Assignment #8

5.2.1(c) Let $f(x) = \sqrt{x}$, $g(x) = 1+x$, $h(x) = |x|$, $j(x) = \sin x$. Then all the f'ns are continuous where defined, and the composite $f \circ g \circ h \circ j$ is defined for all x, $\because$ $1 + |\sin x| \geq 1$ $\forall x$. $\therefore$ $\sqrt{1+|\sin x|}$ is cont. $\forall x \in \mathbb{R}$. $k(x) = x$ is continuous and nonzero $\forall x \neq 0$, so $\therefore$ $\frac{\sqrt{1+|\sin x|}}{x}$ is cont $\forall x \neq 0$.

8. Let $h(x) = f(x) - g(x)$. Then h is cont on $\mathbb{R}$ and $h(r) = 0$ $\forall r \in \mathbb{Q}$. Let $x \in \mathbb{R}$. Then, since every $x \in \mathbb{R}$ is a cluster point of $\mathbb{Q}$, there is a sequence (x_n) in $\mathbb{Q}$ s.t. $(x_n) \to x$. $\because$ h is continuous, $(h(x_n)) \to h(x)$ (sequential criterion). But $\forall n$, $h(x_n) = 0$, so $(h(x_n)) = (0) \to 0$. $\therefore$ $h(x) = 0$, and so $f(x) = g(x)$ $\forall x \in \mathbb{R}$.

5.3.1 $\because$ $I = [a,b]$ is closed and bounded, and f continuous, the Max/Min th'm implies $\exists$ an absolute min. pt x_* in $[a,b]$, so $\forall x \in [a,b]$, $f(x) \geq f(x_*)$. $\because$ $f(x) > 0$ $\forall x \in I$, $f(x_*) > 0$. Let $\alpha = f(x_*)$ to obtain the result.