Fundamentals of Engineering Thermodynamics

Chapter 1 – Getting Started

System: The chosen object to be studied

- Closed System: A fixed quantity of mass
- Control Volume: A region of space where mass can flow
- Isolated System: A special type of closed system which doesn't interact with the surroundings

Surroundings: Anything outside of the system

Boundary: The division of the surroundings and the system

- A system boundary should be chosen such that the calculations are simplified for convenience

Macroscopic Characteristics: Concerned with the overall behaviour of the system

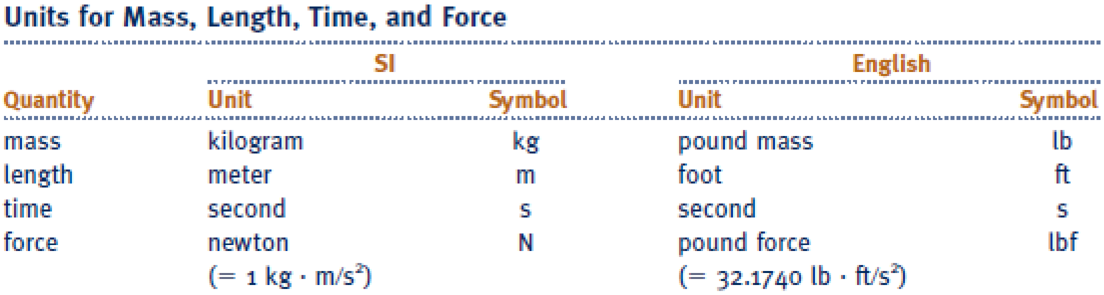
- Property: A numerical value that can be assigned at a given time without knowing the previous behaviour of the system (Ex: Mass, Volume, Energy, Pressure, Temperature)
  - Extensive: If the value of the system is the sum of the values for the parts in the system (Ex: Mass, Volume, Energy, etc…)
  - Intensive: Values that vary from place to place in the system but are not additive (Ex: Specific Volume, Pressure, Temperature, etc…)
- State: The condition of the system described by its properties
- Process: The change of states of a system
- Steady State: If the properties do not change when undergoing a process

Microscopic Characteristics: How matter reacts at a specific point

Equilibrium: When there are no changes in observable properties

- Equilibrium State: There are no changes at an isolated instant

In Thermodynamics, the primary dimensions are mass, length, time, and temperature; sometimes electric current when electricity is involved. Secondary dimensions are expressed in terms of the base units.



Units for Mass, Length, Time, and Force

	SI		English	
Quantity	Unit	Symbol	Unit	Symbol
mass	kilogram	kg	pound mass	lb
length	meter	m	foot	ft
time	second	s	second	s
force	newton (= 1 kg · m/s^2)	N	pound force (= 32.1740 lb · ft/s^2)	lbf

The density (ρ) at a given point is: $\rho = \lim_{V \to V'} \left(\frac{m}{V}\right)$ {Units: kg/m^3}

Chapter 1 – Getting Started

The mass can then be calculated as: $m = \int \rho dV$

Specific Volume (v) is the reciprocal of the density. {Units: m^3/kg}

The number of moles (n) is: $n = \frac{m}{M}$. To signal that a property is on a molar basis a bar is used over its symbol.

The pressure (p) at a given point is defined as:

$$p = \lim_{A \to A'} \left(\frac{F_{\text{normal}}}{A} \right)$$

- In manometers and barometers: The gas pressure is $p = p_{\text{atm}} + \rho g L$ and the atmospheric pressure is $p_{\text{atm}} = p_{\text{vapor}} + \rho_{\text{m}} g L$ which can be approximated as $p_{\text{atm}} = \rho_{\text{m}} g L$.

For a buoyant object, the force is defined as: F = ρgV {ρ is the density of the surrounding liquid}.

For a pressure measuring device, when the system pressure is greater than the local atmospheric pressure $p(\text{gage}) = p(\text{absolute}) - p_{\text{atm}}(\text{absolute})$ and when the atmospheric pressure is greater than the pressure of the system $p(\text{vacuum}) = p_{\text{atm}}(\text{absolute}) - p(\text{absolute})$.

The Rankine scale is a measure of temperature used in the FPS system and the Kelvin scale is used in SI, both scales have an absolute zero. The relationship between the two scales is:

$$T(°R) = 1.8T(K)$$

Celsius is not the SI unit for temperature but is a commonly used measurement. The relationship between Celsius and Kelvin is:

$$T(°C) = T(K) – 273.15$$

Similarly the relationship between Fahrenheit and Rankine is:

$$T(°F) = T(°R) – 459.67$$

Which leads to the relationship between Celsius and Fahrenheit:

$$T°F) = 1.8T(°C) + 32$$

Chapter 2 – Energy and 1st Law of Thermodynamics

Kinetic Energy (KE): The energy is an object as it moves. $\Delta KE = KE_2 - KE_1 = \frac{1}{2}m(V_2^2 - V_1^2)$

The change in kinetic energy is a representation of work done (W) onto the object. Thus the equation becomes:

$$\frac{1}{2}m(V_2^2 - V_1^2) = \int_{s_1}^{s_2} \mathbf{F} \cdot ds$$

- Kinetic Energy is an extensive property.

Potential Energy (PE): The energy store in an object when at rest.

- The gravitational potential energy is the quantity (mgz). The change in gravitational potential energy is:

$$\Delta PE = PE_2 - PE_1 = mg(z_2 - z_1)$$

- Potential energy is an extensive property

The conservation of energy states that:

$$\frac{1}{2}m(V_2^2 - V_1^2) + mg(z_2 - z_1) = 0$$

Thermodynamic Definition of Work: Work is done by a system on its surroundings if the sole effect on everything external to the system could have been the raising of a weight.

$W > 0$: work done *by* the system

$W < 0$: work done *on* the system

- Work is not a property
- When there is a change in the volume of a system, the work done on the system algebraically is

$$W = \int_{V_1}^{V_2} p\,dV$$

Whereas graphically, it is the area of a pressure vs. volume graph.

Power: The rate of energy transfer by work. It is equal to the force multiplied by the velocity.

$$\dot{W} = \mathbf{F} \cdot \mathbf{V}$$

A polytropic process is any process where pV^n = constant or pv^n = constant.

All energy that is not kinetic or potential is considered internal energy (U)

Chapter 2 – Energy and 1st Law of Thermodynamics

- Internal energy is and extensive property

The total change in energy (ΔE) is represented as:

$$\Delta E = \Delta U + \Delta \mathrm{KE} + \Delta \mathrm{PE}$$

$$E_2 - E_1 = (U_2 - U_1) + (\mathrm{KE}_2 - \mathrm{KE}_1) + (\mathrm{PE}_2 - \mathrm{PE}_1)$$

$$\Delta \mathrm{E} = Q - W$$

A transfer of heat (Q) from one object to another is also a transfer of energy from one object to another.

$Q > 0$: heat transfer *to* the system
$Q < 0$: heat transfer *from* the system

Thermodynamic Cycle: A sequence of processes that begins and ends at the same state.

- For any cycle, the energy balance is as follows

$$\Delta E_{\text{cycle}} = Q_{\text{cycle}} - W_{\text{cycle}}$$

- However since the initial and final states are the same, the net change in energy for the system is 0.

$$W_{\text{cycle}} = Q_{\text{cycle}}$$

- In a cycle where a net-work transfer of energy into the surroundings is performed (Power Cycle), the equation is read as:

$$W_{\text{cycle}} = Q_{\text{in}} - Q_{\text{out}}$$

  - The thermal efficiency (η) is: $\eta = \frac{W_{\text{cycle}}}{Q_{\text{in}}}$ or $\eta = \frac{Q_{\text{in}} - Q_{\text{out}}}{Q_{\text{in}}} = 1 - \frac{Q_{\text{out}}}{Q_{\text{in}}}$

- In a cycle where a net-work transfer of energy into the system occurs (Refrigeration/Heat Pump Cycle), the equation is read as:

$$W_{\text{cycle}} = Q_{\text{out}} - Q_{\text{in}}$$

  - The coefficient of performance (β) {Refrigeration Cycle}: $\beta = \frac{Q_{\text{in}}}{W_{\text{cycle}}}$ or $\beta = \frac{Q_{\text{in}}}{Q_{\text{out}} - Q_{\text{in}}}$

Chapter 2 – Energy and 1st Law of Thermodynamics

- The coefficient of performance (γ) {Heat Pump Cycles}: $\gamma = \frac{Q_{\text{out}}}{W_{\text{cycle}}}$ or $\gamma = \frac{Q_{\text{out}}}{Q_{\text{out}} - Q_{\text{in}}}$

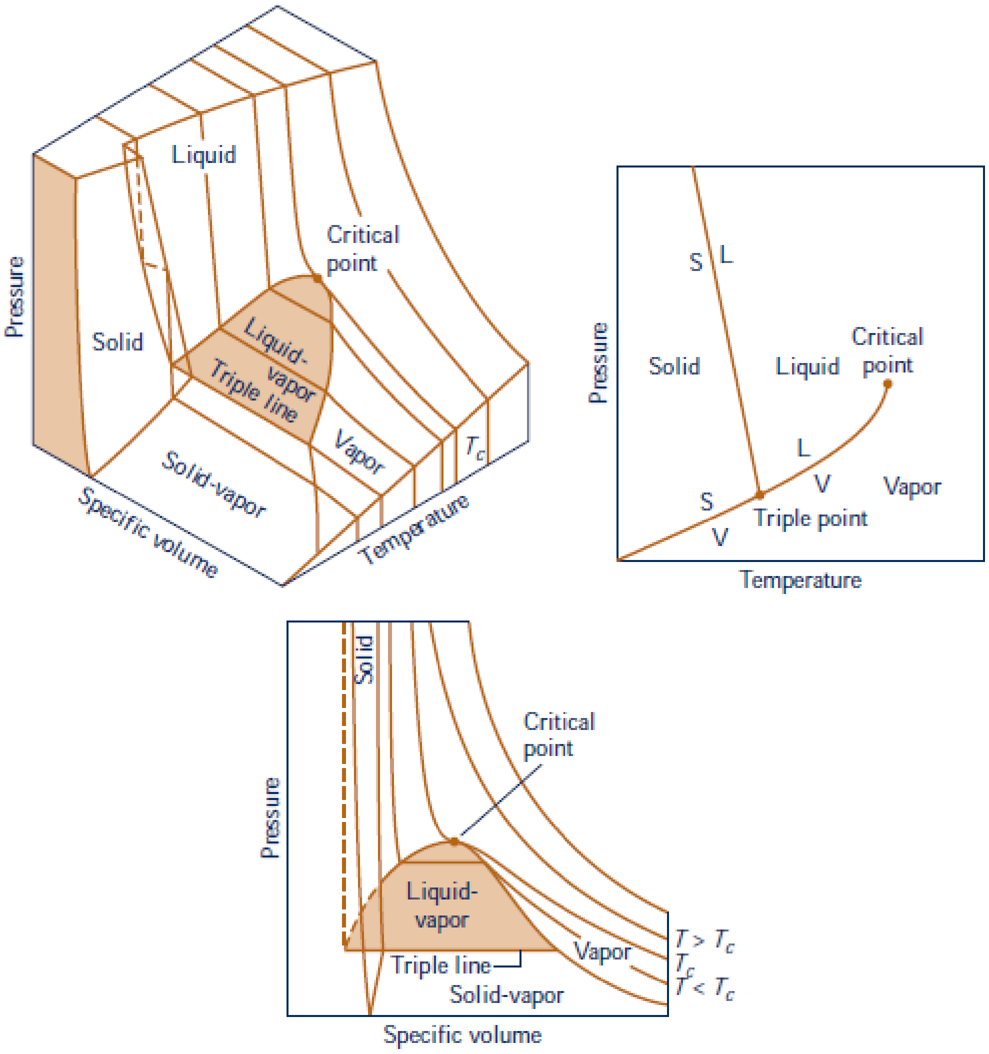
Chapter 3 – Evaluating Properties

Phase: A quantity of matter that is homogenous through in both chemical composition and physical structure (solid, liquid, gas).

- Two phases coexist during the changes in phase (vaporization, melting, and sublimation)
- A pure substance is uniform and invariable in chemical composition.

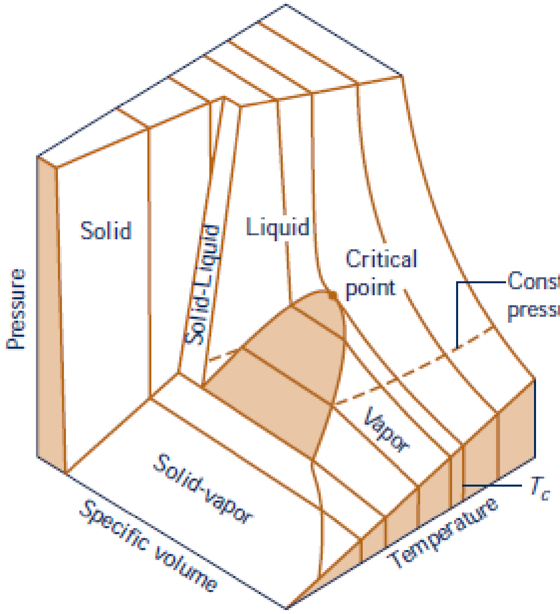
Simple Compressible Systems: Containing water or uniform mixture of non-reacting gases. The specification of the values for any <u>two</u> independent intensive properties will fix the values of all other intensive properties.

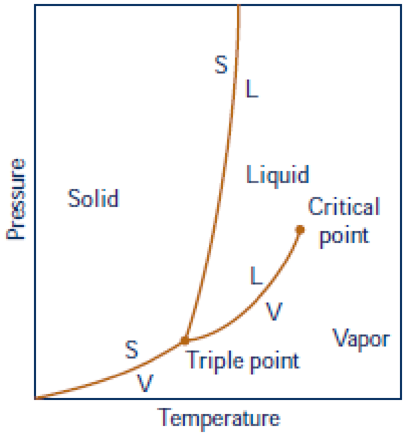
***p–v–T* surface and projections for a substance that expands on freezing.**

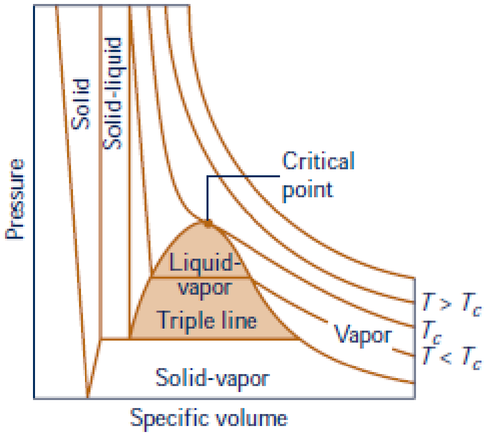


***p–v–T* surface and projections for a substance that contracts on freezing.**

Chapter 3 – Evaluating Properties





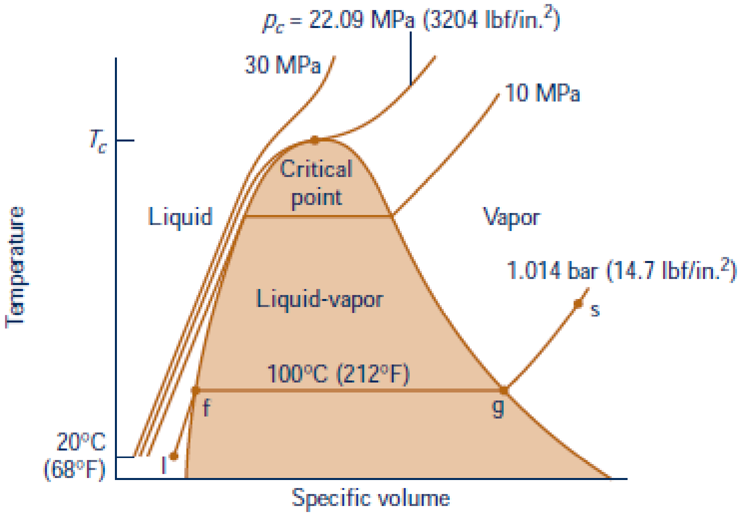


Triple Point: A point where all three phases can exist

Saturation State: A state at which a phase change begins or ends

Vapour Dome: Composed of the two-phase liquid-vapour states. Bordering lines are called Saturation Liquid and Saturation Vapour. Critical Point is at the top of the dome.

Chapter 3 – Evaluating Properties



Subcooled/Compressed Liquid: The states to the left of the vapour dome because the temperature is less than the saturation temperature at the given pressure.

The two-phase liquid-vapour mixture states can be distinguished from one another by the quality (*x*).

$$X = \frac{m_{vapour}}{m_{liquid} + m_{vapour}}$$

Superheated Vapour: States to the right of the vapour dome because the system temperature would be greater than the saturation temperature corresponding to the given pressure.

In the vapour dome, the specific volume can be calculated as:

$$v = (1 - x)v_f + xv_g = v_f + x(v_g - v_f)$$

Enthalpy (H): The sum of internal energy and the product of pressure and volume.

$$H = U + pV$$

Or for a mass basis.

$$h = u + pv$$

Or for a molar basis.

$$\bar{h} = \bar{u} + p\bar{v}$$

From the property tables in the book, internal energy and enthalpy can be calculated as:

$$u = (1 - x)u_f + xu_g = u_f + x(u_g - u_f)$$

$$h = (1 - x)h_f + xh_g = h_f + x(h_g - h_f)$$

Chapter 3 – Evaluating Properties

Where U_{fg} is usually denoted for ($U_g - U_f$) and similarly H_{fg} is denoted for ($H_g - H_f$).

In systems where potential energy and kinetic energy can be ignored, the energy balance equation is:

$$U_2 - U_1 = Q - W$$

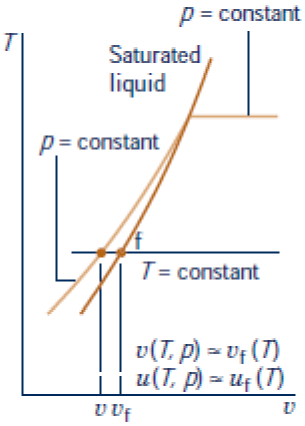
Specific Heats (c_v and c_p): The partial derivative of the functions u(T, *v*) and *h*(*T, p*) respectively. Under certain special conditions they relate the temperature change of a system to the amount of energy added by heat transfer.

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v$$

$$c_p = \left(\frac{\partial h}{\partial T}\right)_p$$

Specific Heat Ration (k): The ratio of the specific heats. $k = \frac{c_p}{c_v}$

Approximations can be made as *u* and *v* vary gradually with pressure changes at a fixed temperature. They can be made using saturated liquid data or incompressible substance model.



$$v(T, p) \approx v_f(T)$$
$$u(T, p) \approx u_f(T)$$

The enthalpy can be approximated using the formula:

$$h(T, p) \approx h_f(T) + \underline{v_f(T)[p - p_{sat}(T)]}$$

- When the underlined term is small, the expression reduces to $h(T, p) \approx h_f(T)$

<u>Incompressible Substance Model</u>

- Specific volume is assumed to be constant
- Specific internal energy is assumed to vary with temperature.

