

The director of patient services of a large health maintenance organization wants to evaluate patient waiting time at a local facility. A random sample of 25 patients is selected from the appoint book. The waiting time is defined as the time from when the patient signs in to when he or she is seen by the doctor. The following data represents the waiting times (in minutes).

x is known:

19.5	30.5	45.6	39.8	29.6
25.4	21.8	28.6	52.0	25.4
26.1	31.1	43.1	4.9	12.7
10.7	12.1	1.9	45.9	42.5
41.3	13.8	17.4	39.0	36.6

Construct a 95% confidence interval estimate of the pop. avg. waiting time.

1. State the necessary assumption(s)/condition(s) required to construct a 95% confidence interval estimate for the population average waiting time.

- It is not normally distributed (n is not equal or greater than 30)

- Apply CLTno. because n = 25

- Assume x is normally distributed

2. State the critical value needed to construct the 95% confidence interval estimate for the population average waiting time.

t critical values = ± 2.064 (Calc: Dist, t, InvT, Area: 0.025, df: 24)

3. State the point estimate and its value used to construct a 95% confidence interval estimate for the population average waiting time. x = 27.892 (Calc: INTR, t, 1-s, List, C-level: 0.95)

4. The 95% confidence interval estimate for the population average waiting time is: $22.1668939 \leq \mu \leq 33.6171261$

Calc: INTR, t, 1-s, List, C-level: 0.95, left and right values

5. State your interpretation of the 95% confidence interval constructed in part 4:

The average waiting time in minutes of all patients is between 22.1669 mins and 32.6378 mins 95% of the time.

6. Now construct the 90% confidence interval for the population average waiting time. The 90% confidence interval:

$23.1461 \leq \mu \leq 32.6379$

7. Which is wider, a 90% or 95% confidence interval? 95%

8. The director of patient services at health maintenance organization wants to tell prospective patients that the average waiting time is 15 minutes. On the basis of the results of (a), can this statement be made? Explain.

No, because 15 minutes is outside of the confidence interval.

The quality control manager at a light bulb factory needs to estimate the average life a large shipment of light bulbs. The process standard deviation is known to be 100 hours. A random sample of 64 light bulbs indicated a sample average life of 350 hours

n = 64 x = 350 $\sigma = 100$

Construct a 95% confidence interval estimate of the true (aka population) average life of light bulbs in this shipment.

μ = population average life of light bulbs x = variable = life span of one light bulb

1. State the necessary assumption(s)/condition(s) required to construct a 95% confidence interval estimate of the true average life of light bulbs in this shipment.

x = lifespan of the light bulb -> x ~ normal distribution? no, x is not normally dist

n = 64 -> verify CLT: n ≥ 30 ? yes, because n = 64, x is normally dist

2. State the critical value needed to construct the 95% confidence interval estimate of the true average life of light bulbs in this shipment. z critical values = ± 1.9600

Calc: Dist, Norm, InvN, Area: 0.95, $\sigma = 1$, $\mu = 0$

3. State the point estimate and its value used to construct a 95% confidence interval estimate of the true average life of light bulbs in this shipment.

x = 350

4. The 95% confidence interval estimate of the true average life of light bulbs in this shipment is: $325.50 \leq \mu \leq 374.50$

5. State your interpretation of the 95% confidence interval constructed in part 4: The average life span of all light bulbs produced in a factory is between 325.50 hours and 374.50 hours 95% of the time.

A sample of 20 students were asked to fill in a survey on gender, how many hours spent studying for the test and what was the grade earned on the same test that they had studied for.

