CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number		Section(s)
Mathematics	208/4		All
Examination	**Date**	**Time**	**Pages**
Midterm	March 2015	1 Hour 30 minutes	2
Instructors			**Course Examiner**
A. Bellahnid, A. Sen, E. Duma, E. Lee, F. Babaee, S. Bettin			D. Sen

FORMULAE:

$$A = P(1+i)^n, \quad A = Pe^{rt}, \quad FV = PMT\frac{(1+i)^n - 1}{i}, \quad PV = PMT\frac{1-(1+i)^{-n}}{i}$$

Special Instructions:

- ▷ Answer all questions.
- ▷ **Only approved calculators are allowed.**

MARKS

[4+3+3] **1.** The management of a company that manufactures skateboards has fixed costs (costs at 0 output) of \$300 per day and a total costs of \$4,300 per day at an output of 100 skateboards per day. Assume that cost C is linearly related to output x.

(A) Find an equation of the line relating output to cost.

(B) Graph the cost equation for $0 \leq x \leq 200$.

(C) Find the equation of the average cost function.

[$2\frac{1}{2} \times 4$] **2.** Solve for x in the following equations:

(A) $25^{2x-1} = 5^{x^2-5x+6}$

(B) $\log_b x = \frac{2}{3}\log_b 27 + 2\log_b 2 - \log_b 3$

(C) $e^{6x^2-8x-32} = e^{5x^2-6x-24}$

(D) $\log_2(x+3) + \log_2(2x+4) = 2$

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[6+4] **3.** (A) A person borrows \$3,600 and agrees to repay the loan in monthly installments over a period of 3 years. The agreement is to pay 1% of the unpaid balance each month for using the money and \$100 each month to reduce the loan. What is the total cost of the loan over the 3 years?

(B) Due to reduced taxes, a person has an extra \$1,200 in spendable income. If we assume that the person spends 65% of this on consumer goods, and the producers of these goods in turn spend 65% on consumer goods, and that this process continues indefinitely, what is the total amount spend (to the nearest dollar) on consumer goods?

[5+5] **4.** George finds a company that charges 0.59\$ per day for each \$1,000 borrowed. If he borrows \$3,000 for 60 days, what amount will he repay, and what annual simple interest rate will he be paying the company?

[5+5] **5.** A company estimates that it will have to replace a piece of equipment at a cost of \$800,000 in 5 years. To have this money available in 5 years, a sinking fund is established by making equal monthly payments into an account paying 6.6% compounded monthly.

(A) How much should each payment be?

(B) How much interest is earned during the last year?

[5+5] **6.** Some friends tell you that they paid \$25,000 down on a new house and are to pay \$525 per month for 30 years. If interest is 7.8% compounded monthly,

(A) What was the selling price of the house?

(B) How much interest will they pay in 30 years?

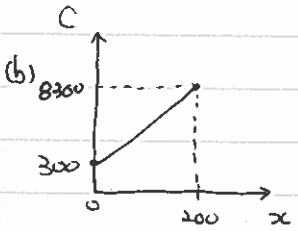
1. (a) Since the cost C is linearly related to output x,

$C = mx + b$.

- $C = 300$ when $x = 0$, $\Rightarrow b = 300$.
- $4300 = m \cdot 100 + 300 \Rightarrow m = 40$.

$\therefore C = 40x + 300$.

(b)



(c) $\bar{C} = \frac{C(x)}{x} = 40 + \frac{300}{x}$

2. (a) $25^{2x-1} = 5^{x^2-5x+6}$

$\Rightarrow 5^{4x-2} = 5^{x^2-5x+6}$

$\Rightarrow 4x - 2 = x^2 - 5x + 6$

$\Rightarrow x^2 - 9x + 8 = 0 \Rightarrow (x-1)(x-8) = 0$

$\therefore x = 1, 8$.

(b) $\log_6 x = \log_6 27^{\frac{2}{3}} + \log_6 2^2 - \log_6 3$

$= \log_6 3^{3 \cdot \frac{2}{3}} + \log_6 4 - \log_6 3$

$= \log_6 \frac{9 \cdot 4}{3} = \log_6 12 \quad \therefore x = 12$.

(c) $6x^2 - 8x - 30 = 5x^2 - 6x - 24$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0. \quad x = 4, -2$.

(d) $\log_2 (x+3) + \log_2 (2x+4) = 2$

$\Rightarrow \log_2 (x+3)(2x+4) = \log_2 2^2 = \log_2 4$

$\Rightarrow (x+3)(2x+4) = 4$

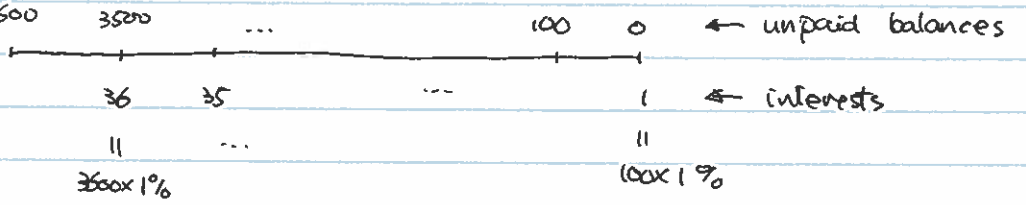
$\Rightarrow 2x^2 + 10x + 8 = 0 \Rightarrow x^2 + 5x + 4 = 0$

$\Rightarrow (x+4)(x+1) = 0 \qquad x = -4, -1.$

$x = -4$ is not a solution of the log equation because $\log_2(-4+3)$ and $\log_2(2\cdot(-4)+4)$ are not defined.

$\therefore$ $x = -1$ is the solution.

3. (a)



$36 + 35 + \cdots + 1 = \frac{36}{2}(1+36) = 666$

(b) $a_1 = 1200 \times 65\% = 780$

$a_2 = a_1 \times 65\%$

$\vdots$

$S_\infty = a_1 + a_2 + \cdots = \frac{780}{1-0.65} = 2228.57.$

4. $P = 3{,}000$, Interest: $0.59 \times 3 \times 60 = 106.2.$

Thus, the total amount repaid is $3000 + 106.2 = 3106.2$

To find the annual interest rate, let $t = 60 \text{ days} = \frac{60}{360} = \frac{1}{6}$ year.

Then $r = \frac{I}{Pt} = \frac{106.2}{3000 \cdot \frac{1}{6}} = 0.2124 = 21.24\%.$

5. $\Rightarrow$ Example 2 in Section 3-3.

6. $PMT = 525, \quad i = \frac{0.078}{12} = 0.0065 \quad n = 30 \times 12 = 360$

$PV = 525 \times \frac{1-(1+0.0065)^{-360}}{0.0065} = 72{,}929.78$

Selling price $= 72{,}929.78 + 25{,}000 = 97{,}929.78.$

Total amount paid: $525 \times 360 = 189{,}000$

Total interest paid: $189{,}000 - 72{,}929.78 = 116{,}070.22$