Chapter 3 – Evaluating Properties

- $c_v(T) = \dfrac{du}{dT}$, $h(T, p) = u(T) + pv$,
- $\left(\dfrac{\partial h}{\partial T}\right)_p = \dfrac{du}{dT}$, $c_p = c_v$
- The change in enthalpy: $h_2 - h_1$ = $\int_{T_1}^{T_2} c(T)\, dT + v(p_2 - p_1)$
- If c is a constant then: $h_2 - h_1 = c(T_2 - T_1) + \underline{v(p_2 - p_1)}$ (incompressible, constant c). The underlined part is often too small and can be dropped.

Universal Gas Constant

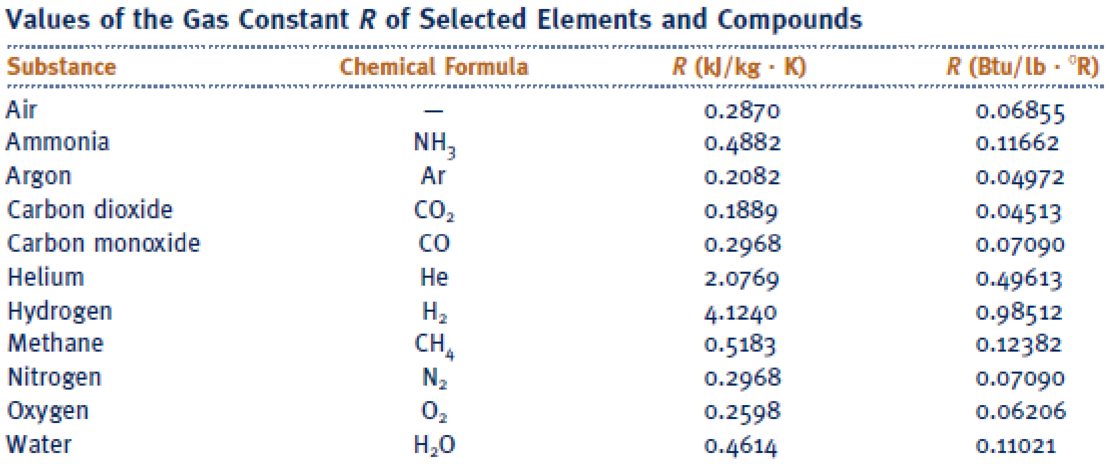
- $\lim\limits_{p\to 0} \dfrac{p\overline{v}}{T} = \overline{R}$ where $\overline{R}$ denotes the common limit for all temperatures. For all gases $\overline{R}$has the same value. $\overline{R}$ represents the universal gas constant. The most common values for $\overline{R}$:

$$\overline{R} = \begin{cases} 8.314 \text{ kJ/kmol} \cdot \text{K} \\ 1.986 \text{ Btu/lbmol} \cdot {}^\circ\text{R} \\ 1545 \text{ ft} \cdot \text{lbf/lbmol} \cdot {}^\circ\text{R} \end{cases}$$

Compressibility Factor

- Z represents the compressibility factor and is expressed as: $Z = \dfrac{p\overline{v}}{\overline{R}T}$
- Z is unitless, $\overline{v} = Mv$ however Z can be represented alternatively as:

$$Z = \frac{pv}{RT} \quad \text{where} \quad R = \frac{\overline{R}}{M}$$



Values of the Gas Constant *R* of Selected Elements and Compounds

Substance	Chemical Formula	*R* (kJ/kg · K)	*R* (Btu/lb · °R)
Air	—	0.2870	0.06855
Ammonia	NH_3	0.4882	0.11662
Argon	Ar	0.2082	0.04972
Carbon dioxide	CO_2	0.1889	0.04513
Carbon monoxide	CO	0.2968	0.07090
Helium	He	2.0769	0.49613
Hydrogen	H_2	4.1240	0.98512
Methane	CH_4	0.5183	0.12382
Nitrogen	N_2	0.2968	0.07090
Oxygen	O_2	0.2598	0.06206
Water	H_2O	0.4614	0.11021

  - Z approaches 1 as pressure approaches 0.
- Z can be plotted versus a dimensionless **reduced pressure (p_R)** and **reduced temperature (T_R)** where p_R and T_R are calculated:

$$p_R = p/p_c$$
$$T_R = T/T_c$$

p_C and T_C are the critical pressure and temperature. Which gives $Z = f(p_R, T_R)$.

- The pseudoreduced specific volume (v'_R): $v'_R = \dfrac{\bar{v}}{\bar{R}T_c/p_c}$ gives a better correlation of the data on a Generalized Compressibility Chart than the reduced specific volume $v_R = \bar{v}/\bar{v}_c$ where $\bar{v}_c$ is the critical specific volume.
- The Generalized Compressibility Chart should only be used when insufficient data has been given in a problem.

Equations of State

The curves in the compressibility chart can be expressed by two theoretical equations:

$Z = 1 + \hat{B}(T)p + \hat{C}(T)p^2 + \hat{D}(T)p^3 + \cdots$ where $\hat{B}, \hat{C}, \hat{D}, \ldots$ depend on temperature only. The dots represent higher-order terms.

$$Z = 1 + \frac{B(T)}{\bar{v}} + \frac{C(T)}{\bar{v}^2} + \frac{D(T)}{\bar{v}^3} + \cdots$$

These equations are known as **virial equations of state** and the coefficients are called **virial coefficients.**

Ideal Gas Equation of State

An ideal gas has a compressibility factor (Z) of 1 and therefore the assumption can be made that pv = RT.

Alternative forms exist: pV = mRT (v = V/m) or on a molar basis $p\bar{v} = \bar{R}T$. ($\bar{v} = V/n$)
$pV = n\bar{R}T$

When a state is given exactly, the internal energy is only dependent on temperature and thus enthalpy can be calculated as:

$$u = u(T)$$
$$h = h(T) = u(T) + RT$$

Δu, Δh, c_v, and c_p Relations

Chapter 3 – Evaluating Properties

For an ideal gas, the specific heat (c_v) is temperature dependent; $c_v(T) = \frac{du}{dT}$, solving for change in internal energy yields $u(T_2) - u(T_1) = \int_{T_1}^{T_2} c_v(T)\, dT$.

Similarly the specific enthalpy (c_p) is also temperature dependent: $c_p(T) = \frac{dh}{dT}$, solving for change in enthalpy yields $h(T_2) - h(T_1) = \int_{T_1}^{T_2} c_p(T)\, dT$.

An important relation between specific enthalpy and specific heat can be obtained using: $\frac{dh}{dT} = \frac{du}{dT} + R$ and substituting in the relations for specific heat and specific enthalpy, $c_p(T) = c_v(T) + R$. A molar basis is represented as $\overline{c}_p(T) = \overline{c}_v(T) + \overline{R}$.

The specific heat ratio (k): $k = \frac{c_p(T)}{c_v(T)}$ which gives us the expressions,

$$c_p(T) = \frac{kR}{k-1}$$

$$c_v(T) = \frac{R}{k-1}$$

Specific enthalpy $\overline{c}_p$ increases for all gases except for Ar, Ne, and He when temperature increases. $\overline{c}_p$ is a constant for these gases determined by $\overline{c}_p = \frac{5}{2}\overline{R}$.

Alternative equations exist such as: $\frac{\overline{c}_p}{\overline{R}} = \alpha + \beta T + \gamma T^2 + \delta T^3 + \varepsilon T^4$ where the coefficients are listed in Table A-21 for several gases in the temperature range 200-1000 K (540-1800°R).

Ideal Gas Tables are A-22 and A-23.

Using Ideal Gas Tables

$h(T) = \int_{T_{\text{ref}}}^{T} c_p(T)\, dT + h(T_{\text{ref}})$ where T_{ref} is an arbitrary reference temperature and $h(T_{\text{ref}})$ is an arbitrary value for enthalpy at the reference temperature. The tables A-22 and A-23 are based on h = 0 at $T_{\text{ref}} = 0$ K. $h(T) = \int_{0}^{T} c_p(T)\, dT$.

Chapter 3 – Evaluating Properties

Using Constant Specific Heats

$$u(T_2) - u(T_1) = c_v(T_2 - T_1)$$
$$h(T_2) - h(T_1) = c_p(T_2 - T_1)$$

when the specific heat and enthalpy are constants.

To determine the constants, $$c_v = \frac{\int_{T_1}^{T_2} c_v(T)\, dT}{T_2 - T_1}, \qquad c_p = \frac{\int_{T_1}^{T_2} c_p(T)\, dT}{T_2 - T_1}$$.

Polytropic Process Relations

A polytropic process is described by pV^n = constant or pv^n = constant. There are two conditions for work: $$\int_1^2 p\, dV = \frac{p_2V_2 - p_1V_1}{1 - n} \qquad (n \neq 1)$$ or $$\int_1^2 p\, dV = p_1V_1 \ln\frac{V_2}{V_1} \qquad (n = 1)$$.

When n is anything but 1 or 0, the process is isometric (constant volume).

However for an ideal gas, the expressions are slightly altered.

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(n-1)/n} = \left(\frac{V_1}{V_2}\right)^{n-1}$$

$$\int_1^2 p\, dV = \frac{mR(T_2 - T_1)}{1 - n} \qquad \text{(ideal gas, } n \neq 1)$$

$$\int_1^2 p\, dV = mRT \ln\frac{V_2}{V_1} \qquad \text{(ideal gas, } n = 1)$$

The case where n = 1 corresponds to an isothermal process (constant temperature).

Chapter 4 – Control Volume Analysis Using Energy

Developing the Mass Rate Balance

A control volume with mass flows in at *i* and flows out at *e*. As such the conservation of mass principle states

$$\left[\begin{array}{c}\text{time } \textit{rate of change} \text{ of} \\ \text{mass contained within the} \\ \text{control volume } \textit{at time t}\end{array}\right] = \left[\begin{array}{c}\text{time } \textit{rate} \text{ of flow of} \\ \text{mass } \textit{in} \text{ across} \\ \text{inlet } \textit{i at time t}\end{array}\right] - \left[\begin{array}{c}\text{time } \textit{rate} \text{ of flow} \\ \text{of mass } \textit{out} \text{ across} \\ \text{exit } \textit{e at time t}\end{array}\right]$$

The conservation of mass with respect to time is denoted as:

$$\frac{dm_{cv}}{dt} = \dot{m}_i - \dot{m}_e$$

where $\frac{dm_{cv}}{dt}$ is the time rate of change of mass within the control volume and $\dot{m}_i$ and $\dot{m}_e$ are the instantaneous mass flow rates at the inlet and exit. The units for this expression are kg/s.

When there are multiple inlets and exits, the expression is a sum: $\frac{dm_{cv}}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e$. This equation is the mass rate balance for control volumes with several inlets and exits.

Evaluating the Mass Flow Rate

To find the mass flowing in or out during a specific time: $\left[\begin{array}{c}\text{amount of mass} \\ \text{crossing } dA \text{ during} \\ \text{the time interval } \Delta t\end{array}\right] = \rho(V_n\,\Delta t)\,dA$

To find the instantaneous mass flowing in or out: $\left[\begin{array}{c}\text{instantaneous rate} \\ \text{of mass flow} \\ \text{across } dA\end{array}\right] = \rho V_n\,dA$

$$\dot{m} = \int_A \rho V_n\,dA$$

One-Dimensional Flow Form of the Mass Rate Balance

Flow is said to be one-dimensional if the following conditions are met:

- The flow is normal to the boundary at locations where mass enters or exits the control volume
- *All* intensive properties, including velocity and density, are *uniform with position* (bulk average values) over each inlet or exit area which matter flows

$$\dot{m} = \rho A V \qquad \text{(one-dimensional flow)}$$

Or in terms of specific volume,

Chapter 4 – Control Volume Analysis Using Energy

$$\dot{m} = \frac{\mathrm{A}\mathrm{V}}{v} \qquad \text{(one-dimensional flow)}$$

(This is called the volumetric flow rate. Units: m^3/s)

$$\frac{dm_{cv}}{dt} = \sum_i \frac{\mathrm{A}_i \mathrm{V}_i}{v_i} - \sum_e \frac{\mathrm{A}_e \mathrm{V}_e}{v_e}$$

Steady-State Form of the Mass Rate Balance

When the system is in a steady state (all properties are unchanging in time):

$$\underset{\text{(mass rate in)}}{\sum_i \dot{m}_i} = \underset{\text{(mass rate out)}}{\sum_e \dot{m}_e}$$

Integral Form of the Mass Rate Balance

$$m_{cv}(t) = \int_V \rho \, dV$$

and incorporating this expression into the equation for mass rate balance yields:

$$\frac{d}{dt}\int_V \rho \, dV = \sum_i \left(\int_{\mathrm{A}} \rho \mathrm{V}_n \, d\mathrm{A} \right)_i - \sum_e \left(\int_{\mathrm{A}} \rho \mathrm{V}_n \, d\mathrm{A} \right)_e$$

($\rho \mathrm{V}_n$ is known as the mass flux)

Developing the Energy Rate Balance for a Control Volume

The *conservation of energy* principle applied to a control volume states:

$$\begin{bmatrix} \text{time } \textit{rate of change} \\ \text{of the energy} \\ \text{contained within} \\ \text{the control volume} \\ \textit{at time t} \end{bmatrix} = \begin{bmatrix} \text{net } \textit{rate} \text{ at which} \\ \text{energy is being} \\ \text{transferred in} \\ \text{by heat transfer} \\ \textit{at time t} \end{bmatrix} - \begin{bmatrix} \textit{net} \text{ rate at which} \\ \text{energy is being} \\ \text{transferred out} \\ \text{by work } \textit{at} \\ \textit{time t} \end{bmatrix} + \begin{bmatrix} \textit{net} \text{ rate of energy} \\ \text{transfer } \textit{into} \text{ the} \\ \text{control volume} \\ \text{accompanying} \\ \text{mass flow} \end{bmatrix}$$

For a one-dimensional flow rate:

$$\frac{dE_{cv}}{dt} = \dot{Q} - \dot{W} + \underline{\dot{m}_i\left(u_i + \frac{\mathrm{V}_i^2}{2} + gz_i\right)} - \underline{\dot{m}_e\left(u_e + \frac{\mathrm{V}_e^2}{2} + gz_e\right)}$$

E_{cv} denotes the energy of the control volume at time t. The terms $\dot{Q}$ and $\dot{W}$ account, respectively, for the net rate of energy transfer by heat and work across the boundary of the control volume at t. The underlined terms account for the rates of transfer or internal, kinetic, and potential energy of the entering and exiting streams. If there is no mass flow in or out, the underlined terms drop out.

Chapter 4 – Control Volume Analysis Using Energy

Evaluating Work for a Control Volume

The work done by the system on the surroundings at an exit is stated as:

$$\begin{bmatrix} \text{time rate of energy transfer} \\ \text{by work } \textit{from} \text{ the control} \\ \text{volume at exit } e \end{bmatrix} = (p_e \mathrm{A}_e)\mathrm{V}_e$$

At the exit, $p_e \mathrm{A}_e$ is the normal force and V_e is the fluid velocity. A similar expression can be written for the work done.

$\dot{W} = \dot{W}_{\text{cv}} + (p_e \mathrm{A}_e)\mathrm{V}_e - (p_i \mathrm{A}_i)\mathrm{V}_i$ (A positive sign means work is done by the control volume and a negative sign indicates work is done onto the control volume), since $\mathrm{AV} = \dot{m}v$ the expression is expressed as:

$$\dot{W} = \dot{W}_{\text{cv}} + \dot{m}_e(p_e v_e) - \dot{m}_i(p_i v_i)$$

The term flow work corresponds to $\dot{m}_e(p_e v_e)$ and $\dot{m}_i(p_i v_i)$ whereas $\dot{W}_{\text{cv}}$ is all other work across the boundary of the control volume.

One-Dimensional Flow Form of the Control Volume Energy Rate Balance

When the expression for work is substituted into the expression for the rate of change in energy we get:

$$\frac{dE_{\text{cv}}}{dt} = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} + \dot{m}_i\left(u_i + p_i v_i + \frac{\mathrm{V}_i^2}{2} + gz_i\right) - \dot{m}_e\left(u_e + p_e v_e + \frac{\mathrm{V}_e^2}{2} + gz_e\right)$$

Since $h = u + pv$, the equation becomes:

$$\frac{dE_{\text{cv}}}{dt} = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} + \dot{m}_i\left(h_i + \frac{\mathrm{V}_i^2}{2} + gz_i\right) - \dot{m}_e\left(h_e + \frac{\mathrm{V}_e^2}{2} + gz_e\right)$$

When there are many inlets and exits the energy rate balance becomes a sum:

$$\frac{dE_{\text{cv}}}{dt} = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} + \sum_i \dot{m}_i\left(h_i + \frac{\mathrm{V}_i^2}{2} + gz_i\right) - \sum_e \dot{m}_e\left(h_e + \frac{\mathrm{V}_e^2}{2} + gz_e\right)$$

Integral Form of the Control Volume Energy Rate Balance

$$E_{\text{cv}}(t) = \int_V \rho e \, dV = \int_V \rho\left(u + \frac{\mathrm{V}^2}{2} + gz\right) dV$$

Chapter 4 – Control Volume Analysis Using Energy

$$\frac{d}{dt}\int_V \rho e\, dV = \dot{Q}_{\mathrm{cv}} - \dot{W}_{\mathrm{cv}} + \sum_i \left[\int_A \left(h + \frac{V^2}{2} + gz\right)\rho V_n\, dA\right]_i - \sum_e \left[\int_A \left(h + \frac{V^2}{2} + gz\right)\rho V_n\, dA\right]_e$$

Steady-State Forms of the Mass and Energy Rate Balances

When the system is fixed at a steady state (the conditions of the mass within the control volume and at the boundary do not vary with time), the mass that flows into the system also flows out at the same rate:

$$\underset{\text{(mass rate in)}}{\sum_i \dot{m}_i} = \underset{\text{(mass rate out)}}{\sum_e \dot{m}_e}$$

As well energy rate in is the same rate as the energy out:

$$\underset{\text{(energy rate in)}}{\dot{Q}_{\mathrm{cv}} + \sum_i \dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i\right)} = \underset{\text{(energy rate out)}}{\dot{W}_{\mathrm{cv}} + \sum_e \dot{m}_e\left(h_e + \frac{V_e^2}{2} + gz_e\right)}$$

For a one inlet, one exit control volume, equation further simplifies to:

$$0 = \dot{Q}_{\mathrm{cv}} - \dot{W}_{\mathrm{cv}} + \dot{m}\left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2)\right]$$

Or dividing by the mass flow rate:

$$0 = \frac{\dot{Q}_{\mathrm{cv}}}{\dot{m}} - \frac{\dot{W}_{\mathrm{cv}}}{\dot{m}} + (h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g(z_1 - z_2)$$

(Units: kJ/kg)

These equations only apply at the boundary of the control volume/

Modeling Considerations for Control Volumes at Steady State

The heat transfer ($\dot{Q}_{\mathrm{cv}}$) is set to zero because it is small relative to other energy transfers across the boundary as a result:

- The outer surface of the control volume is well insulated
- The outer surface area is too small for there to be effective heat transfer
- The temperature difference between the control volume and its surroundings is so small that the heat transfer can be ignored
- The gas or liquid passes through the control volume so quickly that there is not enough time for significant heat transfer to occur

The work term ($\dot{W}_{\mathrm{cv}}$) drops out of the equations when:

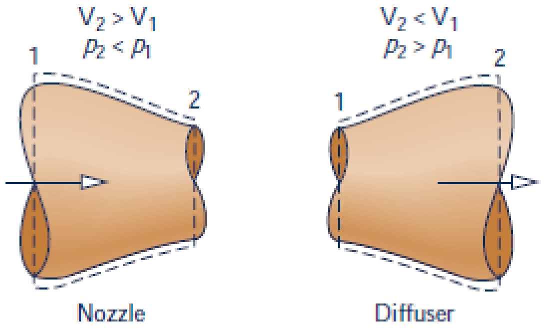
- There are not rotating shafts
- There are no displacements of the boundary
- There are no electrical effects
- Or other work mechanisms

The kinetic and potential energies of the matter entering and exiting the control volume are neglected when they are small relative to other energy transfers.

