0.0 General

0.1 E and Q

$\dot{E}_{\text{IN}} = \dot{E}_{\text{OUT}}$

$E_{\text{ST}} = mc\Delta T \Rightarrow \rho cV\Delta T$

$q = \dot{m}c_{\text{p}}\Delta T \Rightarrow \rho V A c_{\text{p}}\Delta T$

$q = q_{\text{cond}} + q_{\text{conv}} + q_{\text{rad}}$

0.2 Fourier's Law

$q''_{\text{cond}} = -k\frac{dT}{dx}$; $q_{\text{cond}} = -kA\frac{dT}{dx} \Rightarrow -kA\frac{\Delta T}{\Delta x}$

0.3 Newton's Law

$q''_{\text{conv}} = h(T_{\text{surf}} - T_\infty)$; $q_{\text{conv}} = hA(T_{\text{surf}} - T_\infty)$

0.4 Stefan-Boltzman Law

$q''_{\text{rad}} = \frac{q}{A} \Rightarrow \varepsilon\sigma T_{\text{s}}^4 - \alpha G = \varepsilon\sigma A\left(T_{\text{surf}}^4 - T_\infty^4\right)$;

$G_{\text{abs}} = \alpha G$; $q_{\text{abs}} = \alpha GA$

1.0 Conduction

1.1.0 Fourier's Law

1.1.1 Cartesian

$\vec{q}'' = -k\vec{\nabla}T \Rightarrow -k\left(\hat{\imath}\frac{\partial T}{\partial x} + \hat{\jmath}\frac{\partial T}{\partial y} + \hat{k}\frac{\partial T}{\partial z}\right)$

$\vec{q}''_x = -k\frac{\partial T}{\partial x}$ $\quad \vec{q}''_y = -k\frac{\partial T}{\partial y}$ $\quad \vec{q}''_z = -k\frac{\partial T}{\partial z}$

1.1.2 Cylindrical

$\vec{q}'' = -k\vec{\nabla}T \Rightarrow -k\left(\hat{r}\frac{\partial T}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial T}{\partial r} + \hat{k}\frac{\partial T}{\partial z}\right)$

$\vec{q}''_r = -k\frac{\partial T}{\partial r}$ $\quad \vec{q}''_\phi = -\frac{k}{r}\frac{\partial T}{\partial \phi}$ $\quad \vec{q}''_z = -k\frac{\partial T}{\partial z}$

1.1.3 Spherical

$\vec{q}'' = -k\vec{\nabla}T \Rightarrow -k\left(\hat{r}\frac{\partial T}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial T}{\partial r} + \hat{\theta}\frac{1}{r\sin\theta}\frac{\partial T}{\partial \theta}\right)$

$\vec{q}''_r = -k\frac{\partial T}{\partial r}$ $\quad \vec{q}''_\phi = -\frac{k}{r}\frac{\partial T}{\partial \phi}$ $\quad \vec{q}''_\theta = -k\frac{1}{r\sin\theta}\frac{\partial T}{\partial \theta}$

1.2.0 HDE

1.2.1 Cartesian

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{g} = \rho C_{\text{p}}\frac{\partial T}{\partial t}$$

$k = $ const

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

where $\alpha = \frac{k}{\rho C_{\text{p}}}$ (thermal diffusivity)

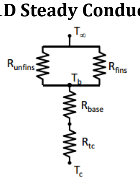
1.2.2 Cylindrical

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{g} = \rho C_{\text{p}}\frac{\partial T}{\partial t}$$

1.2.3 Spherical

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right) + \dot{g} = \rho C_{\text{p}}\frac{\partial T}{\partial t}$$

1.3.0 1D Steady Conduction/Electrical Analog



$R_{\text{cond}} = \frac{L}{kA}$ $\quad R_{\text{conv}} = \frac{1}{hA}$

$R_{\text{rad}} = \frac{1}{h_{\text{r}}A}$

$h_{\text{r}} = \varepsilon\sigma(T_{\text{s}} + T_{\text{surr}})(T_{\text{s}}^2 + T_{\text{surr}}^2)$

