Assignment 10 Solutions

6.1.2 $\frac{f(x)-f(0)}{x-0} = \frac{x^{1/3}-0}{x-0} = x^{-2/3}$. Let $x_n = \frac{1}{n^{3/2}}$. Since $x^{3/2}$ is continuous at 0 and $\frac{1}{n} \to 0$, $\frac{1}{n^{3/2}} \to 0$. $x_n \neq 0$ $\forall n$. But $\left(\frac{1}{n^{3/2}}\right)^{-2/3} = n$, and (n) diverges since it's unbounded.

$\therefore$ $\lim_{x\to 0} \frac{f(x)-f(0)}{x-0}$ D.N.E, so f is not diff'ble at 0.

6.1.8(c) For $x>0$, $h(x)=x^2$ is diff'ble and $h'(x)=2x$. For $x<0$, $h(x) = -x^2$, so h is diff'ble and $h'(x) = -2x$. At $x=0$, the formula for $x\geq 0$ can only gives the right derivative, which may not be the value of the whole derivative. $\therefore$ use the def'n. $\frac{h(x)-h(0)}{x-0} = \frac{x|x|}{x} = |x|$ for $x \neq 0$. $\because$ $\lim_{x\to 0}|x| = 0$, $h'(0) = 0$.

6.2.7 For any $x>1$, $\ln(x)$ is cont on $[1,x]$, diff'ble on $(1,x)$ so by the MVT, $\ln x - \ln 1 = \frac{1}{c}\cdot(x-1)$ for some $c \in (1,x)$. $\therefore$ $1/x < 1/c < 1$, so $\frac{x-1}{x} < \frac{1}{c}(x-1) < x-1$, so $\frac{x-1}{x} < \ln x < x-1$,