Nozzle and Diffuser Modeling Considerations

Nozzle: A flow passage of varying cross-sectional area in which the velocity of a gas or liquid increases in the direction of flow

Diffuser: The gas or liquid decelerates in the direction of flow



The only work is flow work at locations where mass enters and exits the control volume. The change in potential energy from inlet to exit is negligible under most considerations; thus the first law equation becomes:

$$0 = \dot{Q}_{cv} + \dot{m}\left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2}\right]$$

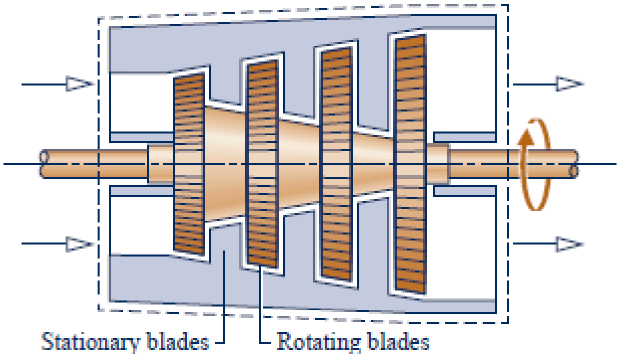
Unless otherwise given the heat transfer with the surroundings is often small enough relative to the enthalpy and kinetic energy terms that it can be neglected and thus:

$$0 = (h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right)$$

Steam and Gas Turbine Modeling Considerations

Turbine: A device in which power is developed as a result of a gas or liquid passing through a set of blades attached to a shaft free to rotate.

Chapter 4 – Control Volume Analysis Using Energy



With a proper selection of the control volume enclosing a steam or gas turbine, the net kinetic energy of the matter flowing across the boundary is usually small enough to be neglected. The net potential energy of the flowing matter also is typically negligible. The first law equation reduces to:

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2)$$

The only heat transfer between the turbine and surroundings is often small enough relative to the power and enthalpy terms that it also can be neglected unless otherwise stated.

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2)$$

Compressor and Pump Modeling Considerations

Compressor and Pump: Devices in which work is done on the substance flowing through them in order to change the state of the substance, typically to increase the pressure and/or elevation. The term *compressor* is used when the substance is a gas (vapour) and the term *pump* is used when the substance is a liquid.

The first law equation is the same for turbines:

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2)$$

For compressors and pumps the value for $\dot{W}_{cv}$ is negative because a power input is required.

Heat Exchanger Modeling Considerations

Heat exchangers can involve multiple inlets and exits. For a control volume enclosing the heat exchanger, the only work is flow work at the places where matter enters and exits. In addition the kinetic and potential energies of the flowing streams can usually be ignored unless otherwise stated.

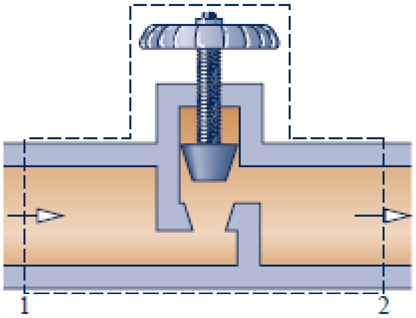
$$0 = \dot{Q}_{cv} + \sum_i \dot{m}_i h_i - \sum_e \dot{m}_e h_e$$

Heat transfer with the surroundings is often small enough to be ignored.

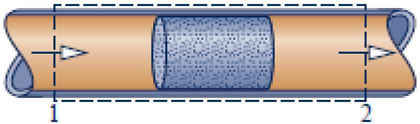
Chapter 4 – Control Volume Analysis Using Energy

Throttling Device Modeling Considerations

A significant reduction in pressure can be achieved simply by introducing a restriction into a line through which a gas or liquid flows. This is commonly done by means of a partially opened valve or a porous plug.



Partially open valve



Porous plug

For a control volume enclosing a throttling device, the only work is flow work at locations where mass enters and exits the control volume, so the work term drops out of the energy rate balance. There is usually no significant heat transfer with the surroundings, and the change in potential energy form inlet to exit is negligible.

$$0 = (h_1 - h_2) + \frac{V_1^2 - V_2^2}{2}$$

In some instances the kinetic energies can be neglected.

$$h_2 = h_1 \qquad (p_2 < p_1)$$ (Throttling Process)

The Mass Balance in Transient Analysis

Transient: When the state changes with time. Because property values, work and heat transfer rates, and mass flow rates may vary with time during transient operation, the steady-state assumption is not appropriate when analyzing such cases.

$$\int_0^t \left(\frac{dm_{cv}}{dt}\right) dt = \int_0^t \left(\sum_i \dot{m}_i\right) dt - \int_0^t \left(\sum_e \dot{m}_e\right) dt$$

$$m_{cv}(t) - m_{cv}(0) = \sum_i \left(\underline{\int_0^t \dot{m}_i \, dt}\right) - \sum_e \left(\underline{\int_0^t \dot{m}_e \, dt}\right)$$

Chapter 4 – Control Volume Analysis Using Energy

$$m_i = \int_0^t \dot{m}_i \, dt \qquad \left\{ \begin{array}{l} \text{amount of mass} \\ \text{entering the control} \\ \text{volume through inlet } i, \\ \text{from time } 0 \text{ to } t \end{array} \right.$$

$$m_e = \int_0^t \dot{m}_e \, dt \qquad \left\{ \begin{array}{l} \text{amount of mass} \\ \text{exiting the control} \\ \text{volume through exit } e, \\ \text{from time } 0 \text{ to } t \end{array} \right.$$

$$m_{\text{cv}}(t) - m_{\text{cv}}(0) = \sum_i m_i - \sum_e m_e$$

The Energy Balance in Transient Analysis

When the energy rate balance is integrated while ignoring the effects of kinetic and potential energy,

$$U_{\text{cv}}(t) - U_{\text{cv}}(0) = Q_{\text{cv}} - W_{\text{cv}} + \sum_i \left(\underline{\int_0^t \dot{m}_i h_i \, dt} \right) - \sum_e \left(\underline{\int_0^t \dot{m}_e h_e \, dt} \right)$$

In the special case where the states at the inlets and exits are constant with time, the respective specific enthalpies are constant.

$$\int_0^t \dot{m}_i h_i \, dt = h_i \int_0^t \dot{m}_i \, dt = h_i m_i$$

$$\int_0^t \dot{m}_e h_e \, dt = h_e \int_0^t \dot{m}_e \, dt = h_e m_e$$

$$U_{\text{cv}}(t) - U_{\text{cv}}(0) = Q_{\text{cv}} - W_{\text{cv}} + \sum_i m_i h_i - \sum_e m_e h_e$$

Another special case is when the intensive properties within the control volume are uniform with position at a particular time *t*. Accordingly, the specific volume and specific internal energy are uniform throughout.

$$m_{\text{cv}}(t) = V_{\text{cv}}(t)/v(t)$$

$$U_{\text{cv}}(t) = m_{\text{cv}}(t)u(t)$$

Chapter 5 – The Second Law of Thermodynamics

Aspects of the Second Law

The second law is useful in:

- Predicting the direction of processes
- Establishing conditions for equilibrium
- Determining the best theoretical performance of cycles, engines, and other devices
- Evaluating quantitatively the factors that preclude the attainment of the best theoretical performance level
- Defining a temperature scale independent of the properties of any thermometric substance
- Developing means for evaluating properties such as *u* and *h* in terms of properties that are more readily obtained experimentally

Clausius Statement of the Second Law

"**It is impossible for any system to operate in such a way that the sole result would be an energy transfer by heat from a cooler to a hotter body.**"

The statement does not say that heat transfer from a cooler body to a hotter body cannot occur, the statement is saying that there is no such system where only a heat transfer happens; there must be other effects within the system or surroundings.

Kelvin-Planck Statement of the Second Law

Thermal Reservoir: A system that always remains at constant temperature even though energy is added or removed by heat transfer.

A thermal reservoir is theoretical as there is no such system in nature; however there are close approximations such as large bodies of water (lakes, oceans).

"**It is impossible for any system to operate in a thermodynamic cycle and deliver a net amount of energy by work to its surroundings while receiving energy by heat transfer form a single thermal reservoir.**"

The statement does not deny the possibility of a system developing a net amount of work from a heat transfer drawn from a single reservoir. It only denies this possibility if the system undergoes a thermodynamic cycle.

A system undergoing a cycle while communicating thermally with a single reservoir cannot deliver a net amount of work to its surroundings. The net work of the cycle cannot be positive.

$$W_{\text{cycle}} \leq 0 \qquad \text{(single reservoir)}$$

Chapter 5 – The Second Law of Thermodynamics

Entropy State of the Second Law

$$\begin{bmatrix}\textit{change}\text{ in the amount}\\ \text{of entropy contained}\\ \text{within the system}\\ \text{during some time}\\ \text{interval}\end{bmatrix} = \begin{bmatrix}\text{net amount of}\\ \text{entropy }\textit{transferred}\\ \textit{in}\text{ across the system}\\ \text{boundary during the}\\ \text{time interval}\end{bmatrix} + \begin{bmatrix}\text{amount of entropy}\\ \textit{produced within}\text{ the}\\ \text{system during the}\\ \text{time interval}\end{bmatrix}$$

Unlike mass and energy, which are conserved, entropy is produced within systems whenever irreversibilities such as friction are present.

The entropy statement of the second law states: “**It is impossible for any system to operate in a way that entropy is destroyed.**”

Entropy production can be zero but never negative.

Irreversible Processes

A process is called **irreversible** if the system and all parts of its surroundings cannot be exactly restored to their respective initial states after the process has occurred.

A process is **reversible** if both the system and surroundings can be returned to their initial states.

Any process involving spontaneous heat transfer form a hotter body to a cooler body is irreversible. The unrestrained expansion of a gas or liquid is also irreversible. Friction, electrical resistance, hysteresis, and inelastic deformation are examples of effects that render a process irreversible.

Irreversible processes include one or more of the following:

- Heat transfer through a finite temperature difference
- Unrestrained expansion of a gas or liquid to a lower pressure
- Spontaneous chemical reaction
- Spontaneous mixing of matter at different compositions or states
- Friction – sliding friction as well as friction in the flow of fluids
- Electric current flow through a resistance
- Magnetization or polarization with hysteresis
- Inelastic deformation

All actual processes are irreversible. Internal irreversibilities are those that occur within the system. External irreversibilities are those that occur within the surroundings.

Chapter 5 – The Second Law of Thermodynamics

Reversible Processes

No process is actually reversible, however there are processes that are approximate. For example, passing a gas through a properly designed nozzle or diffuser.

Interpreting the Kelvin-Planck Statement

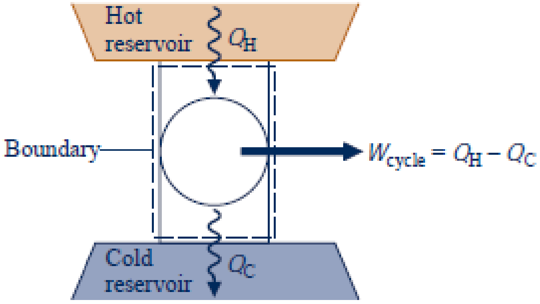
$$W_{\text{cycle}} \le 0 \begin{cases} < 0: & \text{Internal irreversibilities present.} \\ = 0: & \text{No internal irreversibilities.} \end{cases} \quad \text{(single reservoir)}$$

Limit on Thermal Efficiency

The thermal efficiency of a cycle that communicates thermally with a cold and hot reservoir is:

$$\eta = \frac{W_{\text{cycle}}}{Q_{\text{H}}} = 1 - \frac{Q_{\text{C}}}{Q_{\text{H}}}$$

Thermal efficiency must always be less than 100%.



Corollaries of the Second Law for Power Cycles

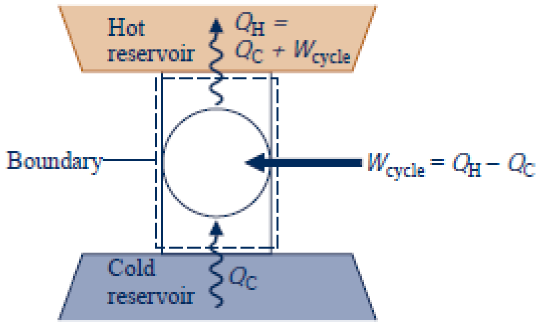
The **Carnot Corollaries** are:

- The thermal efficiency of an irreversible power cycle is always less than the thermal efficiency of a reversible power cycle when each operates between the same two thermal reservoirs.
- All reversible power cycles operating between the same two thermal reservoirs have the same thermal efficiency.

Limits on Coefficients of Performance

The second law of thermodynamics places limits on the performance of refrigeration and heat pump cycles as is does for power cycles.

Chapter 5 – The Second Law of Thermodynamics



For a refrigeration cycle the coefficient of performance is:

$$\beta = \frac{Q_C}{W_{cycle}} = \frac{Q_C}{Q_H - Q_C}$$

The coefficient of performance for a heat pump cycle is:

$$\gamma = \frac{Q_H}{W_{cycle}} = \frac{Q_H}{Q_H - Q_C}$$

Corollaries of the Second Law for Refrigeration and Heat Pump Cycles

The corollaries of the second law:

- The coefficient of performance of an irreversible refrigeration cycle is always less than the coefficient of performance of a reversible refrigeration cycle when each operates between the same two thermal reservoirs
- All reversible refrigeration cycles operating between the same two thermal reservoirs have the same coefficient of performance

The above corollaries hold when discussing heat pump cycles.

The Kelvin Scale

For reversible power cycles, the ratio of the heat transfers $\frac{Q_C}{Q_H}$ depends only on the temperatures of the two reservoirs. That is:

$$\left(\frac{Q_C}{Q_H}\right)_{\substack{\text{rev}\\ \text{cycle}}} = \psi(\theta_C, \theta_H)$$

When representing the temperature of the reservoirs in the Kelvin scale, the ratio is equivalent to:

$$\left(\frac{Q_C}{Q_H}\right)_{\substack{\text{rev}\\ \text{cycle}}} = \frac{T_C}{T_H}$$

Chapter 5 – The Second Law of Thermodynamics

The Kelvin scale is called an absolute temperature scale and has an absolute zero.

The Gas Thermometer

The constant-volume gas thermometer has been adopted internationally as the standard instrument for calibrating other thermometers.

Power Cycles

The maximum thermal efficiency of a *reversible* power cycle is:

$$\eta_{\max} = 1 - \frac{T_C}{T_H}$$

This is also known as the **Carnot efficiency**.

Refrigeration and Heat Pump Cycles

The maximum coefficient of performance for a reversible refrigeration cycle is:

$$\beta_{\max} = \frac{T_C}{T_H - T_C}$$

The maximum coefficient of performance for a reversible heat pump cycle is:

$$\gamma_{\max} = \frac{T_H}{T_H - T_C}$$

Carnot Power Cycle

Carnot Cycle: The system executing the cycle undergoes a series of four internally reversible processes; two adiabatic processes alternated with two isothermal processes.

A sample Carnot gas power cycle:

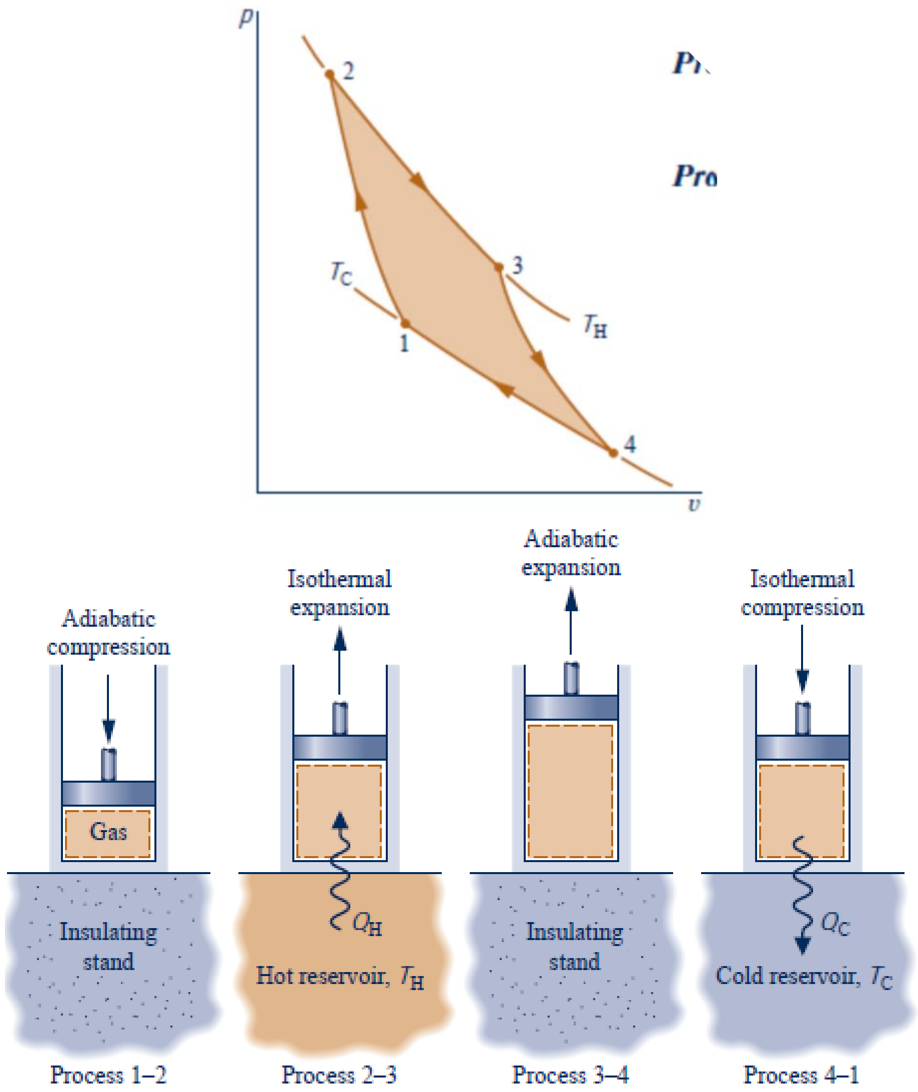
Process 1-2: The gas is compressed adiabatically to state 2, where the temperature is T_H.