$R_{\text{EQ}} = \sum_{i=1}^{N} R_i$ OR $\frac{1}{R_{\text{EQ}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$

$R_{\text{t,c}} = \frac{T_{\text{A}} - T_{\text{B}}}{q_{\text{x}}}$; $R''_{\text{t,c}} = R_{\text{t,c}}A$

1.3.1 Plane Wall

HDE: $\frac{d^2T}{dx^2} = 0$

Temperature Distribution: $T = T_1 - (T_1 - T_2)\frac{x}{L}$

Heat Flux: $\vec{q}''_x = k\frac{T_1 - T_2}{L}$

Heat Rate: $q_x = \frac{kA}{L}(T_1 - T_2)$

Thermal Resistance: $R_{\text{t,cond}} = \frac{L}{kA}$

Electric Analog: $V = IR \Leftrightarrow \Delta T = qR_{\text{t}}$

1.3.2 Cylindrical Wall

HDE: $\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$

Temperature Distribution: $T = T_2 + (T_1 - T_2)\frac{\ln(r/r_2)}{\ln(r_1/r_2)}$

Heat Flux: $\vec{q}''_r = \frac{k(T_1 - T_2)}{r\ln(r_2/r_1)}$

Heat Rate: $q_{\text{r}} = \frac{2\pi Lk(T_1 - T_2)}{r\ln(r_2/r_1)}$

Thermal Resistance: $R_{\text{t,cond}} = \frac{\ln(r_2/r_1)}{2\pi Lk}$

Critical Insulation Thickness: $r_{\text{crit}} = \frac{k}{h}$

1.3.3 Spherical Wall

HDE: $\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$

Temperature Distribution: $T = T_1 - (T_1 - T_2)\left[\frac{1 - r_1/r}{1 - r_1/r_2}\right]$

Heat Flux: $\vec{q}''_r = \frac{k(T_1 - T_2)}{r^2[(1/r_1) - (1/r_2)]}$

Heat Rate: $q_{\text{r}} = \frac{4\pi k(T_1 - T_2)}{(1/r_1) - (1/r_2)}$

Thermal Resistance: $R_{\text{t,cond}} = \frac{(1/r_1) - (1/r_2)}{4\pi k}$

Critical Insulation Thickness: $r_{\text{crit}} = \frac{2k}{h}$

1.4 Conduction with $\dot{g}$

For Cartesian, HDE: $\frac{d^2T}{dx^2} + \frac{\dot{g}}{k} = 0$

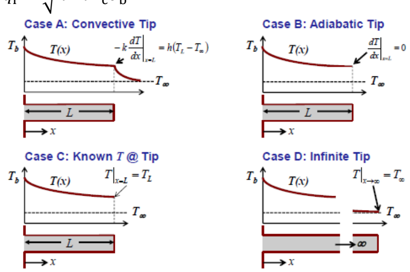
Integrating Twice: $-\frac{\dot{g}}{2k}x^2 + C_1x + C_2$ (can't use elec. analog)

1.5.0 Extended Surfaces and Fins

$L_{\text{C}} = L + \frac{t}{2}$

Infinitely Long Fin (Table 3.4)

$q_{\text{f}} = \sqrt{\bar{h}PkA_{\text{c}}}\theta_{\text{b}}$



1.5.1 Annular Fins

General Fin Equation

$$\frac{\partial^2 T}{\partial x^2} + \left(\frac{1}{A_{\text{c}}}\frac{dA_{\text{c}}}{dx}\right)\frac{\partial T}{\partial x} - \left(\frac{1}{A_{\text{c}}}\frac{h}{k}\frac{dA_{\text{s}}}{dx}\right)(T - T_\infty) = 0$$

Annular Fin:

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} - \frac{2h}{kt}(T - T_\infty) = 0 \Rightarrow \frac{d^2\theta}{dr^2} + \frac{1}{r}\frac{d\theta}{dr} - m^2(\theta) = 0$$

