MAT 1320D Calculus I Solution
Midterm 1

Professor: Wanshun Wong

1. [1 point] Solve for x: $\ln(6x+8)=9$.

Using the exponential function on both sides gives $6x+8=e^9$, therefore $x=\frac{e^9-8}{6}$.

2. [2 points] Find the limit $\displaystyle\lim_{x\to -1}\frac{\sqrt{x+10}-3}{x+1}$.

$$\begin{aligned}
\lim_{x\to -1}\frac{\sqrt{x+10}-3}{x+1} &= \lim_{x\to -1}\frac{\sqrt{x+10}-3}{x+1}\cdot\frac{\sqrt{x+10}+3}{\sqrt{x+10}+3}\\
&= \lim_{x\to -1}\frac{x+10-9}{(x+1)(\sqrt{x+10}+3)}\\
&= \lim_{x\to -1}\frac{1}{\sqrt{x+10}+3}\\
&= \frac{1}{6}.
\end{aligned}$$

3. [1 point] If $\frac{\pi}{2}<x<\frac{3\pi}{2}$, what is $\arcsin(\sin(x))$?

If $\frac{\pi}{2}<x<\frac{3\pi}{2}$, then $-\frac{\pi}{2}<\pi-x<\frac{\pi}{2}$ and it satisfies $\sin(\pi-x)=\sin(x)$. Hence

$$\arcsin(\sin(x))=\arcsin(\sin(\pi-x))=\pi-x.$$

4. [3 points] Use the definition of the derivative to find $f'(x)$ if $f(x)=\dfrac{2x}{3-x}$.

$$\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{1}{h}\cdot\left(\frac{2x+2h}{3-x-h}-\frac{2x}{3-x}\right)\\
&= \lim_{h\to 0}\frac{1}{h}\cdot\frac{6h}{(3-x-h)(3-x)}\\
&= \frac{6}{(3-x)^2}
\end{aligned}$$

5. [2 points] Use known formulas to find the derivative of the function $y = \dfrac{\tan x}{5^x}$.

We can apply Quotient Rule:

$$\begin{aligned} f'(x) &= \frac{(5^x)(\sec^2 x) - (\tan x)(5^x)(\ln 5)}{5^{2x}} \\ &= \frac{\sec^2 x - (\ln 5)(\tan x)}{5^x} \end{aligned}$$

6. [3 points] For which value(s) of x does the function $y = \sqrt{x^3 - 3x + 1}$ have a horizontal tangent line ?

First we compute y' by Chain Rule:

$$\begin{aligned} y' &= \frac{1}{2}(x^3 - 3x + 1)^{-1/2} \cdot (3x^2 - 3) \\ &= \frac{3x^2 - 3}{2\sqrt{x^3 - 3x + 1}} \end{aligned}$$

By solving $y' = 0$, we see that the function has a horizontal tangent line at $x = -1$. Notice that $x = 1$ is not in the domain of y because $1^3 - 3(1) + 1 = -1$ is negative. Another way to think about this is that $x = 1$ is NOT a solution of $y' = 0$ because $y'(1)$ is not defined.

7. [3 points] A table of values for f, g, f', and g' is given.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	1	2	3	4
2	2	1	-1	-2

(a) Find $(f \circ g)(1)$.
(b) Find the derivative of the composite function $(f \circ g)$ at $x = 1$.

(a) $(f \circ g)(1) = f(g(1)) = f(2) = 2$.

(b) We apply Chain Rule: $(f \circ g)'(1) = f'(g(1)) \cdot g'(1) = f'(2) \cdot g'(1) = -4$.