Process 2-3: The assembly is placed in contact with the reservoir at T_H. The gas expands isothermally while receiving energy Q_H from the hot reservoir by heat transfer.

Process 3-4: The assembly is again placed on the insulating stand and the gas is allowed to continue to expand adiabatically until the temperature drops to T_C.

Process 4-1: The assembly is placed in contact with the reservoir at T_C. The gas is compressed isothermally to its initial state while it discharges energy Q_C to the cold reservoir by heat transfer.

Chapter 5 – The Second Law of Thermodynamics



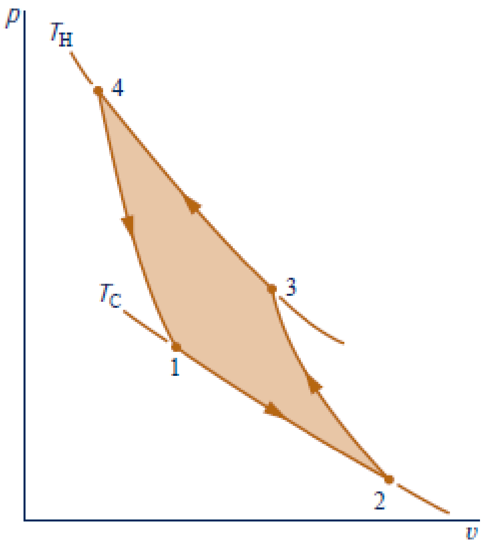
Carnot Refrigeration and Heat Pump Cycles

Process 1-2: The gas expands isothermally at T_C while receiving energy Q_C from the cold reservoir by heat transfer.

Process 2-3: The gas is compressed adiabatically until its temperature is T_H.

Process 3-4: The gas is compressed isothermally at T_H while it discharges energy Q_H to the hot reservoir by heat transfer.

Process 4-1: The gas expands adiabatically until its temperature decreases to T_C.



A refrigeration or heat pump effect can be accomplished in a cycle only if a net work input is supplied to the system executing the cycle.

Chapter 6 – Using Entropy

Defining Entropy Change

The change in entropy is given by:

$$S_2 - S_1 = \left(\int_1^2 \frac{\delta Q}{T} \right)_{\substack{\text{int} \\ \text{rev}}}$$

Entropy is an extensive property. The SI units for entropy are J/K and the FPS units for entropy are Btu/°R. The units for specific entropy are kJ/kg·K for *s* and kJ/kmol·K for $\bar{s}$ for SI and Btu/lb·°R and Btu/lbmol·°R respectively for FPS.

Saturation Data

For saturation states, the values of s_f and s_g are tabulated as a function of either saturation pressure or saturation temperature. The specific entropy of a two-phase liquid-vapour mixture is calculated using the equality:

$$\begin{aligned} s &= (1 - x)s_f + xs_g \\ &= s_f + x(s_g - s_f) \end{aligned}$$

Liquid Data

In the absence of compressed liquid data, the value of the specific entropy can be estimated in the same way as estimates for *v* and *u* are obtained for liquid states, by using the saturated liquid value at the given temperature.

$$s(T, p) \approx s_f(T)$$

Introducing the T ds Equations

The T dS equations are developed by considering a pure, simple compressible system undergoing an internally reversible process.

The first T dS equation is: $TdS = dU + pdV$

The second T dS equation is: $TdS = dH - Vdp$

The equations can be written on a unit mass basis as:

$$Tds = du + pdv$$

$$Tds = dh - vdp$$

Or on a per mole basis:

$$Td\bar{s} = d\bar{u} + pd\bar{v}$$

$$Td\bar{s} = d\bar{h} - \bar{v}dp$$

Chapter 6 – Using Entropy

Entropy Change of an Incompressible Substance

When the specific heat is constant, the change in entropy is:

$$s_2 - s_1 = c \ln\frac{T_2}{T_1} \quad \text{(incompressible, constant } c\text{)}$$

Entropy Change of an Ideal Gas

If temperature and specific volume are given for both steps, the change in entropy can be calculated as:

$$s(T_2, v_2) - s(T_1, v_1) = \int_{T_1}^{T_2} c_v(T)\frac{dT}{T} + R \ln\frac{v_2}{v_1}$$

If the temperature and pressure are given for both step, the change in entropy can be calculated as:

$$s(T_2, p_2) - s(T_1, p_1) = \int_{T_1}^{T_2} c_p(T)\frac{dT}{T} - R \ln\frac{p_2}{p_1}$$

The specific heat and specific enthalpy are related gas constant by:

$$c_p(T) = c_v(T) + R$$

Using Ideal Gas Tables

The absolute entropy [s°(T)] is defined as:

$$s^\circ(T) = \int_{T'}^{T} \frac{c_p(T)}{T} dT$$

Where T is an arbitrary reference temperature.

The expression for the change in temperature can then be simplified if pressure and temperature are given for both steps:

$$s(T_2, p_2) - s(T_1, p_1) = s^\circ(T_2) - s^\circ(T_1) - R \ln\frac{p_2}{p_1}$$

Or on a molar basis as:

$$\bar{s}(T_2, p_2) - \bar{s}(T_1, p_1) = \bar{s}^\circ(T_2) - \bar{s}^\circ(T_1) - \bar{R} \ln\frac{p_2}{p_1}$$

Assuming Constant Specific Heats

When the specific heat and specific enthalpy are constant, the change in entropy is:

Chapter 6 – Using Entropy

$$s(T_2, v_2) - s(T_1, v_1) = c_v \ln\frac{T_2}{T_1} + R \ln\frac{v_2}{v_1}$$
$$s(T_2, p_2) - s(T_1, p_1) = c_p \ln\frac{T_2}{T_1} - R \ln\frac{p_2}{p_1}$$

Area Representation of Heat Transfer

An entropy transfer accompanies heat transfer. The direction of entropy transfer is the same as the direction of heat transfer. In an adiabatic internally reversible process, entropy remains constant. A constant-entropy process is called an isentropic process.

The internally reversible heat transfer is calculated as:

$$Q_{\substack{\text{int}\\\text{rev}}} = \int_1^2 T\,dS$$

Which is the area under a temperature-entropy diagram.

Entropy Rate Balance for Control Volumes

The control volume entropy rate balance is:

$$\frac{dS_{\text{cv}}}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{\sigma}_{\text{cv}}$$

Which can be rewritten as:

$$\frac{d}{dt}\int_V \rho s\,dV = \int_A \left(\frac{\dot{q}}{T}\right)_b dA + \sum_i \left(\int_A s\rho V_n\,dA\right)_i - \sum_e \left(\int_A s\rho V_n\,dA\right)_e + \dot{\sigma}_{\text{cv}}$$

Rate Balances for Control Volumes at Steady State

The steady-state form of the entropy rate balance is:

$$0 = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{\sigma}_{\text{cv}}$$

One-Inlet, One-Exit Control Volumes at Steady State

The steady-state form of the entropy rate balance is thus:

$$0 = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{\text{cv}}$$

In the special case where there is no entropy transfer accompanying heat transfer:

$$s_2 - s_1 = \frac{\dot{\sigma}_{\text{cv}}}{\dot{m}}$$

Chapter 6 – Using Entropy

Isentropic Processes

Isentropic: Constant entropy

Using the Ideal Gas Model

When there is a constant entropy for an ideal gas, the expression for the change in entropy reduces to:

$$0 = s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{p_2}{p_1}$$

$$s^\circ(T_2) = s^\circ(T_1) + R \ln \frac{p_2}{p_1}$$

$$p_2 = p_1 \exp\left[\frac{s^\circ(T_2) - s^\circ(T_1)}{R}\right]$$

The above expressions can also be used on a molar basis.

For the special case of air being used as an ideal gas:

$$\frac{p_2}{p_1} = \frac{\exp[s^\circ(T_2)/R]}{\exp[s^\circ(T_1)/R]}$$

Where $P_r(T) = \exp[s^\circ(T)/R]$:

$$\frac{p_2}{p_1} = \frac{p_{r2}}{p_{r1}} \qquad (s_1 = s_2, \text{air only})$$

P_r is called the relative pressure. Contrary to the name, it is not a real pressure.

The ratio of the specific volumes is:

$$\frac{v_2}{v_1} = \left(\frac{RT_2}{p_2}\right)\left(\frac{p_1}{RT_1}\right)$$

$$\frac{v_2}{v_1} = \left[\frac{RT_2}{p_r(T_2)}\right]\left[\frac{p_r(T_1)}{RT_1}\right]$$

$$\frac{v_2}{v_1} = \frac{v_{r2}}{v_{r1}} \qquad (s_1 = s_2, \text{air only})$$

Where $v_r(T) = RT/P_r(T)$.

Chapter 6 – Using Entropy

When the specific heat and specific enthalpy are constants. The values for the properties can be calculated as:

$$c_p = \frac{kR}{k-1}, \qquad c_v = \frac{R}{k-1}$$

Where k is the specific heat ration and R is the gas constant.

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k} \qquad (s_1 = s_2,\ \text{constant } k)$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1} \qquad (s_1 = s_2,\ \text{constant } k)$$

Which yields

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{k} \qquad (s_1 = s_2,\ \text{constant } k)$$

.

Isentropic Turbine Efficiency

The work developed per unit mass flowing through the turbine is equivalent to the change in enthalpy.

$$\frac{\dot{W}_{\text{cv}}}{\dot{m}} = h_1 - h_2$$

The maximum value for the work per unit mass flowing through the turbine is:

$$\left(\frac{\dot{W}_{\text{cv}}}{\dot{m}}\right)_{\text{s}} = h_1 - h_{2\text{s}}$$

Where the subscript ‘s’ denotes a quantity evaluated for an isentropic process froma specified inlet state to a specified exit pressure.

Thusly, the isentropic turbine efficiency is:

$$\eta_{\text{t}} = \frac{\dot{W}_{\text{cv}}/\dot{m}}{(\dot{W}_{\text{cv}}/\dot{m})_{\text{s}}} = \frac{h_1 - h_2}{h_1 - h_{2\text{s}}}$$

Both the numerator and denominator of this expression are evaluated for the same inlet state and same exit pressure. The value of the efficiency is typically between 0.7-0.9.

Isentropic Nozzle Efficiency

The efficiency is defined as:

$$\eta_{\text{nozzle}} = \frac{\text{V}_2^2/2}{(\text{V}_2^2/2)_{\text{s}}}$$

Typically the efficiency is around 95%.

Isentropic Compressor and Pump Efficiencies

The efficiency is :

$$\eta_c = \frac{(-\dot{W}_{cv}/\dot{m})_s}{(-\dot{W}_{cv}/\dot{m})} = \frac{h_{2s} - h_1}{h_2 - h_1}$$

A typical efficiency for a compressor is between 75-85%.

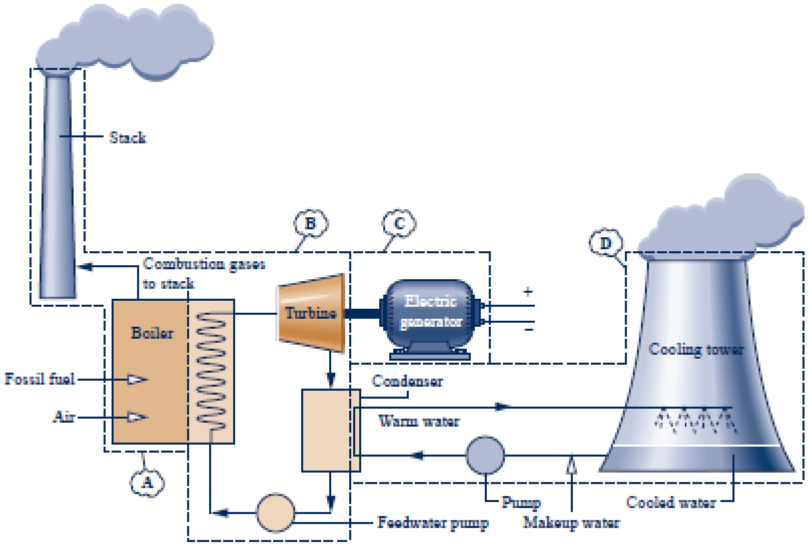
A pump efficiency is written similarly.

Chapter 8 – Vapour Power Systems

Introducing Vapour Power Plants

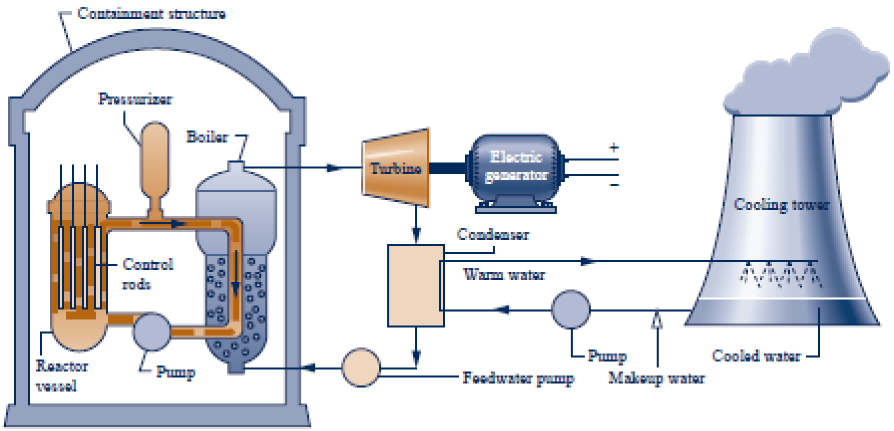
The *Rankine cycle* is the basic building block of vapour plants.

Energy conversion from heat to work occurs in subsystem **B** of the four alternative vapour plant configurations. Subsystem **A** is to supply the energy needed to vaporize the power plant *working fluid* into the vapour required by the turbine of subsystem **B**. The principal difference in the four power plant configurations is the way working fluid vaporization is accomplished by action of subsystem **A**.



(*a*) Fossil-fueled vapor power plant.

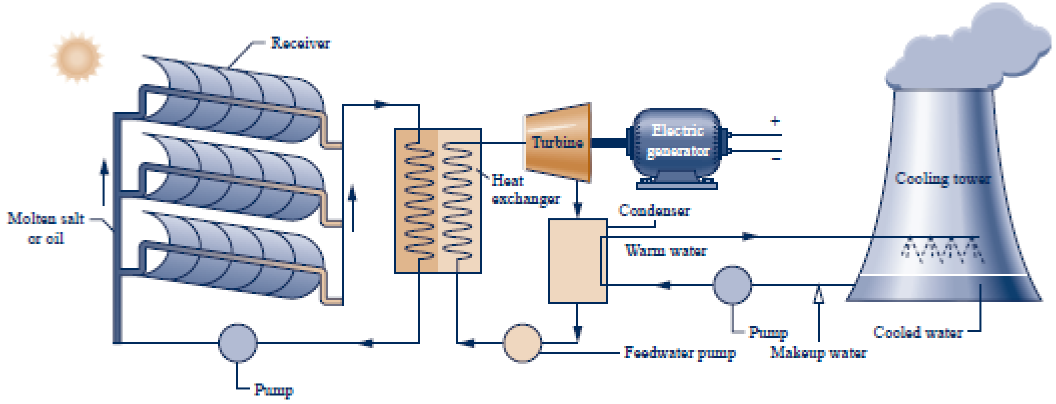
- Vaporization is accomplished in fossil-fueled plants by heat transfer *to* water passing through the boiler tubes *from* hot gases produced in the combustion of the fuel. This is also seen in plants fueled by biomass, municipal waste (trash), and mixtures of coal and biomass.



(*b*) Pressurized-water reactor nuclear vapor power plant.

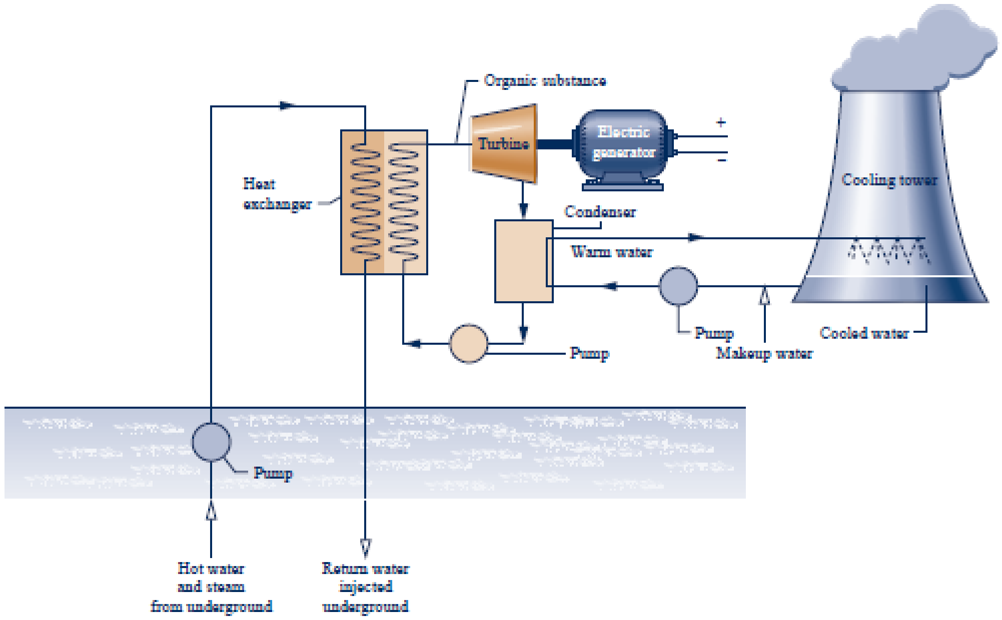
Chapter 8 – Vapour Power Systems

- In nuclear plants, energy required for vaporizing the cycle working fluid originates in a controlled nuclear reaction occurring in a reactor-containment structure. The *pressurized-water* reactor has two water loops. One loop circulates water through the reactor core and a boiler within the containment structure; this water is kept under pressure so it heats but does not boil. A separate loop carries steam from the boiler to the turbine. *Boiling-water* reactors have a single loop that boils water flowing through the core and carries steam directly to the turbine.



