text{m}^3/\text{s}$  and

0.131m<sup>3</sup>/s with an error of 59.5%. For closed-throttle system, the inlet and outlet volume flow rate were found out to be 0.022m<sup>3</sup>/s and 0.058m<sup>3</sup>/s with an error of 62%.

Upon comparing the graphs in figure 2 and 3, a very strong correlation can be concluded. For the flow (closed-throttle) that wasn't affected much by the formation of boundary layers and force of friction, the flow was stabilized at nearly half the length of the mixing tube indicating complete mixing. However, for the flow (open-throttle) that had a significant amount of turbulence due to the formation of boundary layers from friction growing more towards the centerline introduced sheer wall stress and energy lost to the atmosphere and in the form of sound. The flow was found not to be stabilized and resulted in a requirement of a longer mixing tube length for the open flow system to flow stabilized and steadily.

Many sources of error might have existed while performing the experiment. Major sources of error include the human error is measuring the height from the manometers. Also, the height in the water columns were varying with time and were not constant hence generating a possible experimental source of error. Simplified assumptions such as constant density of air through the mixing tube and constant atmospheric pressure value as well as constant density of the liquid in the manometer were made. Such human and experimental error are expected and anticipated.

### **Conclusion**

From the data collected during the experiment and from the flow analysis done, the assumptions made in this lab were found to be nearly accurate. In terms of closed throttle system, the pressure variation and the velocity profile were found to obtain a relatively stabilized path, whereas in the case of open throttle system, the assumptions made did not account for the un-stabilized path of the flow. The assumptions trying to disregard the formation of boundary layers which tend to increase towards the centerline of the mixing tube was found to be incorrect. This concluded that boundary layer does appear in both the open and closed throttle systems. The viscosity and viscous effects of the fluid concluded that the kinetic and potential energy of the system is not constant and hence there is an energy transfer from one system to another.

### **References**

- [1] S.A. Sjolander, *MAAE 2300 Fluid Mechanics 1- Course Manual*, Department of Mechanical and Aerospace Engineering, Carleton University, September 2011
- [2] F.M. White, *Fluid Mechanics*, 5th edition, New York: McGraw-Hill, 2007
- [3] Bruce R. Munson, Donald F. Young, Theodore H. Okiishi, Wade W. Huebsch, *Fundamentals of Fluid Mechanics*, 6th edition, John Wiley & Sons, 2009

**Appendix 1**  
**Data Tables**

Table 1 tabulates the static and stagnation pressures values for Flow 1- open throttle system. The pressures have been measured by taking the height difference of corresponding static taps. Please refer to Appendix 2- Sample Calculations for the calculations carried out. The static taps were placed at 2 inch periods starting from the bell-mouth inlet plane and ending at the pitot tubes rake. Similarly, Table 2 tabulates the static and stagnation pressure values for Flow 2- closed throttle system.

Flow 1: Open						
No.	Height in Static taps (+/- 2 mm)	Height in Pitot Tubes (+/- 2 mm)	Change in Static Height (hATM - hTAP) (in m)	Change in Stagnation Height (hATM - hTAP) (in m)	Static Pressure (Pa)	Stagnation Pressure (Pa)
1	130	82	-0.014	0.048	101187.66	101795.88
2	144	70	-0.028	0.06	101050.32	101913.6
3	146	64	-0.03	0.066	101030.7	101972.46
4	140	60	-0.024	0.07	101089.56	102011.7
5	138	59	-0.022	0.071	101109.18	102021.51
6	138	60	-0.022	0.07	101109.18	102011.7
7	138	60	-0.022	0.07	101109.18	102011.7
8	134	58	-0.018	0.072	101148.42	102031.32
9	128	58	-0.012	0.072	101207.28	102031.32
10	124	56	-0.008	0.074	101246.52	102050.94
11	120	55	-0.004	0.075	101285.76	102060.75
12	116	54	0	0.076	101325	102070.56
13	116	57	0	0.073	101325	102041.13
14	114	59	0.002	0.071	101344.62	102021.51
15	114	60	0.002	0.07	101344.62	102011.7
16	114	62	0.002	0.068	101344.62	101992.08
17	114	62	0.002	0.068	101344.62	101992.08
Atm(18)	116	62	0	0.068	101325	101992.08
Atm(19)	116	64	0	0.066	101325	101972.46
Atm(20)	116	130	0	0	101325	101325

