DGD (circle): 1 2 Last name: Solutions

TA (circle): Andrew Elizabeth First name:

Marks: /16 Student number:

MAT 1348A — First Homework Assignment — Due Jan. 21, 2015 at 11:20am

Instructions: Show all relevant work to receive full credit. Submit a finished product, not a draft. You may write on both sides of the paper or insert additional pages if necessary. Please staple the pages. Submit the assignment to your TA in the DGD or in the appropriate submission box in the Department of Mathematics and Statistics. Late assignments will not be accepted.

1. Write each of the following statements as a compound proposition using correct logical connectives. You must clearly **define the propositional variables** you used in your compound propositions. [5pts]

   (a) Passing the course is a sufficient condition for students to do well on the final exam.
   Compound proposition: $p \rightarrow f$

   (b) Students do well on the final exam only if they do not miss the midterm exam.
   Compound proposition: $f \rightarrow \neg m$

   (c) Not missing the midterm exam and doing the homework is necessary for passing the course.
   Compound proposition: $p \rightarrow \neg m \wedge h$

   (d) Students do not do well on the final exam if and only if they do not do their homework or miss the midterm exam.
   Compound proposition: $\neg f \leftrightarrow (\neg h \vee m)$

   (e) Students do well on the final exam unless they miss the midterm exam.
   Compound proposition: $f \vee m \; (\equiv \neg m \rightarrow f \equiv \neg f \rightarrow m)$

   Your propositional variables:

p: "Students pass the course"

f: "Students do well on the final exam."

m: "Students miss the midterm exam."

h: "Students do the homework."

2. Use a **truth table** to determine whether the given set of four propositions is **consistent**. Clearly explain what feature of the truth table supports your answer. [5pts]

$$\{\, x \vee \neg y \vee z,\ (z \leftrightarrow y) \rightarrow x,\ \neg z \wedge (x \rightarrow y),\ x \wedge \neg z \,\}$$

x	y	z	$x \vee \neg y \vee z$	$(z \leftrightarrow y) \rightarrow x$	$\neg z \wedge (x \rightarrow y)$	$x \wedge \neg z$
T	T	T	T	T	F	F
T	T	F	T	T	T	T
T	F	T	T	T	F	F
T	F	F	T	T	F	T
F	T	T	T	F	F	F
F	T	F	F	T	T	F
F	F	T	T	T	F	F
F	F	F	T	F	T	F

This set of propositions is consistent because there is a row (truth assignment) where all 4 compound propositions are T.

3. On the Island of Knights and Knaves you meet two natives, A and B. Given their statements below, what can you determine about their identities? Fully explain your reasoning. [6pts]

   (a) A says: "B is a knight but I am a knave". What do you conclude?

   (b) A says: "B is a knight only if I am a knight". What do you conclude?

   (c) In Case (b) above, what do you conclude if B adds: "Either A is a knave or I am a knave, but not both"?

Define the following propositions:

a: "A is a knight."

b: "B is a knight."

(a) A says: $b \wedge \neg a$

a	b	$b \wedge \neg a$	
T	T	F	
T	F	F	
F	T	T	←
F	F	F	

must match

Conclusion: A and B are both knaves.

(b) A says: $b \to a$

a	b	$b \to a$	
T	T	T	←
T	F	T	←
F	T	F	←
F	F	T	

Conclusion: A and B are not both knaves

(c) A says: $b \to a$

B says: $\neg a \oplus \neg b$

Conclusion: A is a knave, B is a knight.

a	b	$b \to a$		$\neg a \oplus \neg b$	
T	T	T	←	F	
T	F	T	←	T	
F	T	F	←	T	←
F	F	T		F	←