(c) Concentrating solar thermal vapor power plant.

- Solar power plants have receivers for collecting and concentrating solar radiation. A suitable substance, molten salt or oil, flows through the receiver, where it is heated, directed to an interconnecting heat exchanger that replaces the boiler of the fossil- and nuclear-fueled plants, and finally returned to the receiver. The heated molten salt or oil provides energy required to vaporize water flowing in the other stream of the heat exchanger. This steam is provided to the turbine.



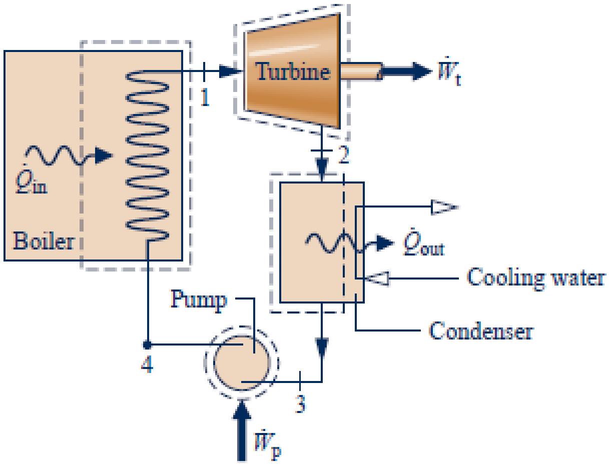
(d) Geothermal vapor power plant.

- The geothermal power plant also uses an interconnecting heat exchanger. In this case hot water and steam from deep below earth's surface flows on one side of the heat exchanger. A *secondary* working fluid having a lower boiling point than the water, such as isobutane or another organic substance, vaporizes on the other side of the heat exchanger. The secondary working fluid vapour is provided to the turbine.

Modeling the Rankine Cycle

- The first law of thermodynamics requires that the net work developed by a system undergoing a power cycle must equal the net energy added by heat transfer to the system
- The second law of thermodynamics requires that the thermal efficiency of a power cycle be less than 100%

When looking at subsystem **B** in the fossil-fueled vapour power plant the principal work and heat transfers are taken to be positive in the directions of the arrows.



Turbine

Vapour from the boiler at state 1, having an elevated temperature and pressure, expands through the turbine to produce work and then is discharged to the condenser at state 2 with relatively low pressure. Neglecting heat transfer with the surroundings, the mass and energy rate balances for a control volume around the turbine reduce at steady state to give

$$\frac{\dot{W}_t}{\dot{m}} = h_1 - h_2$$

Condenser

In the condenser there is heat transfer from the working fluid to cooling water flowing in a separate stream. The working fluid condenses and the temperature of the cooling water increases. At steady state, mass and energy rate balances for a control volume enclosing the condensing side of the heat exchanger give

$$\frac{\dot{Q}_{\text{out}}}{\dot{m}} = h_2 - h_3$$

Pump

The liquid condensate leaving the condenser at 3 is pumped from the condenser into the higher pressure boiler. Taking a control volume around the pump and assuming no heat transfer with the surroundings, mass and energy rate balances give

$$\frac{\dot{W}_{\text{p}}}{\dot{m}} = h_4 - h_3$$

Where $\frac{\dot{W}_p}{\dot{m}}$ is the rate of power input per unit of mass passing through the pump. This energy transfer is positive in the direction of the arrow.

Boiler

The working fluid completes a cycle as the liquid leaving the pump at 4, called the boiler **feedwater**, is heated to saturation and evaporated in the boiler. Taking a control volume enclosing the boiler tubes and drums carrying the feedwater from state 4 to state 1, mass ad energy rate balances give

$$\frac{\dot{Q}_{\text{in}}}{\dot{m}} = h_1 - h_2$$

Performance Parameters

The thermal efficiency of the power cycle is:

$$\eta = \frac{\dot{W}_{\text{t}}/\dot{m} - \dot{W}_{\text{p}}/\dot{m}}{\dot{Q}_{\text{in}}/\dot{m}} = \frac{(h_1 - h_2) - (h_4 - h_3)}{h_1 - h_4}$$

$$\begin{aligned}\eta &= \frac{\dot{Q}_{\text{in}}/\dot{m} - \dot{Q}_{\text{out}}/\dot{m}}{\dot{Q}_{\text{in}}/\dot{m}} = 1 - \frac{\dot{Q}_{\text{out}}/\dot{m}}{\dot{Q}_{\text{in}}/\dot{m}} \\ &= 1 - \frac{(h_2 - h_3)}{(h_1 - h_4)}\end{aligned}$$

Heat Rate: The amount of energy added by heat transfer to the cycle, usually in Btu, to produce a unit of net work output, usually in kW·h. Accordingly the heat rate is inversely proportional to the thermal efficiency and has units of Btu/ kW·h.

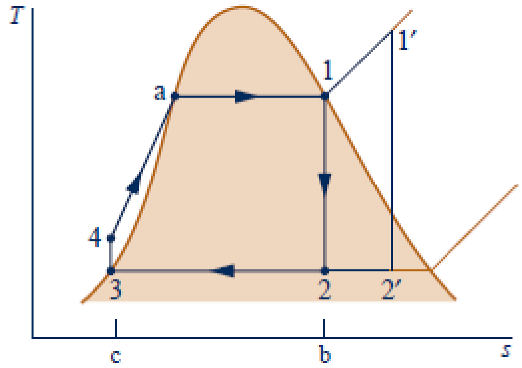
Chapter 8 – Vapour Power Systems

Back Work Ratio: The ratio of the pump work input to the work developed by the turbine.

$$\mathrm{bwr} = \frac{\dot{W}_p/\dot{m}}{\dot{W}_t/\dot{m}} = \frac{(h_4 - h_3)}{(h_1 - h_2)}$$

Ideal Rankine Cycle

Ideal Rankine Cycle: The working fluid passes through the various components of the simple vapour power cycle without irreversibilities, frictional pressure drops would be absent from the boiler and condenser, and the working fluid would flow through these components at constant pressure. Also, in the absence of irreversibilities and heat transfer with the surroundings, the processes through the turbine and pump would be isentropic.



Process 1-2: Isentropic expansion of the working fluid through the turbine from saturated vapour at state 1 to the condenser pressure

Process 2-3: Heat transfer *from* the working fluid as it flows at constant pressure through the condenser with saturated liquid at state 3

Process 3-4: Isentropic compression in the pump to state 4 in the compressed liquid region

Process 4-1: Heat transfer *to* the working fluid as it flows at constant pressure through the boiler to complete the cycle

Because the pump is idealized as operating without irreversibilities, evaluating the pump work is determined by:

$$\left(\frac{\dot{W}_p}{\dot{m}}\right)_{\substack{\text{int}\\ \text{rev}}} = \int_3^4 v\, dp$$

Because the specific volume of the liquid normally varies only slightly as the liquid flows from the inlet to the exit of the pump, a plausible approximation to the value of the integral can be had by taking the specific volume at the pump inlet, v_3 as constant for the process:

$$\left(\frac{\dot{W}_p}{\dot{m}}\right)_s \approx v_3(p_4 - p_3)$$

Where the subscript s signals the *isentropic*-internally reversible *and* adiabatic-process of the liquid flowing through the pump.

Chapter 8 – Vapour Power Systems

Principal Irreversibilities and Losses

Internal Effects

Turbine

The principal internal irreversibility experienced by the working fluid is associated with expansion through the turbine. Eat transfer from the turbine to its surroundings is a loss; but since it is of secondary importance, this loss is ignored. Isentropic efficiency is:

$$\eta_t = \frac{(\dot{W}_t/\dot{m})}{(\dot{W}_t/\dot{m})_s} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

Where the numerator is the actual work developed per unit of mass flowing through the turbine and denominator is the work per unit of mass flowing for an isentropic expansion from the turbine inlet state to the turbine exhaust pressure. Irreversibilities within the turbine reduce the net power output of the plant and thus thermal efficiency.

Pump

The work input to the pump required to overcome irreversibilities also reduces the net power output of the plant. The isentropic pump efficiency is:

$$\eta_p = \frac{(\dot{W}_p/\dot{m})_s}{(\dot{W}_p/\dot{m})} = \frac{h_{4s} - h_3}{h_4 - h_3}$$

$$\eta_p = \frac{(\dot{W}_p/\dot{m})_s}{(\dot{W}_p/\dot{m})} = \frac{v_3(p_4 - p_3)}{h_4 - h_3}$$

External Effects

The most significant source of irreversibility by far for a fossil-fueled vapour power plant is associated with combustion of the fuel and subsequent heat transfer from hot combustion gases to the cycle working fluid. As combustion and subsequent heat transfer occur in the surroundings of the subsystem, they are classified as *external*.

Improving Performance – Superheat, Reheat, and Supercritical

Superheat and reheat are both features that are normally incorporated into vapour power plants.

There is no limit to having saturated vapour at the turbine inlet, further energy can be added by heat transfer to the steam, bringing it to a superheated vapour condition at the turbine inlet. This is accomplished in a separate heat exchanger called a superheater. The combination of boiler and superheater is referred to as a *steam generator*. A cycle with superheat has a higher average temperature of heat addition than a cycle without superheating, so the thermal efficiency is higher. Superheating tends to alleviate the problem of low steam quality at the turbine exhaust.

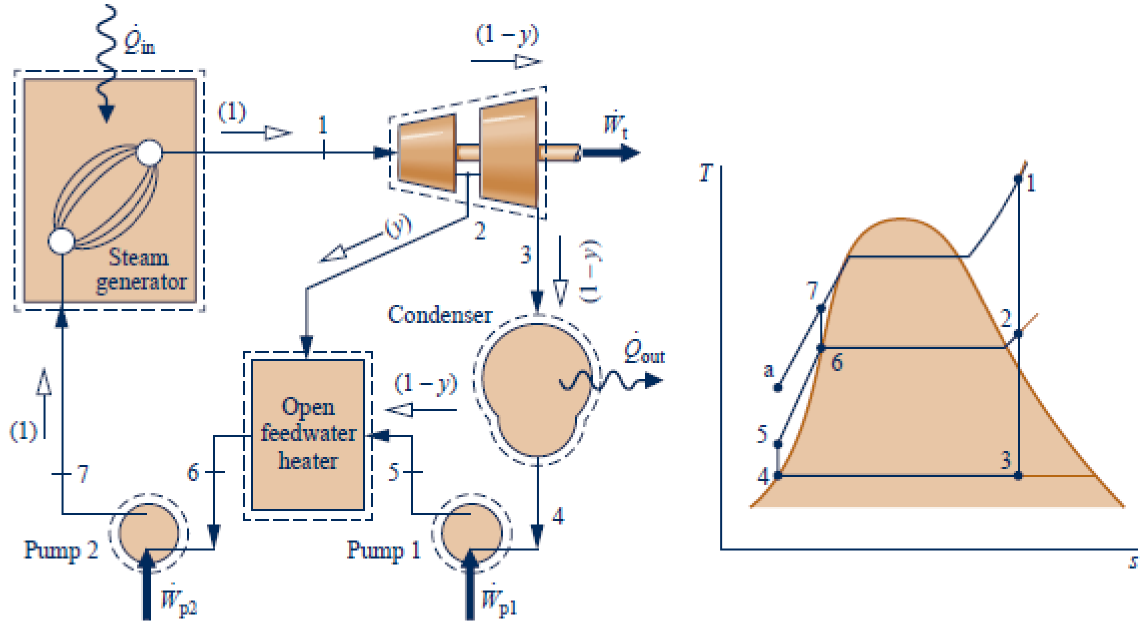
Reheat allows a power plant to take advantage of the increased efficiency that results with higher boiler pressures and avoid low-quality steam at the turbine exhaust. Steam expands through a first-stage turbine to some pressure between the steam generator and condenser pressures. Steam is then reheated in the steam generator (ideally there is no pressure drop at this stage). The steam is then expanded in a second-stage turbine to the condenser pressure. With reheat, the quality of the steam is increased at the turbine exhaust.

The temperature of the steam entering the turbine is restricted by metallurgical limitations imposed by materials used to fabricate the superheater, reheater, and turbine. High pressure in the steam generator also requires piping that can withstand great stresses at elevated temperatures. Improved materials and fabrication methods now allow vapour power plants to operate with steam generator pressures exceeding the critical pressure of water (22.1 MPa). These plants are known as supercritical plants. Steam generation occurs at a pressure above the critical pressure, no pronounced phase change occurs during this process, and a conventional boiler is not used.

Open Feedwater Heaters

Regeneration is also referred to as regenerative feedwater heating. Regeneration yields an average increase in temperature of heat addition, thus a higher thermal efficiency.

An open feedwater heater is a direct contact-type heat exchanger in which streams at different temperatures mix to form a stream at an intermediate temperature. There is ideally no pressure drop in the open feedwater heater. Steam enters the first-stage turbine and expands, where a fraction of the total flow is extracted into an open feedwater heater operating at the extraction pressure. The rest of the steam expands through the second-stage turbine. This portion of the total flow is condensed to saturated liquid, and then pumped to the extraction pressure and introduced into the feedwater heater. A single mixed stream exits the feedwater heater. The working fluid is then pumped to the steam generator.



The total flow passing through the second-stage turbine when taking a control volume enclosing both turbine stages is $\frac{\dot{m}_3}{\dot{m}_1} = 1 - y$ where $y = \frac{\dot{m}_2}{\dot{m}_1}$. It can alternatively be expressed as:

$$y = \frac{h_6 - h_5}{h_2 - h_5}$$

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1 - h_2) + (1 - y)(h_2 - h_3) \quad , \quad \frac{\dot{W}_p}{\dot{m}_1} = (h_7 - h_6) + (1 - y)(h_5 - h_4)$$

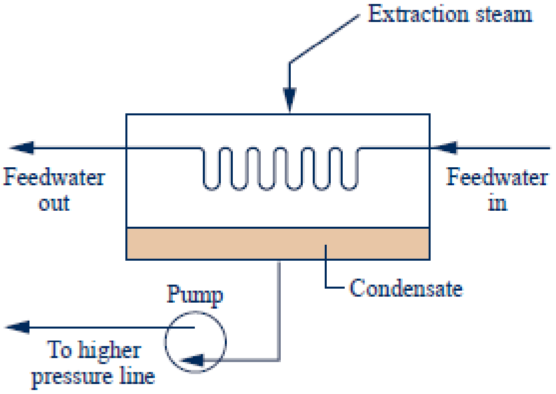
$$\frac{\dot{Q}_{in}}{\dot{m}_1} = h_1 - h_7 \quad , \quad \frac{\dot{Q}_{out}}{\dot{m}_1} = (1 - y)(h_3 - h_4)$$

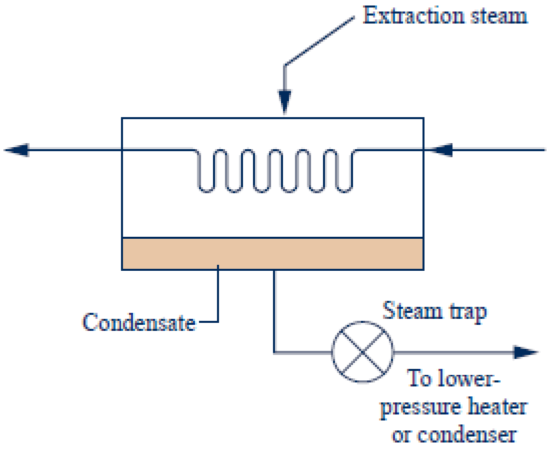
Closed Feedwater Heaters

Closed heaters are shell-and-tube-type recuperators in which the feedwater temperature increases as the extracted steam condenses on the outside of the tubes carrying the feedwater. Since the two steams do not mix, they can be at different pressures.

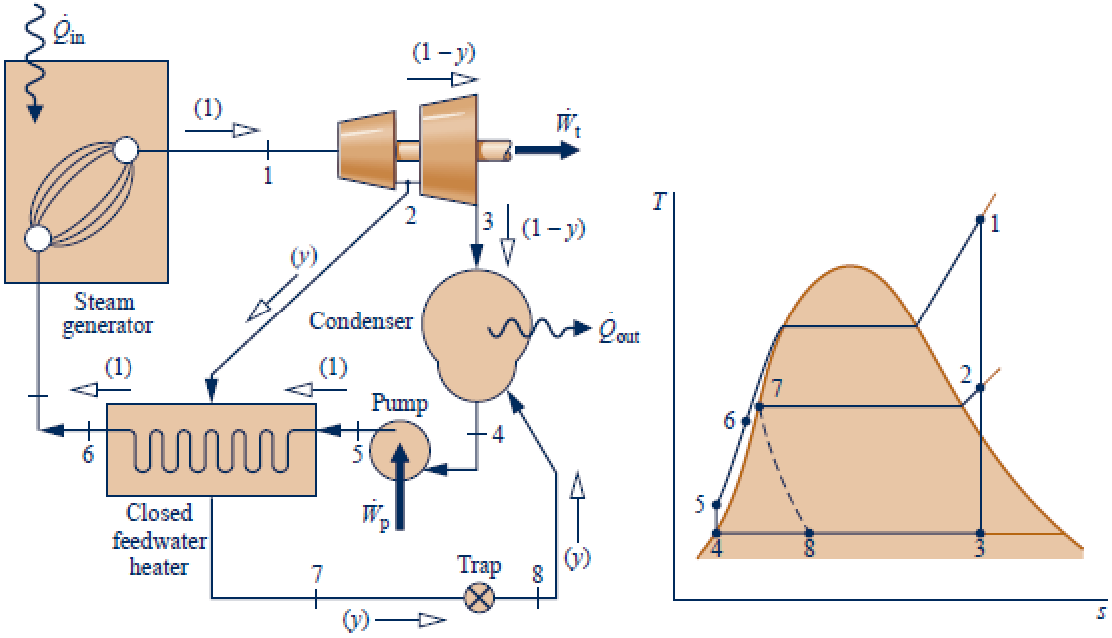
There are two ways to remove the condensate from the heater. The condensate from the closed feedwater heaters is pumped forward to a higher-pressure point in the cycle. Or alternatively, the condensate is allowed to expand through a trap into a feedwater heater operating at a lower pressure or into the condenser. A trap is a type of valve that permits only liquid to pass through to a region of lower pressure.

Chapter 8 – Vapour Power Systems





The working fluid passes isentropically through the turbine stages and pups. Except for expansion through the trap, there are no pressure drops accompanying flow through other components. The total steam flow expands through the first-stage turbine. At this location, a fraction of the flow is bled into the closed feedwater heater, where it condenses. Saturated liquid at the extraction pressure exits the feedwater heater. The condensate is then trapped into the condenser, where it is reunited with the portion of the total flow passing through the second-stage turbine. The expansion is through the trap is irreversible. (This is represented by a dashed line on T-s diagrams). The total flow exiting the condenser as saturated liquid is pumped to the steam generator pressure and enters the feedwater heater. The temperature of the feedwater is increased in passing through the feedwater heater. The cycle is completed as the working fluid is heated in the steam generator at constant pressure.