**Table 1: Static and Stagnation Pressure Values for Flow 1- Open Throttle**

Flow 1: Closed Throttle						
No.	Height in Static taps (+/- 2 mm)	Height in Pitot Tubes (+/- 2 mm)	Change in Static Height (hATM - hTAP) (in m)	Change in Stagnation Height (hATM - hTAP) (in m)	Static Pressure (Pa)	Stagnation Pressure (Pa)
1	104	54	0.024	0.074	101560.44	102050.94
2	127	56	0.001	0.072	101334.81	102031.32
3	130	56	-0.002	0.072	101305.38	102031.32
4	132	56	-0.004	0.072	101285.76	102031.32
5	126	56	0.002	0.072	101344.62	102031.32
6	95	56	0.033	0.072	101648.73	102031.32
7	62	56	0.066	0.072	101972.46	102031.32
8	46	56	0.082	0.072	102129.42	102031.32
9	44	56	0.084	0.072	102149.04	102031.32
10	42	56	0.086	0.072	102168.66	102031.32
11	42	56	0.086	0.072	102168.66	102031.32
12	42	56	0.086	0.072	102168.66	102031.32
13	42	56	0.086	0.072	102168.66	102031.32
14	42	56	0.086	0.072	102168.66	102031.32
15	42	56	0.086	0.072	102168.66	102031.32
16	42	56	0.086	0.072	102168.66	102031.32
17	42	56	0.086	0.072	102168.66	102031.32
18	127	56	0.001	0.072	101334.81	102031.32
19	127	56	0.001	0.072	101334.81	102031.32
Atm(20)	128	128	0	0	101325	101325

**Table 2: Static and Stagnation Pressure values for Flow 2- Closed Throttle**

Table 3 tabulates the measured and the predicted velocity values of the fluid along the radius of the mixing tube for Flow 1- open throttle system. The predicted value of the velocity was determined by the analysis done to calculate the velocity at the outlet plane of the control volume. The measured velocity was calculated using equation 12. Please refer to Appendix 2- Sample calculations for the velocity calculations carried out. Table 4 tabulates the measured and the predicted velocity values of the fluid for Flow 2- closed throttle system. Negative values of radius indicate the distance of pitot tubes below the centerline of the mixing tube.

Flow 1- Open Throttle				
No.	Stagnation Height Change (m)	Radius along the centerline (mm)	Measured Velocity (m/s)	Predicted Velocity (m/s)
		38.1		
1	0.048	37.195	22.51	20.00
2	0.06	34.945	26.82	20.00
3	0.066	32.695	28.01	20.00
4	0.07	30.445	27.72	20.00
5	0.071	28.195	27.57	20.00
6	0.07	25.945	27.42	20.00
7	0.07	23.695	27.42	20.00
8	0.072	21.445	27.12	20.00
9	0.072	19.195	26.20	20.00
10	0.074	16.945	25.89	20.00
11	0.075	14.695	25.41	20.00
12	0.076	12.445	24.93	20.00
13	0.073	10.195	24.43	20.00
14	0.071	7.945	23.75	20.00
15	0.07	5.695	23.58	20.00
16	0.068	3.445	23.23	20.00
17	0.068	1.195	23.23	20.00
18	0.068	-1.055	23.58	20.00
19	0.066	-3.305	23.23	20.00
20	0	-5.555	23.58	20.00

