Michelle: part 1: 20pts × 289 exams = 5780 points.

(slow)

TA #1: Part 2: 20pts × 113 exams = 2260 points

TA #2: Q1 + ~~Q2a) 13pts × 402 exams~~ Q2 d) e) f): 10 × 402 exams = 4020 pts

TA #3: Q2 a), b), c): 15 × 402 exams = 6030 p

(fast)

DATE OF EXAM	Thursday, October 3, 2013
EXAM TYPE Solutions.	Special Materials
ADDITIONAL MATERIALS ALLOWED	Pink Tie Calculator

Notes:

1. Answer all questions in the space provided. If you run out of space, use the back page provided for rough work. Clearly indicate where your solution continues.
2. Check that there are 13 sheets.
3. **Your grade will be influenced by how clearly you express your ideas, and how well you organize your solutions.**

Question	Mark	Out of	Question	Mark	Out of	Question	Mark	Out of
1		5	Part 1 Q1		5	Part 2 Q1		4
2		20	Part 1 Q2		7	Part 2 Q2		2
×	×	×	Part 1 Q3		8	Part 2 Q3		5
×	×	×	×	×	×	Part 2 Q4		9
×	×	×	Total		45	×	×	×

1. Solve for all values of x and state your solution. Show your work.

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$$\left|\frac{2x-3}{4}\right| > 1$$

case $\frac{2x-3}{4} > 1$

$\Rightarrow 2x-3 > 4$

$\Rightarrow 2x > 7$

$\Rightarrow x > \frac{7}{2}$

case $-\left(\frac{2x-3}{4}\right) > 1$

$\Rightarrow \frac{3-2x}{4} > 1$

$\Rightarrow 3-2x > 4$

$\Rightarrow -2x > 1$

$\Rightarrow x < -\frac{1}{2}$

solution: $\{x \in \mathbb{R} : x < -\frac{1}{2} \text{ or } x > \frac{7}{2}\}$

2. Consider the functions: $y_1 = x^2 - 5x$, $y_2 = -x^2 - x + 6$.

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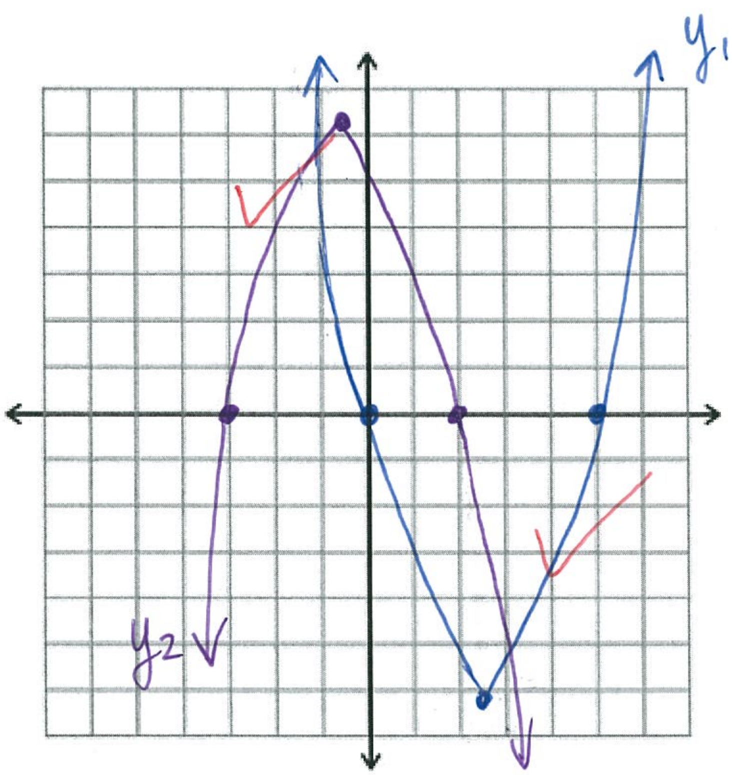
(a) Sketch both functions on the grid below by finding the vertex and x-intercepts of each. There is additional space on this page to show your work.