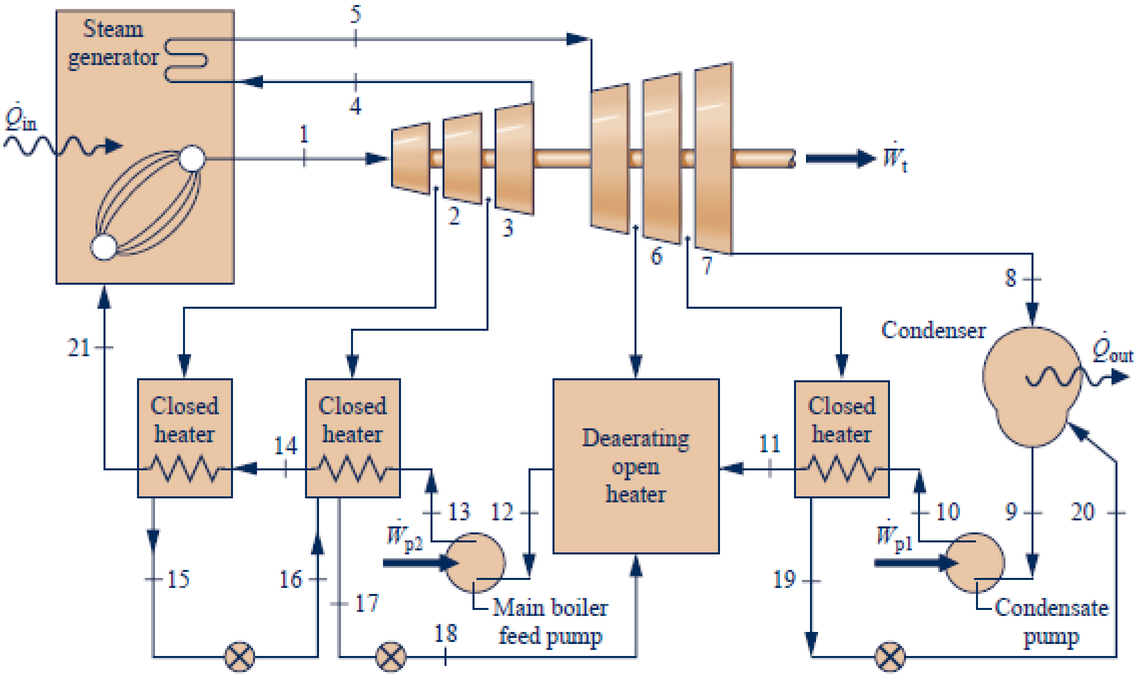


$$y = \frac{h_6 - h_5}{h_2 - h_7}$$

Chapter 8 – Vapour Power Systems

Multiple Feedwater Heaters

The thermal efficiency of the regenerative cycle can be increased by incorporating several feedwater heaters at suitably chosen pressures. Power plants with multiple feedwater heaters ordinarily have at least one open feedwater heater operating at a pressure greater than atmospheric pressure so that oxygen and other dissolved gases can be vented from the cycle, this procedure is known as **deaeration**. It is needed to maintain the purity of the working fluid in order to minimize corrosion.

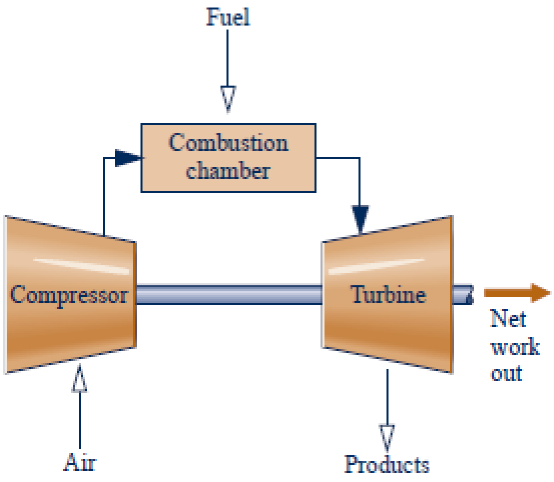


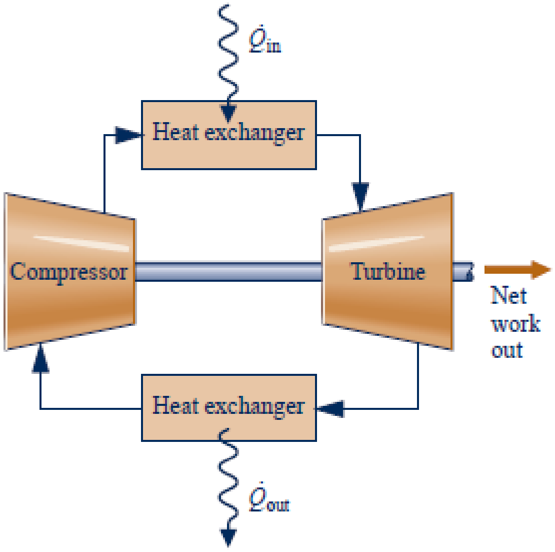
Chapter 9 - Gas Power Systems

Modelling Gas Turbine Power Plants

Gas turbine power plants may operate on either an open or closed basis. The open mode picture is more common. This is an engine in which atmospheric air is continuously drawn into the compressor, where it is compressed to a high pressure. The air then enters a combustion chamber, combustor, where it is mixed with fuel and combustion occurs, resulting in combustion products at an elevated temperature. The combustion products expand through the turbine and are subsequently discharged to the surroundings. Part of the turbine work developed is used to drive the compressor; the remainder is available to generate electricity, to propel a vehicle, or for other purposes.

In the closed mode, the working fluid receives an energy input by heat transfer from an external source, for example a gas-cooled nuclear reactor. The gas exiting the turbine is passed through a heat exchanger, where it is cooled prior to re-entering the compressor.





The air-standard analysis is often used when studying open gas turbine power plants. There are two assumptions in the analysis:

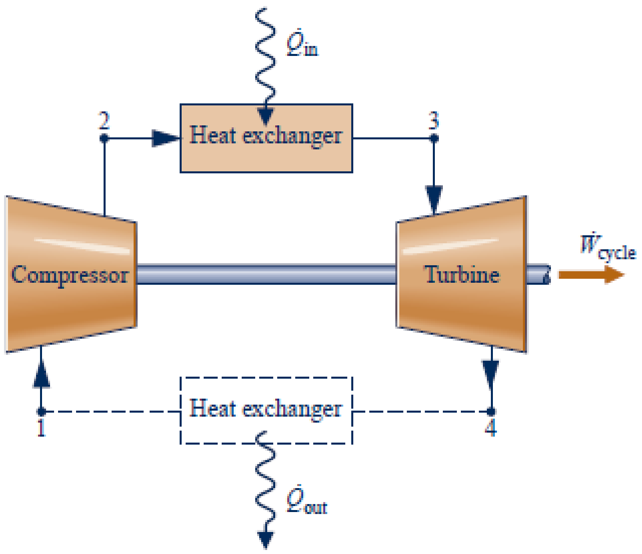
- The working fluid is air, which behaves as an ideal gas
- The temperature rise that would be brought about by combustion is accomplished by heat transfer form an external source

With an air-standard analysis, we avoid dealing with the complexities of the combustion process and the change of composition during combustion.

Evaluating Principal Work and Heat Transfers

A simplified representation of the states visited by the air in such a cycle can be devised by regarding the turbine exhaust air as restored to the compressor inlet state by passing through a heat exchanger where heat rejection to the surroundings occurs. This is called a **Brayton** cycle.

Chapter 9 - Gas Power Systems



Assuming the turbine operates adiabatically and with negligible effects of kinetic and potential energy, the work developed per unit of mass flowing is:

$$\frac{\dot{W}_t}{\dot{m}} = h_3 - h_4$$

With the same assumptions, the compressor work input per unit of mass flowing is:

$$\frac{\dot{W}_c}{\dot{m}} = h_2 - h_1$$

The heat added to the cycle per unit of mass is:

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_2$$

The heat rejected per unit of mass is:

$$\frac{\dot{Q}_{out}}{\dot{m}} = h_4 - h_1$$

The thermal efficiency of the cycle is:

$$\eta = \frac{\dot{W}_t/\dot{m} - \dot{W}_c/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_2}$$

The back work ratio for the cycle is:

$$\text{bwr} = \frac{\dot{W}_c/\dot{m}}{\dot{W}_t/\dot{m}} = \frac{h_2 - h_1}{h_3 - h_4}$$

For the same pressure rise, a gas turbine compressor would require a much greater work input per unit of mass flow than the pump of a vapour power plant because the average specific volume of the gas flowing through the compressor would be many times greater than that of the liquid passing through the pump. Hence a relatively, large portion of the work developed by the turbine is required to drive the compressor. Typical back work ratios of gas turbines range from 40 to 80%. In comparison, the back work ratios of vapour power plants are normally only 1 or 2%.

Ideal Air-Standard Brayton Cycle

Ignoring irreversibilities as the air circulates through the various components of the Brayton cycle, there are no frictional pressure drops, and the air flows at constant pressure through the heat exchangers. If stray heat transfers to the surroundings are also ignored, the processes through the turbine and compressor are isentropic.

For isentropic processes 1-2 and 3-4 in a Brayton cycle:

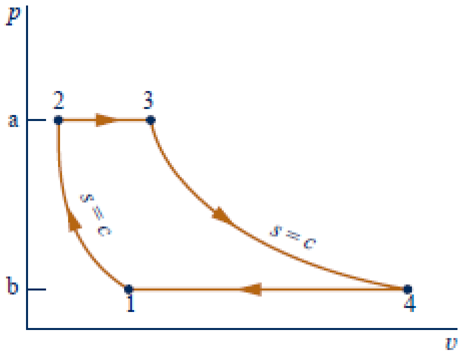
$$p_{r2} = p_{r1}\frac{p_2}{p_1}\quad , \quad p_{r4} = p_{r3}\frac{p_4}{p_3} = p_{r3}\frac{p_1}{p_2}$$

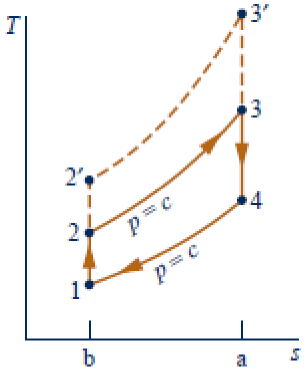
Where p_2/p_1 is the compressor ratio. Since air flows through the heat exchangers of the ideal cycle at constant pressure, it follows that $p_4/p_3 = p_2/p_1$.

When an ideal Brayton cycle is analyzed on a cold air-standard basis, the specific heats are taken as constant.

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(k-1)/k} \quad , \quad T_4 = T_3\left(\frac{p_4}{p_3}\right)^{(k-1)/k} = T_3\left(\frac{p_1}{p_2}\right)^{(k-1)/k}$$

Where k is the specific heat ratio, $k = c_p/c_v$.





Thermal efficiency increases with increasing pressure ration across the compressor. The thermal efficiency is thus:

$$\eta = 1 - \frac{1}{(p_2/p_1)^{(k-1)/k}} \qquad \text{(cold-air standard basis)}$$

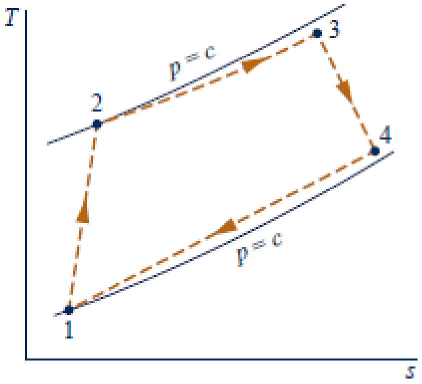
Considering Gas Turbine Irreversibilities and Losses

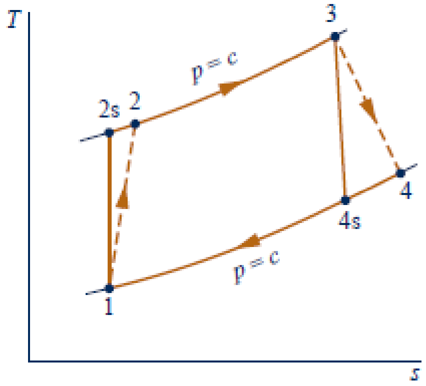
Because of frictional effects within the compressor and turbine, the working fluid would experience increases in specific entropy across these components. Owing to friction, there also would be pressure drops as the working fluid passes through the heat exchangers. However, because frictional pressure drops in the heat exchangers are less significant sources of irreversibility, they can be ignored and show the flow through the heat exchangers as occurring at constant pressure.

The isentropic efficiencies of the turbine and compressor are:

$$\eta_t = \frac{(\dot{W}_t/\dot{m})}{(\dot{W}_t/\dot{m})_s} = \frac{h_3 - h_4}{h_3 - h_{4s}}, \quad \eta_c = \frac{(\dot{W}_c/\dot{m})_s}{(\dot{W}_c/\dot{m})} = \frac{h_{2s} - h_1}{h_2 - h_1}$$

Among the irreversibilities of actual gas turbine power plants, irreversibilities within the turbine and compressor are important, but the most significant is combustion irreversibility.

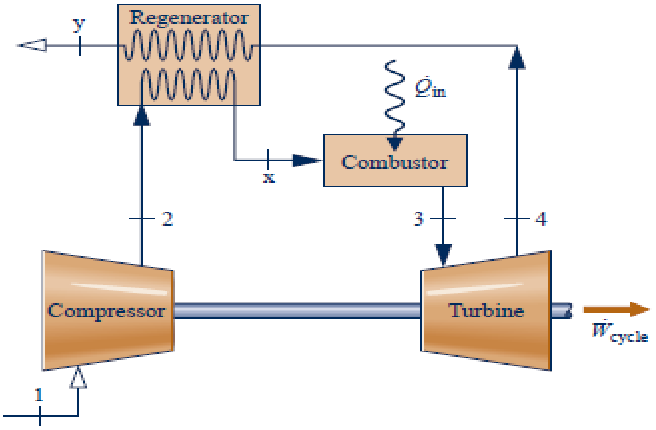


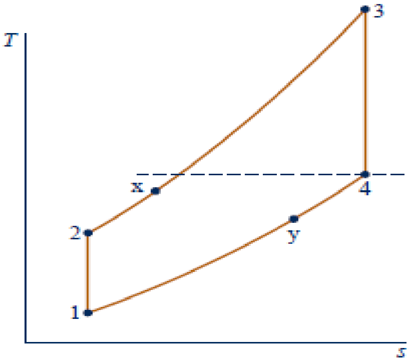


Regenerative Gas Turbines

The turbine exhaust temperature of a simple gas turbine is normally well above the ambient temperature.

Regenerator: A heat exchanger which allows the air exiting the compressor to be preheated before entering the combustor.



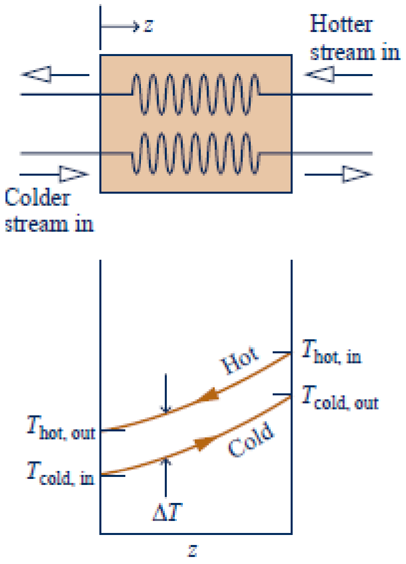


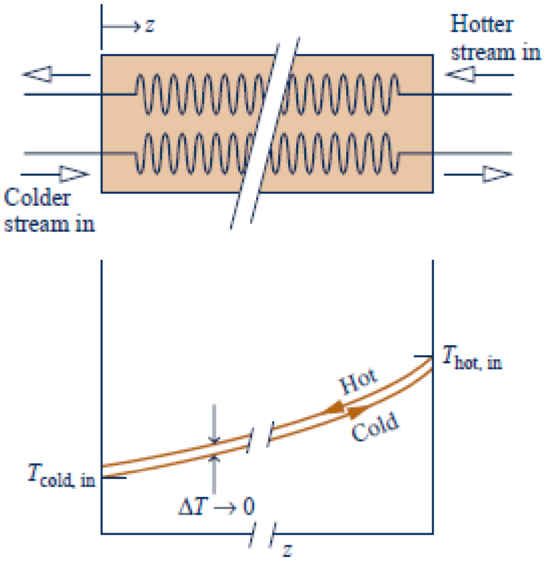
The heat added per unit of mass is then given by:

$$\frac{\dot{Q}_{\text{in}}}{\dot{m}} = h_3 - h_x$$

The net work developed per unit of mass flow is not altered by the addition of a regenerator. Thus, since the heat added is reduced, the thermal efficiency increases.

It can be concluded that the external heat transfer required by a gas turbine power plant decreases as the specific enthalpy h_x increases and thus as the temperature T_x increases.





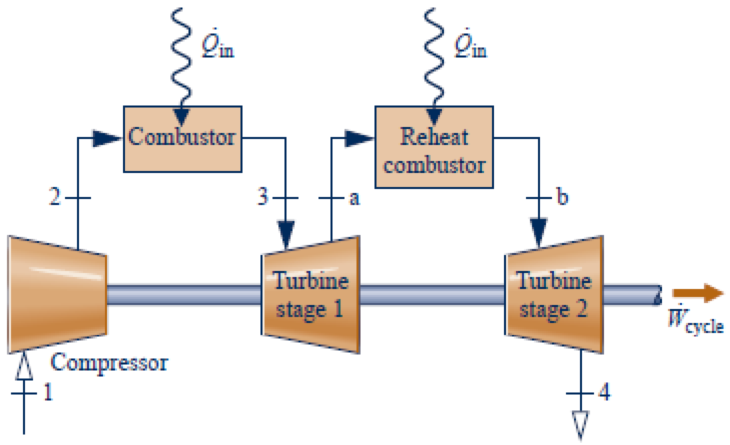
The regenerator effectiveness is a parameter that gauges the departure of an actual regenerator from such an ideal generator. It is defined as the ratio of the actual enthalpy increase of the air flowing through the compressor side of the regenerator to the maximum theoretical enthalpy increase.