$m^2 = \frac{2h}{kt}$

$\theta(r) = C_1I_0(mr) + C_2K_0(mr)$

$R_{\text{t,f}} = \frac{1}{hA_f\eta_{\text{f}}}$

1.5.2 Fin Arrays

Single Fin

$\eta_{\text{f}} = \frac{q_{\text{f}}}{q_{\text{f ideal}}} \Rightarrow \frac{q_{\text{fin}}}{hA_f\theta_b}$; $\quad q_{\text{f}} = \frac{\theta_b}{R_{\text{f}}}$;

$\varepsilon_{\text{fin}} = \frac{q_{\text{f}}}{q_{\text{no f}}} \Rightarrow \frac{\theta_b}{R_{\text{f}}}\left(\frac{1}{hA_b\theta_b}\right) = \frac{R_b}{R_{\text{f}}}$

Fin Array

$\eta_{\text{array}} = \frac{q_{\text{a}}}{q_{\text{a,ideal}}} \Rightarrow \frac{q_{\text{a}}}{hA_{\text{a}}\theta_b}$; $\quad q_{\text{f}} = \frac{\theta_b}{R_{\text{f}}} \Rightarrow Nq_{\text{f}}$; $\quad \varepsilon_{\text{a}} = \frac{q_{\text{w/a}}}{q_{\text{no a}}}$

$R_{\text{array}} = R_{\text{EQ}} \Rightarrow R_{\text{base}} + R_{\text{t,c}} + \left(R_{\text{fins}}^{-1} + R_{\text{unfins}}^{-1}\right)^{-1}$

$q_{\text{array}} = \frac{\Delta T}{R_{\text{array}}}$

1.6.0 Transient Conduction

1.6.1 Lumped Capacitance

$\frac{T_{\text{s,1}} - T_{\text{s,2}}}{T_{\text{s,2}} - T_\infty} = \frac{L/kA}{1/hA} \Rightarrow \frac{R_{\text{t,cond}}}{R_{\text{t,conv}}} = \frac{hL}{k} \equiv \text{Bi}$

for spheres, $\text{Bi} = \frac{hL^*}{k} \Rightarrow \frac{hV}{kA_{\text{s}}}$

$\text{Bi} < 0.1$, the error for lumped capacitance method is small

$\text{Bi} \ll 1$: $\frac{\theta}{\theta_{\text{i}}} = \frac{T - T_\infty}{T_i - T_\infty} \Rightarrow \exp\left[-\left(\frac{hA_{\text{s}}}{\rho Vc}\right)t\right]$

$\theta^* = \exp(-\text{Bi}\cdot\text{Fo})$; $\text{Fo} = \frac{\alpha t}{L^{*2}}$

No $\dot{g}$, $q_{\text{conv}} << q_{\text{rad}}$, no heat flux:

$$t = \frac{\rho Vc}{4A_{\text{s}}\varepsilon\sigma T_{\text{surr}}^3}\left[\ln\left|\frac{T_{\text{surr}} + T}{T_{\text{surr}} - T}\right| - \ln\left|\frac{T_{\text{surr}} + T_{\text{i}}}{T_{\text{surr}} - T_{\text{i}}}\right| + 2\left(\tan^{-1}\left(\frac{T}{T_{\text{surr}}}\right) - \tan^{-1}\left(\frac{T_{\text{i}}}{T_{\text{surr}}}\right)\right)\right]$$

No q_{rad}; just $\dot{g}$, q_{conv}, and flux:

$a = \frac{hA_{\text{s}}}{\rho Vc}$; $b = \frac{q''_{\text{s}}A_{\text{s}} + \dot{g}}{\rho Vc}$; $\frac{T - T_\infty}{T_i - T_\infty}$

$\frac{T - T_\infty}{T_i - T_\infty} = \exp(-at) + \frac{b/a}{T_i - T_\infty}[1 - \exp(-at)]$

