Assignment # 7 Sol'ns

(4.2.5) Let $\varepsilon > 0$ be given. $\because$ f is bounded in a nbhd of c, $\exists \delta_1 > 0$, $\exists M > 0$ s.t. $\forall x \in V_{\delta_1}(c) \cap A$, $|f(x)| \le M$. Since $\lim_{x \to c} g = 0$, $\exists \delta_2 > 0$ s.t. $\forall x \in A$ s.t. $0 < |x-c| < \delta_2$, $|g(x)| < \varepsilon/M$. Then for $\delta = \min\{\delta_1, \delta_2\}$, $\forall x \in A$ s.t. $0 < |x-c| < \delta$, $|f(x)g(x)|$ $= |f(x)||g(x)| < M \cdot \frac{\varepsilon}{M} = \varepsilon$. $\therefore \lim_{x \to c} fg = 0$.

(4.2.11 c) let $x_n = \frac{1}{\pi/2 + n\pi}$, $\forall n \in \mathbb{N}$. $x_n \neq 0$ $\forall n$ and since $0 < x_n < \frac{1}{n}$ $\forall n$, $(x_n) \to 0$ by the Squeeze th'm. But $\forall n$, sgn $\sin\left(\frac{1}{x_n}\right)$ $=$ sgn $(-1)^n = (-1)^n$. As $((-1)^n)$ is divergent, a Divergence Criterion proves $\lim_{x \to 0}$ sgn $\sin\left(\frac{1}{x}\right)$ diverges.

(5.1.6) Let $\varepsilon > 0$ be given. $\because$ f is continuous at c, $\exists \delta > 0$ s.t. $\forall x \in V_\delta(c) \cap A$, $|f(x) - f(c)| < \varepsilon/2$. Then, for any $x, y \in V_\delta(c) \cap A$,

$$|f(x) - f(y)| = |f(x) - f(c) + f(c) - f(y)| \underset{\Delta \text{ ineq.}}{\le} |f(x) - f(c)| + |f(y) - f(c)| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon.$$