Hours: Male 14, Female 17, Female 3, Female 6, Male 17, Male 3, Male 8, Male 4, Female 20, Male 15, Female 7, Male 9, Male 0, Male 5, Female 11, Female 15, Male 18, Female 13, Male 8, Male 4

Construct a 91% confidence interval estimate of the proportion of all female students.

a) What parameter are you estimating here? π

b) State/check the necessary assumptions required to construct the confidence interval:

$np \geq 5? \rightarrow 20 (8/20) = 8$

$n(1-p) \rightarrow 20 (1 - 8/20) = 12$

The conditions are met, therefore, it is normally distributed.

c) Before you construct a 91% confidence interval estimate of the proportion of all female students, state the critical value: z = ± 1.6954

InvN, Area: 0.91, $\sigma = 1$, $\mu = 0$

d) The point estimate for a 91% confidence interval estimate of the proportion of all female students is: x = 9.85

e) State the 91% confidence interval estimate of the population proportion of all female students: $0.2143 \leq \pi \leq 0.5857$

Calc: INTR, z, 1-p, C-level: 0.91, x = 8, n = 20

A survey is planned to determine the mean annual family medical expenses of employees of a large company. The management of the company wishes to be 95% confident that the sample mean is correct to within ± 50 of the population mean annual family medical expenses. A previous study indicates that the standard deviation is approx. \$400.

a) How large a sample size is necessary? $245.8 \rightarrow 246$

z = 1.9599 e = 50 $\sigma = 400$

* In this question, you are estimating μ (the mean)

$n = \frac{z^2 \sigma^2}{e^2}$

b) If management wants to be correct with ± 25 , how many employees need to be selected? $983.3 \rightarrow 984$

z = 1.9599 e = 25 $\sigma = 400$

What proportion of people hit snags with online transactions? According to a poll, 89% hit snags with online transactions.

a) To conduct a follow up study that would provide 95% confidence that the point estimate is correct to within ± 0.04 of the population proportion, how large a sample size is required? $235.0339 \rightarrow 234$

$\pi = 0.89$ z = 1.9599 e = 0.04

*In this question, you are estimating π (the proportion)

$n = \frac{\pi (1-\pi) z^2}{e^2}$

IMPORTANT INFORMATION:

*The critical value is either a z-value or a t-value based on the confidence level. You use the INVN function to obtain the z-value with $\sigma = 1$, $\mu = 0$. To obtain the t-value, use the INVT function.

*If you are asked to construct a 95% confidence interval for population parameter, you use INTR.

*sometimes variance will be give; for instance $\sigma^2 = 9$ but $\sigma = 3$

Census: A set of data that includes ALL members of a population.

Sample: A subset of a population.

Sample size: number of elements in a sample, is denoted as n

Population size: number of elements in a population, is denoted as N.

Population Proportion : is denoted as π .



Sample proportion: is denoted as p
Population Mean: is denoted as μ
Sample Mean: is denoted as \bar{X}
Parameter: measurement describing the population (e.g. μ or π)
Estimation: using sample data to estimate the unknown population parameter (μ or π)
Confidence interval: an interval that is likely to include a population parameter (μ or π) given a confidence level (usually expressed in percentage).
The interval is (point estimate) \pm (the margin of error) which is (point estimate) \pm (critical value) (standard error)
The **critical value** is either a z-value or a t-value based on the confidence level. You use the "INVN" function to obtain the z-value with mean equals zero and standard deviation equals one. Two ways to obtain the t-value, (1) use the "INVT" function and (2) use the t-table.
Margin of error: measures the uncertainty in estimating the population parameter.
Margin of error= (critical value) x (standard error).
Point estimate: Using the sample statistic (e.g. \bar{X} or p) to estimate the corresponding population parameter (μ or π). For example p is the point estimate of π and \bar{X} is the point estimate of μ .
Confidence Level : the probability that the constructed interval will contain the population value, usually expressed in percentage.

Central Limit Theorem
Assume that the population from which we will randomly select a sample of n measurements has a mean μ and standard deviation σ . Then the population of all possible sample means has:
1. Mean: \bar{X}
2. Standard deviation : σ
3a. if population of individual measurements is normal, \bar{X} is normal
3b. If population of individual measurements is not normal, \bar{X} is normal if sample size is large enough ($n \geq 30$)