1.6.2.0 Transient Conduction w/ Spatial Temp. Variation:

$$\theta^* = \sum_{n=1}^{\infty} C_{\text{n}}\exp(-\zeta_{\text{n}}^2\text{Fo})\cos(\zeta_n x^*)$$

where: $x^* = \frac{x}{L}$; $C_{\text{n}} = \frac{4\sin\zeta_{\text{n}}}{2\zeta_{\text{n}} + \sin(2\zeta_{\text{n}})}$ and $\zeta_{\text{n}}\tan\zeta_{\text{n}} = \text{Bi}$

if $\text{Fo} > 0.2$ only need 1st term of the infinite series

Semi-Infinite Solid

1.6.2.1 Plane Wall Fo > 0.2:

$\theta^* = \frac{\theta}{\theta_{\text{i}}} \Rightarrow \frac{T(x,t) - T_\infty}{T_i - T_\infty} = C_1\exp(-\zeta_1^2\text{Fo})\cos(\zeta_1 x^*)$

$\frac{Q}{Q_0} = 1 - \frac{\sin\zeta_1}{\zeta_1}\theta^*_{0,\text{t}}$

$C_1 = \frac{4\sin\zeta_1}{2\zeta_1 + \sin(2\zeta_1)}$; $\zeta_1\tan\zeta_1 = \text{Bi}$

1.6.2.2 Cylindrical Geometries Fo > 0.2:

$\theta^* \Rightarrow \frac{T - T_\infty}{T_i - T_\infty} = C_1\exp(-\zeta_1^2\text{Fo})J_0(\zeta_1 r^*)$

$\theta_0^* = \frac{T_{0,t} - T_\infty}{T_i - T_\infty}$, $r^* = \frac{r}{r_0}$

$\frac{Q}{Q_0} = 1 - \frac{2\theta_0^*}{\zeta_1}J_1(\zeta_1)$

$\zeta_n\frac{J_1(\zeta_n)}{J_0(\zeta_n)} = \text{Bi}$

1.6.2.3 Spherical Geometries Fo > 0.2:

$\theta^* = C_1\exp(-\zeta_1^2\text{Fo})\left[\frac{1}{\zeta_1 r^*}\sin(\zeta_1 r^*)\right]$

$\theta_0^* = \frac{T_{0,t} - T_\infty}{T_i - T_\infty}$, $r^* = \frac{r}{r_0}$

$\frac{Q}{Q_0} = 1 - \frac{3\theta_0^*}{\zeta_1^3}[\sin(\zeta_1) - \zeta_1\cos(\zeta_1)]$

$1 - \zeta_n\cot\zeta_n = \text{Bi}$

1.6.3.0 Semi-Infinite Solid

1.6.3.1 Case 1 — Constant Surface Temperature

$T(0,t) = T_{\text{s}}$

$\frac{T(x,t) - T_{\text{s}}}{T_{\text{i}} - T_{\text{s}}} = \text{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)$; $q''_{\text{s}}(t) = \frac{k(T_{\text{s}} - T_{\text{i}})}{2\sqrt{\alpha t}}$

1.6.3.2 Case 2 — Constant Surface Heat Flux

$q''_{\text{s}} = q''_0$

$$T(x,t) - T_{\text{i}} = \frac{2q''_0\left(\frac{\alpha t}{\pi}\right)^{1/2}}{k}\exp\frac{-x^2}{(4\alpha t)} - \frac{q''_0 x}{k}\text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

1.6.3.3 Case 3 — Surface Convection

$-k\frac{\partial T}{\partial x}\Big|_{x=0} - h[T_\infty - T(0,t)]$

$$\frac{T(x,t) - T_{\text{i}}}{T_\infty - T_{\text{i}}} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \left[\exp\left(\frac{hx}{k} + \frac{h^2\alpha t}{k^2}\right)\right]\left[\text{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)\right]$$