y_1 vertex: $\frac{-b}{2a} = \frac{5}{2} = 2.5$

$y_1(2.5) = (2.5)^2 - 5(2.5)$
$= -6.25$

vertex: $(2.5, -6.25)$

$y = 0$: $x^2 - 5x = 0$
$\Rightarrow x(x-5) = 0$
$\Rightarrow x = 0, x = 5$

y_2 vertex: $\frac{-b}{2a} = \frac{1}{-2} = -0.5$

$y_2(-0.5) = -(-0.5)^2 - (-0.5) + 6$
$= -0.25 + 0.5 + 6$
$= 6.25$

vertex: $(-0.5, 6.25)$

$y = 0$: $-x^2 - x + 6 = 0$
$\Rightarrow x^2 + x - 6 = 0$
$\Rightarrow (x+3)(x-2) = 0$
$\Rightarrow x = -3, x = 2$

(b) Find the equation of the line passing through the intersection points of $y_1 = x^2 - 5x$ and $y_2 = -x^2 - x + 6$. Show your steps.

Find intersection points: $y_1 = y_2$

$\Rightarrow x^2 - 5x = -x^2 - x + 6$

$\Rightarrow 2x^2 - 4x - 6 = 0$

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow (x-3)(x+1) = 0$

$x = 3, \ x = -1.$

/6

Find y-values (can use y_1 or y_2)

$y_1(3) = 3^2 - 5(3) = -6$ $\quad$ $y_1(-1) = (-1)^2 - 5(-1) = 6$

Points: $(3, -6), (-1, 6)$

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{6+6}{-1-3} = \dfrac{12}{-4} = -3$

$y = mx + b \Rightarrow 6 = -3(-1) + b \Rightarrow b = 2$

$\therefore \ y = -3x + 2$

(c) State the domain of y_2 below.

$D(y_2) = \{x : x \in \mathbb{R}\}$

/1

or $= \mathbb{R}$

or $= (-\infty, \infty)$

(d) Is $y_2 = -x^2 - x + 6$ a one-to-one function? Why or why not?

no, since parabolas are never one-to-one

/2

or no, y_2 fails the horizontal line test.

or no, consider $y_2(-3)$, $y_2(2)$. these are equal but $-3 \neq 2$

(e) Find a restricted domain in order to form a new function from y_2 that is one-to-one.

y_2 vertex at $(-0.5, 6.25)$

$\therefore$ axis of symmetry at $x = -\frac{1}{2}$.

New domain: $\{x \in \mathbb{R} : x \geq \frac{-1}{2}\}$

/2

or $\{x \in \mathbb{R} : x \leq \frac{-1}{2}\}$

there are other answers.

(f) Give one more example of a function that is one-to-one.

$y = x$

/1

or $y = x^3$

or $y = \sqrt{x}$

there are many answers.

or $y = \frac{1}{x}$

This section of the test has two parts. You must complete PART 1 or PART 2, but not both.

PART 1 is mainly word problem based. PART 2 is mainly calculation based.

Circle the part you will complete: PART 1 PART 2

PART 1

PART 1 Q1: wording: confusing

At the beginning of 2009, you opened a TFSA (Tax-Free Savings Account) and deposited some funds. The bank pays 2% annual interest, compounded quarterly. At the beginning of 2013, you have a total of $5415.36 in interest rounded to the nearest cent. Find the amount invested in 2009 to the nearest cent. $S = P(1 + i)^n$

$S = 5415.36$, $i = \frac{0.02}{4} = 0.005$, $n = 4 \times 4 = 16$

2009-2013 quarterly

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$$5415.36 = P(1.005)^{16}$$

$$P = \frac{5415.36}{(1.005)^{16}}$$

$$= 5000.00$$

the amount invested was $5000.00

$S = P + 5415.36$, $i = 0.005$, $n = 16$.

$$P = \frac{5415.36}{1.005^{16} - 1} \doteq \$65,189.42.$$

the amount invested was $65,189.42.

PART 1 Q2:

For a certain product the demand function is $p(q) = \dfrac{12}{\sqrt{q}}$, and the cost function is $c(q) = 4q$. If revenue is given by: $r = pq$, find the profit equation associated with the product.

profit = total revenue − total cost

$$P = r - c = pq - c(q)$$

$$= \frac{12}{\sqrt{q}} q - 4q$$

/3

$$P(q) = 12\sqrt{q} - 4q$$

Find the numbers q where profit is at \$0, and explain the significance of both of these numbers.

$$P = 0 \Rightarrow 12\sqrt{q} - 4q = 0$$

$$\Rightarrow 3\sqrt{q} - q = 0$$

$$\Rightarrow \sqrt{q}(3 - \sqrt{q}) = 0$$

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$$\Rightarrow q = 0 \text{ or } q = 9$$

At $q = 0$, $P = 0$ since no items are sold, or manufactured.