**Table 3: Measured and Predicted velocity values of fluid along the mixing tube for Flow 1- Open Throttle System**

Flow 2- Closed Throttle				
No.	Stagnation Height Change (m)	Radius along the centerline (mm)	Measured Velocity (m/s)	Predicted Velocity (m/s)
		38.1		
1	0.074	37.195	5.89	11.20
2	0.072	34.945	6.12	11.20
3	0.072	32.695	8.67	11.20
4	0.072	30.445	9.77	11.20
5	0.072	28.195	9.77	11.20
6	0.072	25.945	9.77	11.20
7	0.072	23.695	9.77	11.20
8	0.072	21.445	9.77	11.20
9	0.072	19.195	9.77	11.20
10	0.072	16.945	9.77	11.20
11	0.072	14.695	9.77	11.20
12	0.072	12.445	9.77	11.20
13	0.072	10.195	9.77	11.20
14	0.072	7.945	9.77	11.20
15	0.072	5.695	9.77	11.20
16	0.072	3.445	9.77	11.20
17	0.072	1.195	9.77	11.20
18	0.072	-1.055	9.77	11.20
19	0.072	-3.305	9.77	11.20
20	0	-5.555	0.00	11.20

**Table 4: Measured and Predicted velocity values of fluid along the mixing tube for Flow 2- Closed Throttle System**

## Appendix 2

### Sample Calculations of Flow Analysis

Sample calculations have been done to show how each value in the data tables were obtained. Please refer to Appendix 1- Data Tables for the calculated values. Flow 1 refers to open throttle flow whereas Flow 2 refers to closed throttle flow.

#### 1. Velocity and Static Pressure at the mixing tube outlet

##### Stagnation and static pressure due to static taps and pitot tubes

The static pressure at each static tap was calculated by implementing equation 1 and stagnation pressure at each pitot tube was calculated using equation 2 for both flows.

##### ➤ **Flow 1**

From equation 1

$$P_{static} = P_{atm} + \rho_{water} g \Delta h_{static}$$

$$P_{static} = 101325Pa + 1000kg/m^3 (9.81m/s)(0.116m - 0.146m) = 101030.7Pa(a)$$

From equation 2

$$P_{stagnation} = P_{atm} + \rho_{water} g \Delta h_{stagnation}$$

$$P_{static} = 101325Pa + 1000kg/m^3 (9.81m/s)(0.130m - 0.064m) = 101972.5Pa(a)$$

##### ➤ **Flow 2**

From equation 1

$$P_{static} = P_{atm} + \rho_{water} g \Delta h_{static}$$

$$P_{static} = 101325Pa + 1000kg/m^3 (9.81m/s)(0.128m - 0.130m) = 101305.4Pa(a)$$

From equation 2

$$P_{stagnation} = P_{atm} + \rho_{water} g \Delta h_{stagnation}$$

$$P_{static} = 101325Pa + 1000kg/m^3 (9.81m/s)(0.128m - 0.056m) = 102031.3Pa(a)$$

Static pressure at the upstream of the primary nozzle was calculated using the U-tube manometer height which recorded the pressure head at the upstream. The pressure downstream of the nozzle was calculated by the static pressure head due to static tap 1.