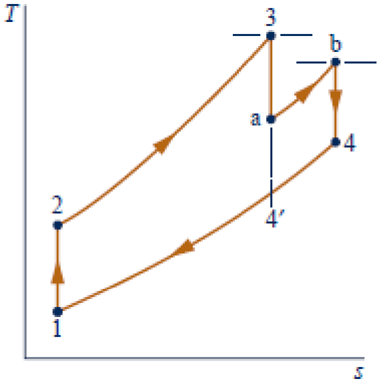
$$\eta_{\text{reg}} = \frac{h_x - h_2}{h_4 - h_2}$$

Gas Turbines with Reheat

The temperature of the gaseous combustion products entering the turbine must be limited. This temperature can be controlled by providing air in excess of the amount required to burn the fuel in the combustor. As a consequence, the gases exiting the combustor contain sufficient air to support the combustion of additional fuel. Some gas turbine power plants take advantage of the excess air by means of a multistage turbine with a reheat combustor between the stages. This increases the net work per unit of mass flow.

After expansion from the combustor to the first-stage turbine, the gas is reheated at constant pressure in the reheat combustor. The expansion is then completed in the second-stage turbine.



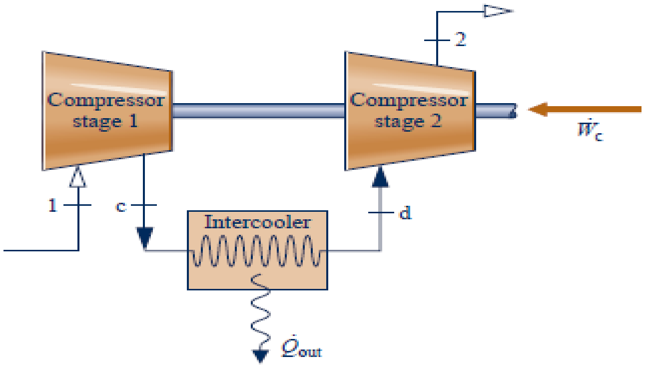


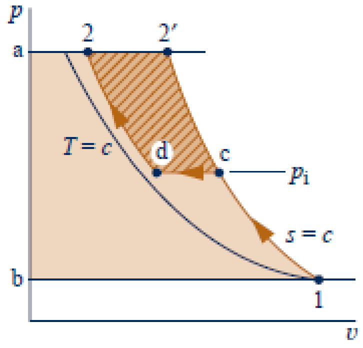
Compression with Intercooling

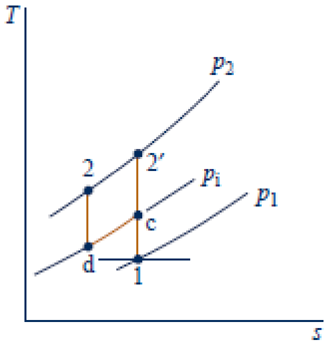
The net work output of a gas turbine also can be increased by reducing the compressor work input. This can be accomplished by means of multistage compression with intercooling. A practical alternative is to separate the work and heat interactions into separate processes by letting compression take place in stages with heat exchangers, called **intercoolers**, cooling the gas between stages.

Looking at the following diagram for intercooling, it can be seen that:

- Process 1-c is an isentropic compression from state 1 to state c where the pressure is p_i
- Process c-d is constant-pressure cooling from temperature T_c to T_d
- Process d-2 is an isentropic compression to state 2

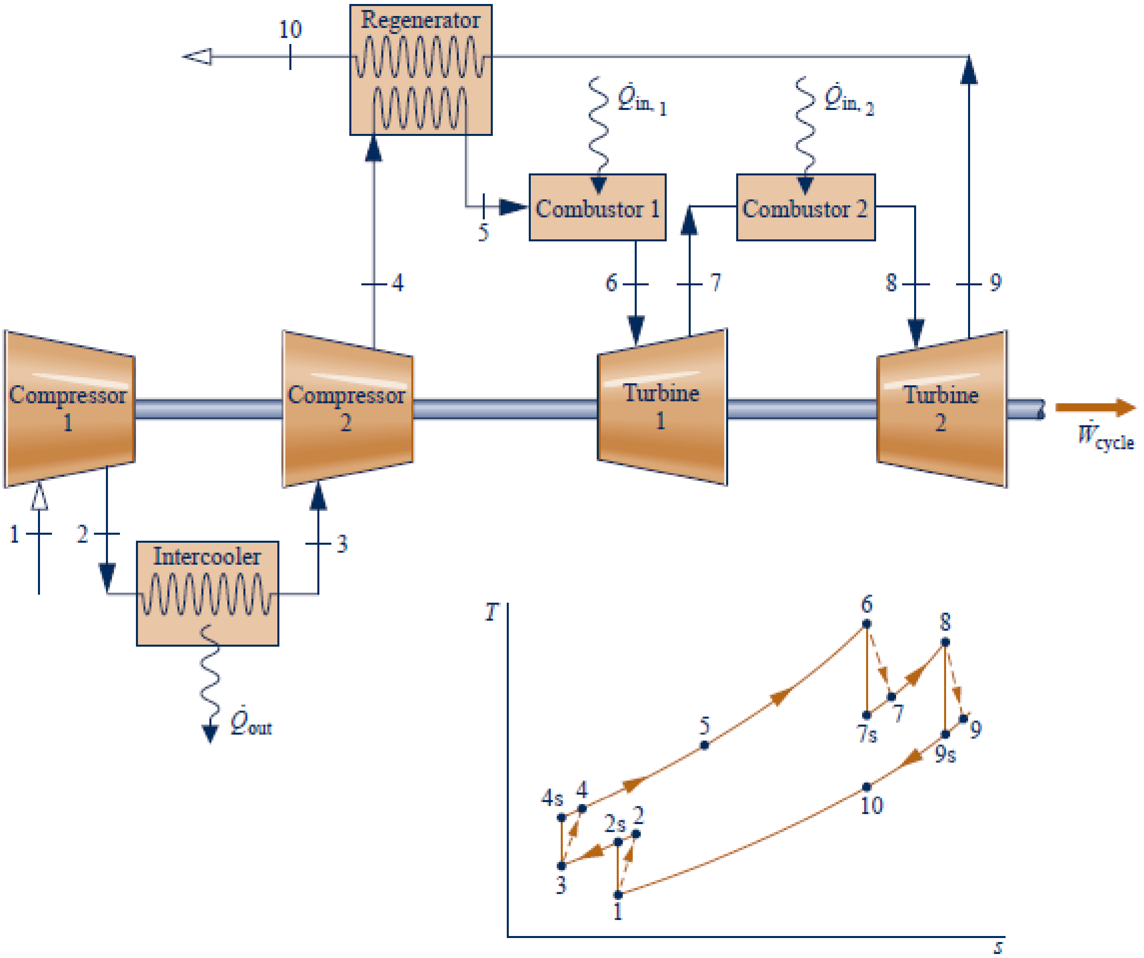






Reheat and Intercooling

Reheat between turbine stages and intercooling between compressor stages provide two important advantages: The net work output is increased, and the potential for regeneration is enhanced.



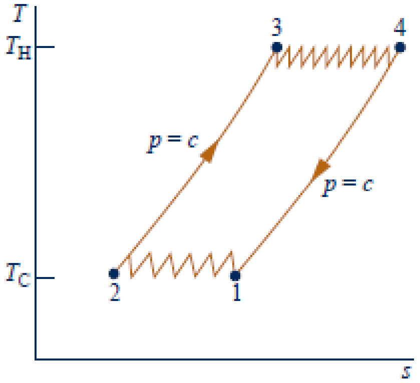
Ericsson and Stirling Cycles

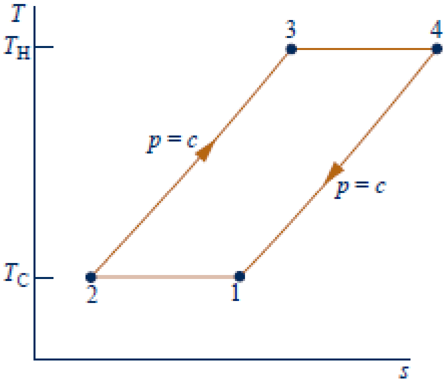
Significant increases in the thermal efficiency of gas turbine power plants can be achieved through intercooling, reheat, and regeneration.

Ericsson Cycle

Each intercooler is assumed to return the working fluid to the temperature T_C at the inlet to the first compression stage and each reheater restores the working fluid to the temperature T_H at the inlet to the first turbine stage. Accordingly, all the heat added externally occurs in the reheaters, and all the heat rejected to the surroundings takes place in the intercoolers.

Since irreversibilities are presumed absent and all heat is supplied and rejected isothermally, the thermal efficiency of the Ericsson cycle equals that of any reversible power cycle operating with heat addition at the temperature T_H and the that rejection at the temperature T_C: $\eta_{max} = 1 - T_C/T_H$.

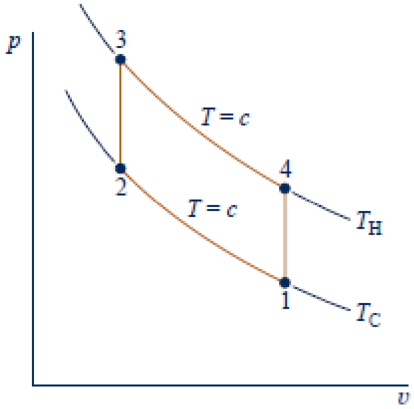


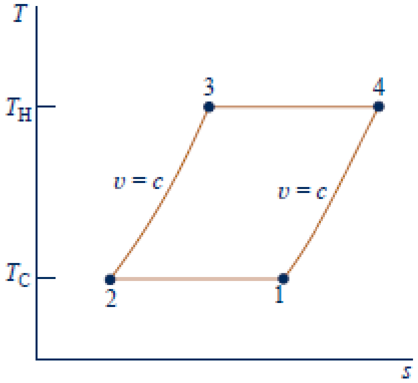


Stirling Cycle

Another cycle that employs a regenerator is the Stirling cycle, the cycle consists of four internally reversible processes in series: isothermal compression from state 1 to state 2 at temperature T_C, constant-volume heating from state 2 to state 3, isothermal expansion from state 3 to state 4 at temperature T_H, and constant-volume cooling from state 4 to state 1 to complete the cycle.

Accordingly, all the heat added to the working fluid externally takes place in the isothermal process 3-4 and all the heat rejected to the surroundings occurs in the isothermal process 1-2. It can be concluded that the thermal efficiency of the Stirling cycle is given by the same expression as for the Carnot and Ericsson cycles.





Combined Gas Turbine – Vapour Power Cycle

A combined cycle couples two proper cycles such that the energy discharged by heat transfer form one cycle is used partly or wholly as the heat input for the other cycle.

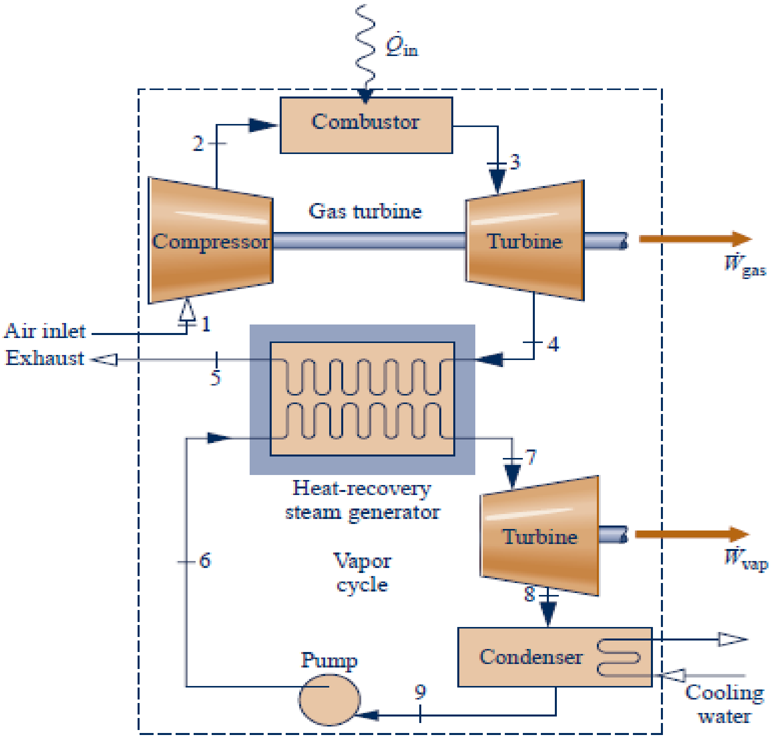
The thermal efficiency of the combined cycle is:

$$\eta = \frac{\dot{W}_{\text{gas}} + \dot{W}_{\text{vap}}}{\dot{Q}_{\text{in}}}$$

Chapter 9 - Gas Power Systems

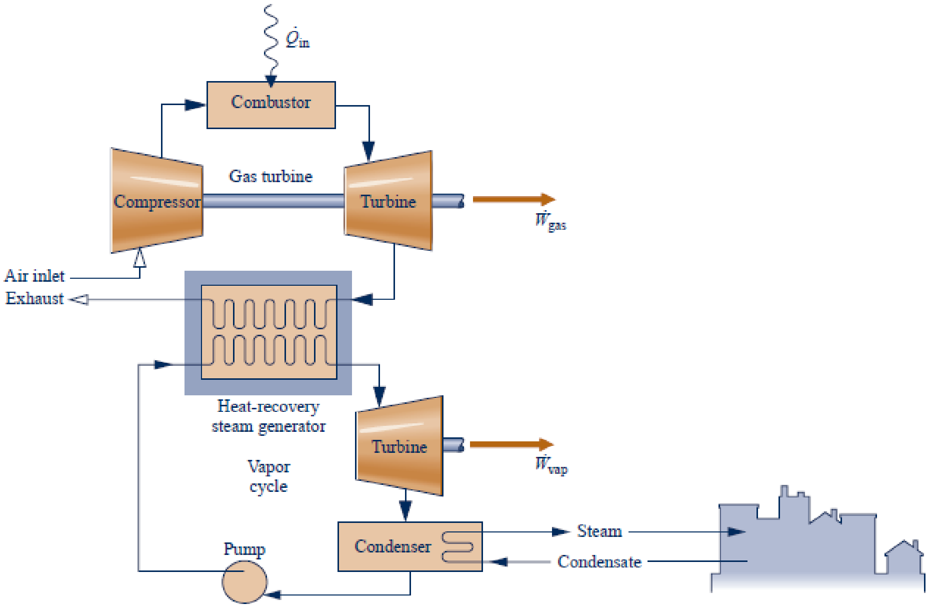
For steady-state operation, negligible heat transfer with the surroundings, and no significant changes in kinetic and potential energy, the result is:

$$\dot{m}_v(h_7 - h_6) = \dot{m}_g(h_4 - h_5)$$



Cogeneration

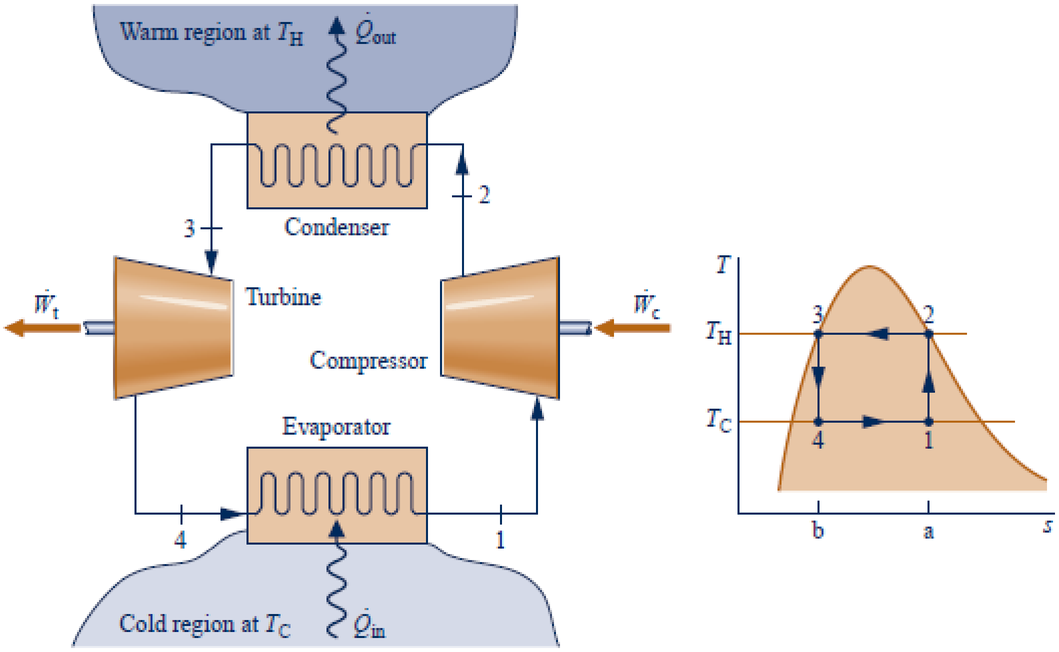
Cogeneration systems are integrated systems that yield two valuable products simultaneously forma single fuel input, electricity and steam (or hot water), achieving cost savings.



Chapter 10 - Refrigeration and Heat Pump Systems

Carnot Refrigeration Cycle

This cycle is obtained by reversing the Carnot vapour power cycle. The cycle is executed by a refrigerant circulating steadily through a series of components. All processes are internally reversible. Also, since heat transfers between the refrigerant and each region occur with no temperature differences, there are no external irreversibilities.



The refrigerant enters the evaporator as a two-phase liquid-vapour mixture. In the evaporator some of the refrigerant changes phase from liquid to vapour as a result of heat transfer from the region at temperature T_C to the refrigerant. The temperature and pressure of the refrigerant remain constant during the process. The refrigerant is then compressed adiabatically, where it is a two-phase, liquid-vapour mixture to a saturated vapour. During this process, the temperature of the refrigerant increases from T_C to T_H, and the pressure also increases. The refrigerant passes form the compressor into the condenser, where it changes phase from saturated vapour to saturated liquid as a result of heat transfer to the region at temperature T_H. The temperature and pressure remain constant in the process. The refrigerant returns to the state at the inlet of the evaporator by expanding adiabatically through a turbine. In this process, the temperature decreases from T_H to T_C, and there is a decrease in pressure.

The net heat transfer *from* the refrigerant equals the net work done *on* the refrigerant. The net work is the difference between the compressor work input and the turbine work output.

The coefficient of performance (β) of any refrigeration cycle is the ratio of the refrigeration effect to the net work input required to achieve that effect.

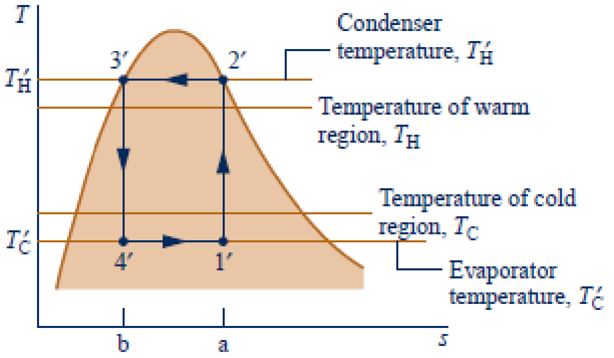
$$\beta_{\max} = \frac{\dot{Q}_{\text{in}}/\dot{m}}{\dot{W}_c/\dot{m} - \dot{W}_t/\dot{m}} = \frac{T_C}{T_H - T_C}$$

This represents the *maximum* theoretical coefficient of performance of any refrigeration cycle operating between regions at T_C and T_H.