2.0 Convection

2.1 External Flows (T7.2, 7.3, 7.4, 7.7)

Local Convective Heat Transfer Coefficient

$-k_f \frac{\partial T}{\partial n}\Big|_{n=0} = h(T_s - T_\infty) \Rightarrow h = \frac{-k_f \frac{\partial T}{\partial n}|_{n=0}}{(T_s - T_\infty)}$

Total convective heat transfer from a surface:

$q = \int_{A_s} h(T_s - T_\infty) dA_s \Rightarrow (T_s - T_\infty) \int_{A_s} h dA_s$

Avg. convective heat transfer from, $\bar{h}$:

$q = \bar{h} A_s (T_s - T_\infty) = (T_s - T_\infty) \int_{A_s} h dA_s \Rightarrow \bar{h} = \frac{1}{A_s} \int_{A_s} h dA_s$

Flow over a heated, flat plate:

$\mathrm{Re} = \frac{U_\infty x}{\nu} \Rightarrow \frac{\rho V D}{\mu}$

- $\mathrm{Re} < 5 \times 10^5$: Laminar
- $\mathrm{Re} > 3 \times 10^6$: Fully Turbulent

Boundary Layer Equations

Mass: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

Momentum: $\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2}$

Energy: $u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$

Laminar Boundary Layer Solution for a Heated Flat Plate

Blasius solution:

$\delta = \frac{5x}{\sqrt{\mathrm{Re}_x}}$

Local shear stress at wall:

$\tau_s = \mu \frac{\partial u}{\partial y}\Big|_{y=0} \Rightarrow 0.332 u_\infty \sqrt{\frac{\rho \mu u_\infty}{x}}$

Local friction coefficient:

$\frac{\overline{C_f}}{2} = \overline{\mathrm{St}} \mathrm{Pr}^{2/3} \quad \overline{C_f} \equiv \frac{\overline{\tau_s}}{\frac{1}{2}\rho u^2} = 0.664 \mathrm{Re}_x^{1/2} \quad \overline{\mathrm{St}} = \frac{\bar{h}}{\rho u c_p}$

$\mathrm{Pr} \equiv \frac{\nu}{\alpha} = \frac{\text{momentum diffusivity (a.k.a. kinematic viscosity)}}{\text{thermal diffusivity}}$

For laminar flow over a flat plate:

$h = \frac{-k(\partial t/\partial y)|_{y=0}}{T_s - T_\infty} \Rightarrow 0.332 k \mathrm{Pr}^{1/3} \left(\frac{U_\infty}{\nu_x}\right)^{1/2}$

$\frac{\delta_t}{\delta} \approx \mathrm{Pr}^{1/3} \qquad \delta = \frac{5x}{\sqrt{\mathrm{Re}_x}}$

For flat plate (laminar flow) with fixed plate temperature boundary condition:

$\mathrm{Nu} \equiv \frac{hx}{k} = 0.332 \mathrm{Pr}^{1/3} \mathrm{Re}^{1/2}$

For flat plate (laminar flow) with fixed uniform heat flux:

$\mathrm{Nu} \equiv \frac{hx}{k} = 0.453 \mathrm{Pr}^{1/3} \mathrm{Re}^{1/2} \}$ (valid for $\mathrm{Pr} > 0.6$)

How to calculate average h over a plate?

$\overline{\mathrm{Nu}} = \frac{\bar{h} L}{k} \Rightarrow \frac{2 h_{x=L} L}{k} = 2 \mathrm{Nu}_{x=L} \Rightarrow 2 \mathrm{Nu}_L$

Film Temperature

$\frac{T_s}{T_\infty} \gg 1$ OR $\frac{T_s}{T_\infty} \ll 1$; $T_{\text{film}} = \frac{T_s + T_\infty}{2}$; When T_s or T_∞ are not specified; 1. Guess T_{film}, 2. Evaluate property data, 3. Solve. 4. Calculate new T_{film} and compare with previous guess, 5. Repeat until T_{film} and solution data converges.