##### ➤ **Flow 1**

From equation 6,

$$P_{C1} = P_{atm} + \rho_{water} g \Delta h_{U-tube}$$

$$P_{C1} = 101325Pa + (1000kg/m^3)(9.81m/s^2)(0.836m)$$

$$P_{C1} = 109526.2Pa(a)$$

From equation 7,

$$P_{C2} = P_{atm} + \rho_{water} g \Delta h_{staticTap1}$$

$$P_{C2} = 101325Pa + (1000kg/m^3)(9.81m/s^2)(0.116m - 0.130m)$$

$$P_{C2} = 101795.9Pa(a)$$

➤ **Flow 2**

From equation 6,

$$P_{C1} = P_{atm} + \rho_{water} g \Delta h_{U-tube}$$

$$P_{C1} = 101325Pa + (1000kg/m^3)(9.81m/s^2)(0.786m)$$

$$P_{C1} = 109035.7Pa(a)$$

From equation 7,

$$P_{C2} = P_{atm} + \rho_{water} g \Delta h_{staticTap1}$$

$$P_{C2} = 101325Pa + (1000kg/m^3)(9.81m/s^2)(0.128m - 0.104m)$$

$$P_{C2} = 101560.5Pa(a)$$

Primary Velocity and Static Pressure

➤ **Flow 1**

From equation 3,

$$v_p = \sqrt{\frac{2C_q(P_{C1} - P_{C2})}{\rho_{air}}}$$

$$v_p = \sqrt{\frac{2(0.93)(109526.2Pa - 101795.9Pa)}{1.2kg/m^3}} = 109.5m/s$$

From equation 5,

$$P_p = P_{C2} + C_p(P_{C1} - P_{C2})$$

By substituting the values, the equation reduces to

$$P_p = 101795.9Pa - 0.045(109526.2Pa - 101795.9Pa) = 101448Pa(a)$$

➤ **Flow 2**

From equation 3,

$$v_p = \sqrt{\frac{2C_q(P_{C1} - P_{C2})}{\rho_{air}}}$$

$$v_p = \sqrt{\frac{2(0.93)(109035.7Pa - 101560.5Pa)}{1.2kg/m^3}} = 107.6m/s$$

From equation 5,

$$P_p = P_{C2} + C_p(P_{C1} - P_{C2})$$

By substituting the values, the equation reduces to

$$P_p = 101560.5Pa - 0.045(109035.7Pa - 101560.5Pa) = 101224.1Pa(a)$$

### Secondary Velocity and Static Pressure

#### ➤ **Flow 1**

From equation 8,

$$P_S = P_{atm} + \rho_{water} g \Delta h_{staticTap2}$$

$$P_S = 101325Pa + 1000kg/m^3 (9.81m/s)(0.116m - 0.144m) = 101050.3Pa(a)$$

From equation 9,

$$v_S = \sqrt{\frac{2(P_{atm} - P_S)}{\rho_{air}}}$$

$$v_S = \sqrt{\frac{2(101325Pa - 101050.3Pa)}{1.2kg/m^3}} = 21.4m/s$$

#### ➤ **Flow 2**

From equation 8,

$$P_S = P_{atm} + \rho_{water} g \Delta h_{staticTap2}$$

$$P_S = 101325Pa + 1000kg/m^3 (9.81m/s)(0.128m - 0.127m) = 101334.8Pa(a)$$

From equation 9,

$$v_S = \sqrt{\frac{2(P_{atm} - P_S)}{\rho_{air}}}$$

$$v_S = \sqrt{\frac{2(101325Pa - 101334.8Pa)}{1.2kg/m^3}} = 4.04m/s$$

### Velocity and Static Pressure at the mixing tube outlet plane (Pitot tube rake)

#### ➤ **Flow 1**

From equation 10,

$$v_{outlet} = \frac{A_P v_P + A_S v_S}{A_{outlet}}$$

$$v_{outlet} = \frac{(0.0222m)^2 109.5m/s + (0.0539m)^2 21.4m/s}{(0.0762m)^2} = 20m/s$$

From equation 11,

$$P_{outlet} = \frac{P_P A_P + P_S A_S - \rho_{air} (A_{out} v_{out}^2 - A_P v_P^2 - A_S v_S^2)}{A_{outlet}}$$

$$P_{outlet} = \frac{101448Pa(0.0222m)^2 + 101050.3Pa(0.0539m)^2 - 1.2((0.0762m)^2(20m/s)^2 - (0.0222m)^2(109.5m/s)^2 - (0.0539m)^2(21.4m/s)^2)}{(0.0762m)^2}$$

$$P_{outlet} = 101661.6Pa(a)$$

➤ **Flow 2**

From equation 10,

$$v_{outlet} = \frac{A_P v_P + A_S v_S}{A_{outlet}}$$

$$v_{outlet} = \frac{(0.0222m)^2 107.5m/s + (0.0539m)^2 4.04m/s}{(0.0762m)^2} = 11.2m/s$$

