Concordia University
Department of Building, Civil and Environmental Engineering
Civil Engineering Systems (CIVI-341) Winter – 2013
Midterm I TYPE A (Open book) [25 marks in total]

Instructor: Dr. L. Amador **NAME:_____** **ID#:_____**

Tuesday, February 26th, 2013 **Start 8:45 am. - End 10:00 am**

1. Find out if the following subsets have isolated points, interior points and/or limit points. Please explicitly identify which are such numbers in every case. [3 marks]
   a. Let $A \subset R, A = Irrationals$

      Isolated = YES all of them, Interior none, limit none

2. Is the following set: open?, closed?, bounded?, compact?.
   Explain why, all marks given to the explanation [7 marks]
   a. **Let** $A \subset R, A = \{x \in Z\}$

      (in other words the set A is a subset of the real numbers and include all integers)

      SOL:

      a) Open NO: Definition says: for every element x of A, there exist an open ball that is fully contained in the set. The set contain all integers, there are no open balls around any integer that will be fully contained in the integers (you will always touch space with rationals or irrationals). Therefore is not open

      b) Closed YES: Definition says a set is closed if its complement is open. Is A^c open?. A^c = all numbers in gaps between two integers, i.e., A^c = …(-3,-2) U (-2,-1) U (-1,0) U (0,1)… therefore A^c is open, then A is closed.

      c) Bounded NO, it is not possible to find r such that A is fully contained in B(0,r)
      d) Compact NO, because is not bounded

3. A construction company exists to produce profit (**π**), which is given by income minus cost. Income comes from the number of facilities (x_0) the company build per month and the amount it charges to build a facility (p_0). However a company needs to pay all its employees (x_1) a monthly wage (p_1), purchase materials (x_2) at prices (p_2) every month and rent machinery (x_3) at lease rate (p_3) per month. The company has a maximum amount of expenditure given by **B** per month. The amounts of labor, materials and machinery (x_1, x_2, x_3) for ***part a.*** are unlimited, the company can obtain as much as it needs at any given month. [TOTAL 10 marks]

   Part a. Define the objective function (please indicate from where to where it maps), define the constraint set. Is the constraint set closed?, is it bounded? is it compact? Is there a solution for this problem?. [3 marks]

Part b. Now take the objective and turn it into a constraint, take the previous constraint from part a) and use it as an objective. [Hint: Assume ***W*** is the minimum required level of profit and minimize cost]. Additionally, suppose we know that labor is limited to 160 hours per month, material is limited to 25 units, and machinery is limited to 10 units. Also we know that: 16 hours of labor plus 5 units of material plus 2 machines are used to build each facility and that the company cannot build more than 5 facilities at the same time. Is there a solution for this new problem?

$$\pi : R^4 \rightarrow R$$
$$MAX \quad \pi = p_0 x_0 - (p_1 x_1 + p_2 x_2 + p_3 x_3)$$
$$x_0, x_1, x_2, x_3$$
$$s.t.$$
$$p_1 x_1 + p_2 x_2 + p_3 x_3 \leq B$$
$$x_0, x_1, x_2, x_3 \geq 0$$

$$MIN \quad C = p_1 x_1 + p_2 x_2 + p_3 x_3$$
$$x_1, x_2, x_3$$
$$s.t.$$
$$p_0 x_0 - (p_1 x_1 + p_2 x_2 + p_3 x_3) \geq W$$
$$16x_1 + 5x_2 + 2x_3 = x_0$$
$$x_0, x_1, x_2, x_3 \geq 0$$
$$x_1 \leq 160$$
$$x_2 \leq 25$$
$$x_3 \leq 10$$
$$x_0 \leq 5$$

ORIGNAL: Constraint set is not closed, x_0 has no limitation, therefore is not bounded, no compact, there is no solution

MODIFIED: Constraint set is closed, not open, bounded, compact, there is a solution

4. The following linear programming optimization is/has?:
   You may use ANY METHOD, [5 marks, 4 given to explanation]

a. Unique Optima

b. Alternate Optima

c. Infeasible

d. Unbounded

$$Maximize \quad Z = 2x_2 - x_1$$
$$x_1 + x_2 \geq 1$$
$$3x_1 + 4x_2 \leq 12$$
$$x_1 \geq 0$$