At $q = 9$, $P = 0$ since cost of production equals revenue.

(This is called the break-even point).

PART 1 Q3: In 2007, two Blackberry (formerly RIM) employees, Marcus and Jen, invested in company stock at a reduced price of \$20 per share. Marcus bough 72 shares more than Jen, and a total of 214 shares was purchased by both. Find the total amount that Marcus invested.

Let x rep # stocks purchased by marcus

Let y .... by Jen.

6

then: $x + y = 214 \Rightarrow y = 214 - x$

$x = y + 72$. ← sub in.

$x = 214 - x + 72$.

$2x = 286$

$x = 143$

at \$20 per share:

$143 \times \$20 = \$2860$.

∴ Marcus invested \$2860

In 2013 Blackberry stock plummets to \$8. How much was lost of Marcus' investment?

at \$8 per share, investment is worth $143 \times \$8 = \$1144$

2

∴ Marcus lost $\$2860 - \$1144 = \$1716$.

PART 2

PART 2 Q1:

Given $f(x) = \dfrac{x^2 - 2}{x^3}$, can we find the x-intercepts and the y-intercepts? If so, find them and if not, explain why not.

x-int $y=0 \Rightarrow 0 = \dfrac{x^2-2}{x^3} \Rightarrow x^2-2=0$

(x^3 can be any value except 0)

$\Rightarrow x^2 = 2$

$\Rightarrow x = \pm\sqrt{2}$

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y-int $x=0 \Rightarrow y = \dfrac{0-2}{0}$ ← undefined.

$\therefore$ no y-intercepts.

PART 2 Q2:

Suppose the total cost to manufacture a product is:
$c(q) = 0.04q^2 + 7q + 600$. When $q = 0$, what is c, and what is its meaning?

$c(0) = 0 + 0 + 600$.

Means fixed costs to manufacture are at \$600

2

PART 2 Q3:

Consider the multivariable function $f(x, y, z) = \dfrac{xy^2\sqrt{z}}{y - z}$. State the independent and dependent variables.

independent: x, y, z

$\frac{}{2}$

dependent: f or $f(x, y, z)$

Find the domain of f, that is: all points (x, y, z) in $\mathbf{R}^3$ that can be input for f.

1. $\sqrt{z} \leftarrow z \geq 0$

2. $y - z \neq 0 \Rightarrow y \neq z$

$\frac{}{3}$ $\therefore D(f) = \{(x, y, z) \in \mathbb{R}^3 : z \geq 0, y \neq z\}$

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PART 2 Q4:

Suppose we have the expression:

$$\frac{x+3}{8-4x} \geq 0$$

There are two cases we must examine to solve this inequality.

Case 1: Both the numerator (top) and the denominator (bottom) are positive.

State Case 2: Both the numerator and denominator are negative.

Solve Case 1:

First line: $x + 3 \geq 0$ AND $8 - 4x \geq 0$

$\Rightarrow x \geq -3$ AND $8 \geq 4x$

$\Rightarrow 2 \geq x$

$x \leq 2$

sol'n: $\{x \in \mathbb{R} : -3 \leq x \leq 2\}$

or $[-3, 2]$

PART 2 Q4 continued:

Solve Case 2:

$x+3 \le 0$ AND $8-4x \le 0$

$\Rightarrow x \le -3$ AND $8 \le 4x$

$\Rightarrow 2 \le x$

$x \ge 2$

solution: no x solve case 2.

or $\{ \}$

What do you notice about Case 2?

No solution.

Does this mean $\dfrac{x+3}{8-4x} \ge 0$ has no solutions? Explain your reasoning.

The solution is the solution of case1 only.

2

$\dfrac{x+3}{8-4x} \ge 0$ only when $x \in [-3, 2]$

This page is for your rough work. You may continue a solution here, but please indicate clearly that your solution continues on this page.