From equation 11,

$$P_{outlet} = \frac{P_P A_P + P_S A_S - \rho_{air} (A_{out} v_{out}^2 - A_P v_P^2 - A_S v_S^2)}{A_{outlet}}$$

$$P_{outlet} = \frac{101224.1Pa(0.0222m)^2 + 101334.8Pa(0.0539m)^2 - 1.2((0.0762m)^2(11.2m/s)^2 - (0.0222m)^2(107.5m/s)^2 - (0.0539m)^2(4.04m/s)^2)}{(0.0762m)^2}$$

$$P_{outlet} = 102619.8Pa(a)$$

Percentage Errors

The percentage errors were calculated based on the below mentioned formula.

$$\%error = \left| \frac{predicted - experimental}{predicted} \right| \times 100\%$$

➤ **Flow 1**

$$\%velocity_{error} = \left| \frac{20m/s - 22.51m/s}{20m/s} \right| \times 100\% = 12.55\%$$

$$\%pressure_{error} = \left| \frac{336Pa - 240Pa}{336Pa} \right| \times 100\% = 28.57\%$$

➤ **Flow 2**

$$\%velocity_{error} = \left| \frac{11.2m/s - 9.77m/s}{11.2m/s} \right| \times 100\% = 12.76\%$$

$$\%pressure_{error} = \left| \frac{1294Pa - 706Pa}{1294Pa} \right| \times 100\% = 45.44\%$$

### Appendix 3

#### Bonus Analysis

To check the internal consistency of data, the ideal volume flow rate was compared to the calculated outlet volume flow rate.

To calculate the experimental volume flow rate, 1-D steady state flow was assumed and the velocity of the fluid after mixing was assumed to have the maximum value at the centerline of the mixing tube. The velocity as a function of cross-sectional radius for any cylindrical duct is given by

$$v = v_{\max} \left[ 1 - \left( \frac{r}{R} \right)^2 \right]$$

The mass flow rate is given by

$$\frac{dm}{dt} = \rho A v$$

Over a very infinitesimal small ring of radius  $dr$ , the above equation can be written as

$$d\left(\frac{dm}{dt}\right) = \rho_{air} v dA = \rho_{air} v_{\max} \left[ 1 - \left( \frac{r}{R} \right)^2 \right] 2\pi r dr$$

On integrating the above equation over the cross-sectional area,

$$\frac{dm}{dt} = \frac{1}{2} \rho_{air} v_{\max} \pi R^2$$

Hence, the volume flow rate can be given as

$$Q = \frac{dm}{dt} \times \frac{1}{\rho_{air}} = \frac{v_{\max} \pi R^2}{2}$$

The value of maximum velocity taken is at the static tap 17 for better analysis since the velocity will be highest here.

The ideal volume flow rate for any system is given as

$$Q_{ideal} = v_{inlet} A_{inlet}$$

### For Open Throttle System

$$Q_{openSystem} = \frac{(23.23m/s)\pi(0.0381m)^2}{2} = 0.053m^3/s$$

$$Q_{ideal} = 0.131m^3/s$$

$$\%error = \left| \frac{(0.053 - 0.131)m^3/s}{0.131m^3/s} \right| \times 100\% = 59.5\%$$

### For Closed Throttle System

$$Q_{openSystem} = \frac{(9.77m/s)\pi(0.0381m)^2}{2} = 0.0222m^3/s$$

$$Q_{ideal} = 0.0585m^3/s$$

$$\%error = \left| \frac{(0.0222 - 0.0585)m^3/s}{0.0585m^3/s} \right| \times 100\% = 62\%$$