Departures form the Carnot Cycle

Actual vapour refrigeration systems depart significantly form the Carnot cycle and have coefficients of performance lower than would be calculated. Three ways actual systems depart from the Carnot cycle are:

- One of the most significant departures is related to the heat transfers between the refrigerant and the two regions. In actual systems, these heat transfers are not accomplished reversibly as presumed. In particular, to achieve a rate of heat transfer sufficient to maintain the temperature of the cold region at T_C with a practical-sized evaporator requires the temperature of the refrigerant in the evaporator, T'_C, to be several degrees *below* T_C. Similarly, to obtain a sufficient heat transfer rate from the refrigerant to the warm region requires that the refrigerant temperature in the condenser, T'_H, be several degrees *above* T_H. Maintaining the refrigerant temperatures in the heat exchangers at T'_C and T'_H rather than T_C and T_H has the effect of reducing the coefficient of performance.



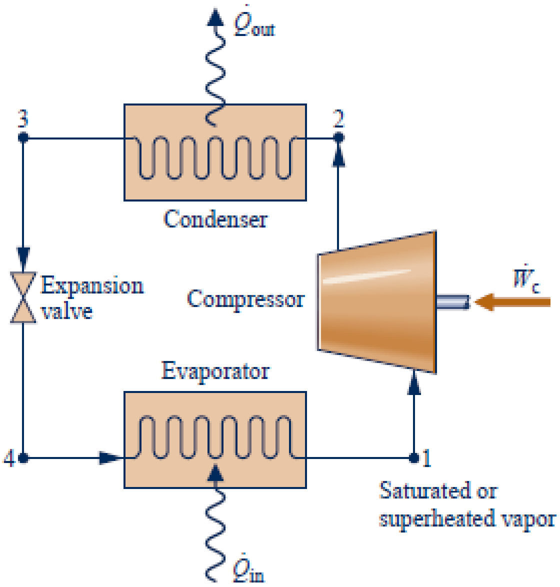
$$\beta' = \frac{T'_C}{T'_H - T'_C}$$

- Even when the temperature differences between the refrigerant and warm and cold regions are taken into consideration, there are other features that make the vapour refrigeration cycle impractical as a prototype. The compression process form state 1′ to state 2′ occurs with the refrigerant as a two-phase liquid-vapour mixture. This is commonly referred to as *wet compression*. Wet compression is normally avoided because the presence of liquid droplets in the flowing liquid-vapour mixture can damage the compressor. In actual systems, the compressor handles vapour only. This is known as *dry compression*.
- Another feature that makes the cycle impractical is the expansion process from the saturated liquid state 3′ to the low-quality, two-phase liquid-vapour mixture state 4′.

This expansion typically produces a relatively small amount of work compared to the work input in the compression process. The work developed by an actual turbine would be smaller yet because turbines operating under these conditions have low isentropic efficiencies. Accordingly, the work output of the turbine is normally sacrificed by substituting a simple throttling valve for the expansion turbine, with consequent savings in initial and maintenance costs.

Evaluating Principal Work and Heat Transfers

Vapour-compression refrigeration systems are the most common refrigeration systems in use.



- As the refrigerant passes through the evaporator, heat transfer from the refrigerated space results in the vaporization of the refrigerant. For a control volume enclosing the refrigerant side of the evaporator, the mass and energy rate balances reduce to give the rate of heat transfer pre unit mass of refrigerant flowing as:

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_1 - h_4$$

The heat transfer rate $\dot{Q}_{in}$ is referred to as the **refrigeration capacity**. In the SI unit system, the capacity is normally expressed in kW. In the English unit system, the refrigeration capacity may be expressed in Btu/h. Another commonly used unit for the refrigeration capacity is the **ton of refrigeration**, which is equal to 200 Btu/min or about 211 kJ/min.

- The refrigerant leaving the evaporator is compressed to a relatively high pressure and temperature by the compressor. Assuming no heat transfer to or from the compressor, the mass and energy rate balances for a control volume enclosing the compressor give:

$$\frac{\dot{W}_c}{\dot{m}} = h_2 - h_1$$

Which is the rate of power input per unit pass of refrigerant flowing.

- Next, the refrigerant passes through the condenser, where the refrigerant condenses and there is heat transfer from the refrigerant to the cooler surroundings. For a control volume enclosing the refrigerant aside of the condenser, the rate of heat transfer from the refrigerant per unit mass of refrigerant flowing is:

$$\frac{\dot{Q}_{out}}{\dot{m}} = h_2 - h_3$$

- Finally, the refrigerant at state 3 enters the expansion valve and expands to the evaporator pressure. This process is usually modeled as a *throttling* process for which:

$$h_4 = h_3$$

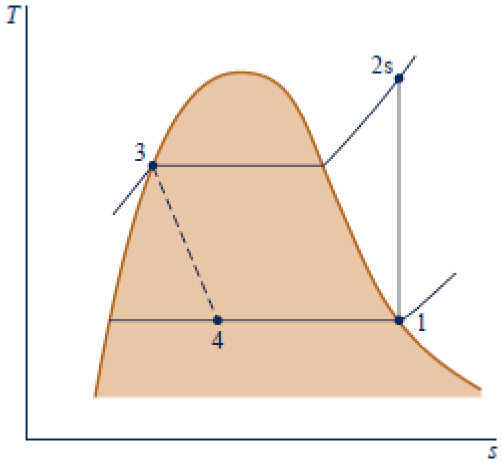
The refrigerant pressure decreases in the irreversible adiabatic expansion, and there is an accompanying increase in specific entropy. The refrigerant exits the valve at state 4 as a two-phase liquid-vapour mixture.

In the vapour-compression system, the net power input is equal to the compressor power, since the expansion valve involves no power input or output.

$$\beta = \frac{\dot{Q}_{in}/\dot{m}}{\dot{W}_c/\dot{m}} = \frac{h_1 - h_4}{h_2 - h_1}$$

Performance of Ideal Vapour-Compression Systems

If irreversibilities within the evaporator and condenser are ignored, there are no frictional pressure drops, and the refrigerant flows at constant pressure through the two heat exchangers. If compression occurs without irreversibilities, and stray heat transfer to the surroundings is also ignored, the compression process is isentropic.



Process 1-2s: *Isentropic* compression of the refrigerant from state 1 to the condenser pressure at state 2s

Process 2s-3: Heat transfer *from* the refrigerant as it flows at constant pressure through the condenser. The refrigerant exits as a liquid at state 3

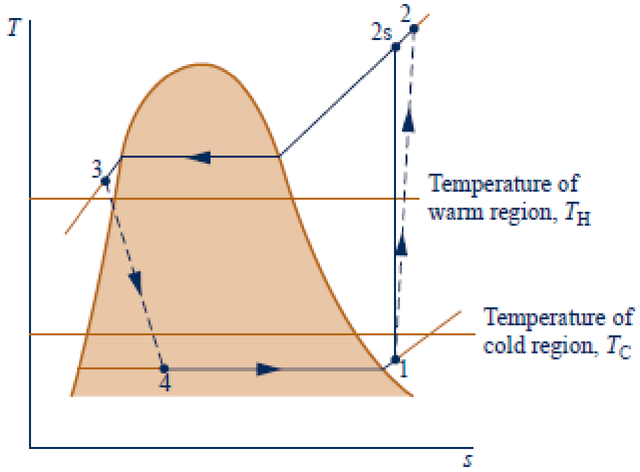
Process 3-4: *Throttling* process from state 3 to a two-phase liquid-vapour mixture at 4

Process 4-1: Heat transfer *to* the refrigerant as it flows at constant pressure through the evaporator to complete the cycle

All processes of the cycle are internally reversible except for the throttling process. Despite the inclusion of this irreversible process, the cycle is commonly referred to as the **ideal vapour-compression cycle**.

Performance of Actual Vapour-Compression Systems

The heat transfers between the refrigerant and the warm and cold regions are not accomplished reversibly: the refrigerant temperature in the evaporator is less than the cold region temperature, T_C, and the refrigerant temperature in the condenser is greater than the warm region temperature, T_H. Such irreversible heat transfers have a significant effect on performance. In particular, the coefficient of performance decreases as the average temperature of the refrigerant in the evaporator decreases and as the average temperature of the refrigerant in the condenser increases.



Another key feature of actual vapour-compression system performance is the effect of irreversibilities during compression, suggested by the use of a dashed line for the compression process form state 1 to state 2. The dashed line is drawn to show the increase in specific entropy that accompanies an *adiabatic irreversible* compression.

The isentropic efficiency for the compressor is:

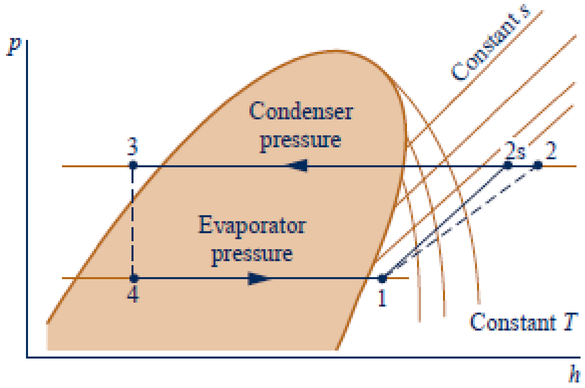
$$\eta_c = \frac{(\dot{W}_c/\dot{m})_s}{(\dot{W}_c/\dot{m})} = \frac{h_{2s} - h_1}{h_2 - h_1}$$

Additional departures from ideality stem from frictional effects that result in pressure drops as the refrigerant flows through the evaporator, condenser, and piping connecting the various components. These pressure drops are not shown on the *T-s* diagram.

Chapter 10 - Refrigeration and Heat Pump Systems

The *p-h* Diagram

A thermodynamic property diagram widely used in the refrigeration field is the pressure enthalpy or ***p-h* diagram**. The principal states of the vapour-compression cycles are located on the *p-h* diagram. Property tables and *p-h* diagrams for many refrigerants are given in handbooks dealing with refrigeration.



Selecting Refrigerants

Refrigerant selection for a wide range of refrigeration and air-conditioning applications is generally based on three factors: performance, safety, and environmental impact. The term *performance* refers to providing the required cooling or heating capacity reliably and cost effectively. Safety refers to avoiding hazards such as toxicity and flammability. Finally, environmental impact primarily refers to using refrigerants that do not harm the stratospheric ozone layer or contribute significantly to global climate change.

It is generally desirable to avoid excessively low pressures in the evaporator and excessively high pressures in the condenser.

Refrigerant Types and Characteristics

Two classes of synthetic refrigerants were developed, each containing chlorine and possessing highly stable molecular structures: CFCs (chlorofluoriocarbons) and HCFCs (hydrochlorofluorocarbons). These refrigerants were widely known as “freons”.

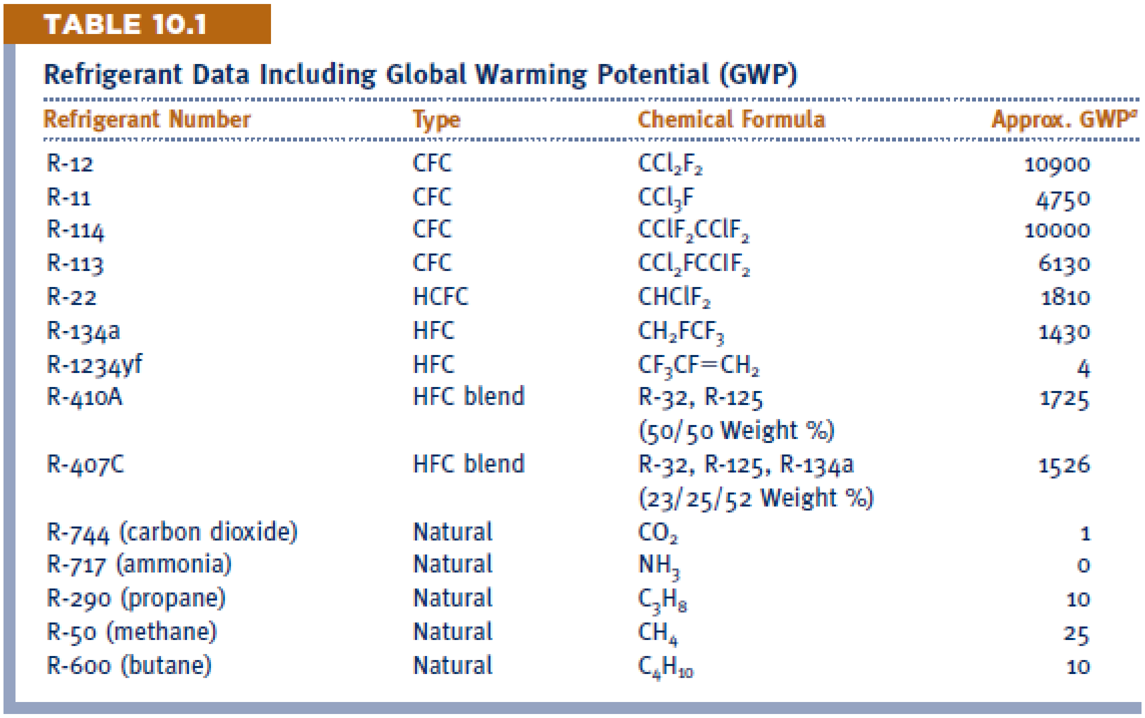


TABLE 10.1

Refrigerant Data Including Global Warming Potential (GWP)

Refrigerant Number	Type	Chemical Formula	Approx. GWP[a]
R-12	CFC	CCl_2F_2	10900
R-11	CFC	CCl_3F	4750
R-114	CFC	$CClF_2CClF_2$	10000
R-113	CFC	CCl_2FCClF_2	6130
R-22	HCFC	$CHClF_2$	1810
R-134a	HFC	CH_2FCF_3	1430
R-1234yf	HFC	$CF_3CF{=}CH_2$	4
R-410A	HFC blend	R-32, R-125 (50/50 Weight %)	1725
R-407C	HFC blend	R-32, R-125, R-134a (23/25/52 Weight %)	1526
R-744 (carbon dioxide)	Natural	CO_2	1
R-717 (ammonia)	Natural	NH_3	0
R-290 (propane)	Natural	C_3H_8	10
R-50 (methane)	Natural	CH_4	25
R-600 (butane)	Natural	C_4H_{10}	10

Environmental Considerations

After decades of uses, compelling scientific data indicating that release of chlorine containing refrigerants into the atmosphere is harmful became widely recognized. Thus HFCs (hydrofluorocarbons) were created.

Natural Refrigerants

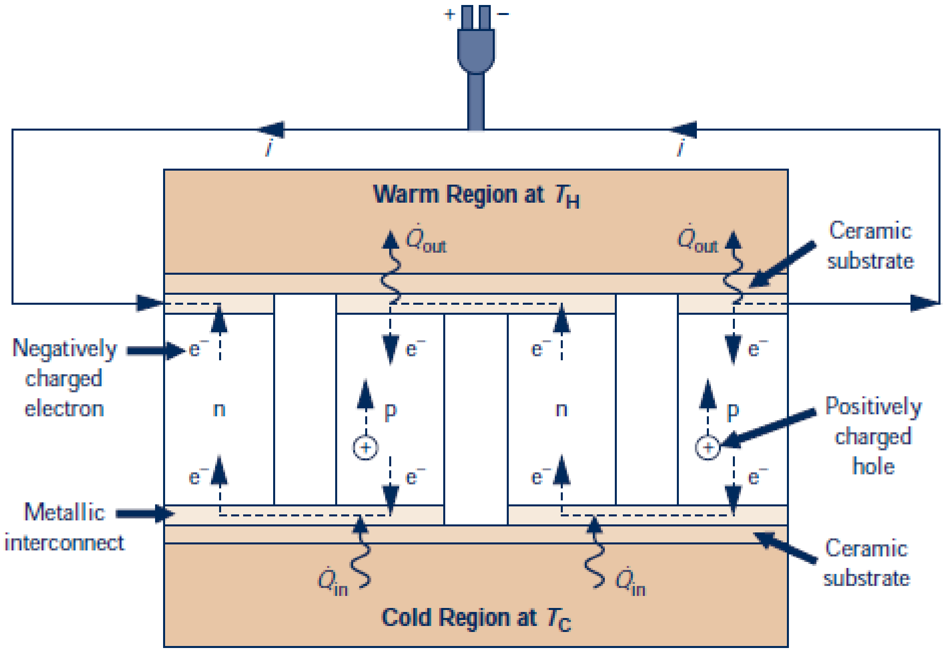
Nonsynthetic, naturally occurring substances also can be used as refrigerants. Called *natural* refrigerants, they include carbon dioxide, ammonia, and hydrocarbons.

Natural refrigerants typically have low Global Warning Potentials.

Refrigeration with No Refrigerant Needed

Alternative cooling technologies aim to achieve a refrigerating effect without use of refrigerants, thereby avoiding adverse effects associated with release of refrigerants to the atmosphere. One such technology is thermoelectric cooling.

Chapter 10 - Refrigeration and Heat Pump Systems



Carnot Heat Pump Cycle

At steady state, the rate at which energy is supplied to the warm region by heat transfer is the sum of the energy supplied to the working fluid from the cold region, $\dot{Q}_{in}$, and the net rate of work input to the cycle, $\dot{W}_{net}$:

$$\dot{Q}_{out} = \dot{Q}_{in} + \dot{W}_{net}$$

The *coefficient of performance* of *any* heat pump cycle is defined as the ratio of the heating effect to the net work required to achieve that effect.

$$\gamma_{max} = \frac{\dot{Q}_{out}/\dot{m}}{\dot{W}_c/\dot{m} - \dot{W}_t/\dot{m}} = \frac{T_H}{T_H - T_C}$$

Vapour-Compression Heat Pumps

Actual heat pump systems depart significantly from the Carnot cycle model. Most systems in common use are of the vapour-compression type. The method of analysis of *vapour-compression heat pumps* is the same as that of vapour-compression refrigeration cycles.

A typical **vapour-compression heat pump** for space heating has the same basic components as the vapour-compression refrigeration system: compressor, condenser, expansion valve, and evaporator.

The coefficient of performance of a simple vapour-compression heat pump is:

$$\gamma = \frac{\dot{Q}_{out}/\dot{m}}{\dot{W}_c/\dot{m}} = \frac{h_2 - h_3}{h_2 - h_1}$$

In the most common type of vapour-compression heat pump for space heating, the evaporator communicates thermally with the outside air. Such **air-source heat pumps** also can be used to provide cooling in the summer with the use of a reversing valve. The solid lines show the flow path of the refrigerant in the heating mode.