Non-dimensionalized Solutions

$h = \frac{k_f}{L} \frac{\partial T^*}{\partial y^*}\Big|_{y=0}; \qquad \mathrm{Nu} \equiv \frac{hL}{k_f} = \frac{\partial T^*}{\partial y^*}\Big|_{y=0}$

Heated Flat Plate Solutions

Laminar flow only:

$\mathrm{Nu}_x = 0.332 \mathrm{Re}_x^{1/2} \mathrm{Pr}^{1/3} \}$ (valid for $\mathrm{Pr} \geq 0.6$)

$\mathrm{Nu}_x = \frac{0.3387 \mathrm{Re}_x^{1/2} \mathrm{Pr}^{1/3}}{[1 + (0.0468/\mathrm{Pr})^{2/3}]^{1/4}} \}$

(valid for Peclet Number ≥ 100)
(where Peclet Number $\equiv \mathrm{Pe} \equiv \mathrm{Re}_x \mathrm{Pr}$)

Fully turbulent boundary layer:

$\delta = 0.37 x \mathrm{Re}_x^{-1/5}$

$\delta = \delta_t$

$\mathrm{Nu}_x = \mathrm{St} \mathrm{Re}_x \mathrm{Pr} \Rightarrow 0.0296 \mathrm{Re}_x^{4/5} \mathrm{Pr}^{1/3} \}$

(valid for $0.6 \leq \mathrm{Pr} \leq 60$)

Mixed laminar and turbulent boundary layer:

$\overline{\mathrm{Nu}}_L = \left[0.664 \mathrm{Re}_{x_c}^{1/2} + 0.037(\mathrm{Re}_L^{4/5} - \mathrm{Re}_{x_c}^{4/5})\right] \mathrm{Pr}^{1/3}$

If transition to turbulent BL is at: $\mathrm{Re}_{x_c} = 5 \times 10^5$

$\overline{\mathrm{Nu}}_L = \left(0.037 \mathrm{Re}_L^{4/5} - 871\right) \mathrm{Pr}^{1/3}$

valid for: $0.6 \leq \mathrm{Pr} \leq 60$; $5 \times 10^5 < \mathrm{Re}_L \leq 10^8$

2.2 Forced Convection in Internal Pipe Flow

$\mathrm{Re}_D = \frac{u_{\text{mean}} D}{\nu} \Rightarrow \frac{\rho u_{\text{mean}} D}{\mu}$

Hydrodynamic entry length:

$\left(\frac{x_{\text{FD,hydro}}}{D}\right)_{\text{lam}} \approx 0.05 \mathrm{Re}_D \mathrm{Pr}$

$\overline{\mathrm{Nu}} = f(\mathrm{Re}_L, \mathrm{Pr})$

Velocity Field in Laminar Pipe Flow

$u(r) = \frac{1}{4\mu}\left(\frac{dp}{dx}\right) r_0^2 \left[1 - \frac{r^2}{r_0^2}\right]$

$u_{\text{mean}} = -\frac{r_0^2}{8\mu} \frac{dp}{dx}$

$\frac{u(r)}{u_{\text{mean}}} = 2\left[1 - \frac{r^2}{r_0^2}\right]$

Mixing Cup Temperature (incompressible, circular tube, c = const)

$T_m = \frac{2}{u_m r_0^2} \int_0^{r_0} u T r dr$

Forced Convection in Laminar Pipe Flow for laminar, fully-developed pipe flow:

$\mathrm{Nu}_D \equiv \frac{hD}{k} = \frac{48}{11} \Rightarrow 4.36 \quad (q_s'' = \text{constant})$

$\mathrm{Nu}_D \equiv \frac{hD}{k} \Rightarrow 3.66 \quad (T_s = \text{constant})$

Heat Transfer in Entry Regions (Laminar)

Laminar thermal entry length:

$\left(\frac{x_{\text{FD,therm}}}{D}\right)_{\text{lam}} \approx 0.05 \mathrm{Re}_D \mathrm{Pr}$

Fully developed velocity and thermal boundary layers:

Graetz Number $\equiv \mathrm{Gz}^{-1} = \frac{x/D}{\mathrm{Re}_D \mathrm{Pr}} > 0.05$

If $\mathrm{Re}_D = 2\,300$, $x/D = 80.5$ for air; $x/D = 500$ for water

Combined Laminar Entry Length (both velocity and thermal B.L. developing)

$\overline{\mathrm{Nu}}_D = 1.86 \left(\frac{\mathrm{Re}_D \mathrm{Pr}}{L/D}\right)^{1/3} \left(\frac{\mu}{\mu_s}\right)^{0.14}$ (valid for: T_s = const) $0.48 < \mathrm{Pr} < 16\,700$; $0.0044 < (\mu/\mu_s) < 9.75$

Thermal Entry Length (or unheated starting length)

$\overline{\mathrm{Nu}}_D = 3.66 + \frac{0.0668(D/L)\mathrm{Re}_D \mathrm{Pr}}{1 + 0.04[(D/L)\mathrm{Re}_D \mathrm{Pr}]^{2/3}}$ (valid for T_s = const)

Flow in Circular Tubes

$(\mathrm{Re}_D)_{\text{inlet/outlet}} = \frac{4\dot{m}}{\pi D \mu}$; incompressible, steady, A = const

Dittus-Boelter Equation

$\mathrm{Nu}_D = 0.023 \mathrm{Re}_D^{4/5} \mathrm{Pr}^n$ where: $n = 0.4$ for heating $T_s > T_m$; $n = 0.3$ for cooling $T_s < T_m$

valid for: $0.7 \leq \mathrm{Pr} \leq 160$, $L/D \geq 10$, $\mathrm{Re}_D \geq 10\,000$, $T_s - T_m$ is moderate

Sider Tate (larger temp variations)

$\mathrm{Nu}_D = 0.027 \mathrm{Re}_D^{4/5} \mathrm{Pr}^{1/3} \left(\frac{\mu}{\mu_{\text{surf}}}\right)^{0.14}$

valid for: $0.7 \leq \mathrm{Pr} \leq 16\,700$, $L/D \geq 10$, $\mathrm{Re}_D \geq 10\,000$, large variations in T

μ_s at surface temp; other properties at $\overline{T}_m$

Transitionally Turbulent Flows:

$\mathrm{Nu}_D = \frac{(f/8)\mathrm{Re}_D \mathrm{Pr}}{1.07 + 12.7(f/8)^{1/2}(\mathrm{Pr}^{2/3} - 1)}$

$f = (0.790 \ln \mathrm{Re}_D - 1.64)^{-2} \quad 3\,000 \lesssim \mathrm{Re}_D \lesssim 5 \times 10^6$

in general $10 \leq (x/D)_{\text{FD}} \leq 60$

Turbulent Hydraulic Diameter

$D_h \equiv \frac{4A_c}{P}$

Energy Balance in Internal Flows

$dq_{\text{conv}} = q_s'' P dx \Rightarrow \dot{m} c_P dT_m$

$\frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m} c_P} \Rightarrow \frac{P}{\dot{m} c_P} h(T_s - T_m)$

Constant Heat Flux

$T_{m,o} = T_{m,i} + \frac{q_s'' P}{\dot{m} c_P} x$

Constant Surface Temp

$\frac{\Delta T_o}{\Delta T_i} = \frac{T_s - T_m(x)}{T_s - T_{m,i}} \Rightarrow \exp\left(-\frac{Px}{\dot{m} c_P} \bar{h}\right)$

$q_s = \bar{h} A_s \Delta T_{\text{LM}}; \quad \Delta T_{\text{LM}} = \frac{\Delta T_o - \Delta T_i}{\ln \frac{\Delta T_o}{\Delta T_i}}$ (T_s = const)

$\frac{\Delta T_o}{\Delta T_i} = \frac{T_\infty - T_o}{T_\infty - T_i} \Rightarrow \exp\left(-\frac{\overline{U} A_s}{\dot{m} c_P}\right) = \exp\left(-\frac{1}{\dot{m} c_P R_{\text{tot}}}\right)$

$q_s = \bar{h} A_s \Delta T_{\text{LM}} \Rightarrow \frac{\Delta T_{\text{LM}}}{R_{\text{tot}}}; \quad R_{\text{tot}} = \frac{1}{\overline{U} A_s}$

2.3 Natural Convection

Grashof Number (ratio of buoyancy to viscous forces):

$\mathrm{Gr} = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} \Rightarrow \frac{g\beta \Delta T L^3}{\nu^2}$

Rayleigh Number:

$\mathrm{Ra} \equiv \mathrm{Gr}\mathrm{Pr} = \frac{g\beta\Delta T L^3}{\nu^2}\left(\frac{\nu}{\alpha}\right) \Rightarrow \frac{g\beta\Delta T L^3}{\nu\alpha}$

Transition to turbulence: $\mathrm{Ra} \approx 10^9$

$\overline{\mathrm{Nu}}_L = \frac{\bar{h} L}{k} C \mathrm{Ra}_L^n$

2.4.0 Heat Exchangers (T11.3, 11.4)

$(\mathrm{Re}_D)_h = \frac{u_{m,h}(D_o - D_i)}{\nu}$

$\frac{1}{\overline{U}} = \frac{1}{h_o} + \frac{1}{h_i}$

$q \Rightarrow C_c[(T_c)_o - (T_c)_i] = C_h[(T_h)_i - (T_h)_o]$

$q = UA \frac{\Delta T_o - \Delta T_i}{\ln \frac{\Delta T_o}{\Delta T_i}} \Rightarrow UA \Delta T_{\text{LM}}; \quad UA = \frac{1}{R_{\text{tot}}}$

parallel flow: $\Delta T_1 \equiv (T_h)_i - (T_c)_i$; $\Delta T_2 \equiv (T_h)_o - (T_c)_o$

counter flow: $\Delta T_1 \equiv (T_h)_i - (T_c)_o$; $\Delta T_2 \equiv (T_h)_o - (T_c)_i$

2.4.1 Heat Exchanger Effectiveness – NTU Method

$\varepsilon = \frac{\text{actual heat transfer}}{\text{max. possible heat transfer}}; \dot{C}_{\min} = (\dot{m} c_P)_{\min}$

$= \frac{q}{q_{\max}} \Rightarrow \frac{\dot{C}_h[(T_h)_i - (T_h)_o]}{\dot{C}_{\min}[(T_h)_i - (T_c)_i]} = \frac{\dot{C}_c[(T_c)_o - (T_c)_i]}{\dot{C}_{\min}[(T_h)_i - (T_c)_i]}$

$q = \varepsilon q_{\max} \Rightarrow \varepsilon \dot{C}_{\min}[(T_h)_i - (T_c)_i]$

$\varepsilon = f\left(\frac{\dot{C}_{\min}}{\dot{C}_{\max}}, NTU\right);$

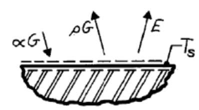
(number of transfer units) $NTU \equiv \frac{UA}{\dot{C}_{\min}}$

3.0 Radiation

$c = \lambda v; e = hv \Rightarrow \frac{hc}{\lambda}$

$h = 6.626 \times 10^{-24}$ J s

$\alpha = 1 - \rho$



Net energy exchange between surfaces:

$q_{ij} = [A_i(E_b)_i]F_{ij} - [A_j(E_b)_j]F_{ji}; \quad A_i F_{ij} = A_j F_{ji}$

$q_{ij} = A_i F_{ij}[(E_b)_i - (E_b)_j] \Rightarrow A_i F_{ij} \sigma (T_i^4 - T_j^4)$

Net exchange from i in multi-surface system:

$q_i = \sum_{j=1}^{N} A_i F_{ij} \sigma (T_i^4 - T_j^4)$