UNIVERSITY OF OTTAWA
Principles of Physics
PHY1321/1331 FALL 2014
Dr. A. Czajkowski

CLASS: PHY_____

STUDENT #: ________________

NAME: ____________________

Assignment 1:
KINEMATICS 1-D Motion

Assigned: Sept 8 *Due: September15 18:00 (Monday)*

1 By algebraic manipulation of the first two kinematic equations for one-dimensional motion:

$$1) v_f = v_i + at \qquad 2) x_f = x_i + v_i t + \tfrac{1}{2} a t^2$$

Obtain the other two kinematic equations: $3) v_f^2 - v_i^2 = 2a\Delta x \qquad 4) x_f = x_i + \tfrac{1}{2}(v_i + v_f)t$

SOLUTION:

$$1) v_f = v_i + at \Rightarrow t = \frac{(v_f - v_i)}{a}$$

$$2) x_f = x_i + v_i \frac{(v_f - v_i)}{a} + \tfrac{1}{2} a \frac{(v_f - v_i)^2}{a^2} \Rightarrow x_f - x_i = v_i \frac{(v_f - v_i)}{a} + \tfrac{1}{2a}\left(v_f^2 - 2v_i v_f + v_i^2\right) \Rightarrow$$

$$\Rightarrow 2a(x_f - x_i) = 2v_i(v_f - v_i) + \left(v_f^2 - 2v_i v_f + v_i^2\right) \Rightarrow 2a(x_f - x_i) = 2v_i v_f - 2v_i v_i + v_f^2 - 2v_i v_f + v_i^2 \Rightarrow 2a(x_f - x_i) = v_f^2 - v_i^2$$

$$1) v_f = v_i + at \Rightarrow t = \frac{(v_f - v_i)}{a}$$

$$2) x_f = x_i + v_i t + \tfrac{1}{2} a \frac{(v_f - v_i)}{a} t \Rightarrow x_f = x_i + v_i t + \tfrac{1}{2}(v_f - v_i)t \Rightarrow x_f = x_i + v_i t + \tfrac{1}{2} v_f t - \tfrac{1}{2} v_i t \Rightarrow x_f = x_i + \tfrac{1}{2}(v_f + v_i)t$$

2 A fast car, driving at 30.0 m/s, enters a one-lane tunnel. The driver observes a slow-moving truck 140 m ahead traveling at 6.00 m/s. She applies her brakes but can accelerate only at –2.00 m/s^2 because the road is wet. Will there be a collision? If yes, determine how far into the tunnel and at what time the collision occurs. If no, determine the distance of closest approach between Sue's car and the van. (4p)

Take the original moment in time to be when car drive notices the van. Choose the origin of the *x*-axis as the position of the car at that moment. We have $x_{is} = 0$, $v_{is} = 30.0\ \text{m/s}$, $a_s = -2.00\ \text{m/s}^2$ so car's position is given by

$$x_s(t) = x_{is} + v_{is}t + \frac{1}{2}a_s t^2 = (30.0\ \text{m/s})t + \frac{1}{2}\left(-2.00\ \text{m/s}^2\right)t^2.$$

For the van, $x_{iv} = 140\ \text{m}$, $v_{iv} = 6.00\ \text{m/s}$, $a_v = 0$ and

$$x_v(t) = x_{iv} + v_{iv}t + \frac{1}{2}a_v t^2 = 140 + (6.00\ \text{m/s})t.$$

To test for a collision, we look for an instant t_c when both are at the same place:

$$30.0t_c - t_c^2 = 140 + 6.00t_c \Rightarrow 0 = t_c^2 - 24.0t_c + 140.$$

From the quadratic formula

$$t_c = \frac{24.0 - \sqrt{(24.0)^2 - 4(140)}}{2} = \frac{24-4}{2} = 10.0\ \text{s} \quad \text{or} \quad t_c = \frac{24.0 + \sqrt{(24.0)^2 - 4(140)}}{2} = \frac{24+4}{2} = 14.0\ \text{s}$$

The smaller value is the collision time.
(The larger value tells when the van would rear end the car again if the vehicles could move through each other).
The wreck happens at position

$$140\ \text{m} + (6.00\ \text{m/s})(10\ \text{s}) = \boxed{200\ \text{m}}.$$

Collision takes place at 200m
From the moment the car's driver applies the brakes

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Assignment 1: KINEMATICS 1-D Motion CONT

Assigned: Sept 8 14:30 Due: September15 19:00

3. The height of a helicopter above the ground is given by $h = 2.00t^3$, where h is in meters and t is in seconds. After 2.00 s, the helicopter releases a small mailbag. How long after its release does the mailbag reach the ground? (4p)

SOLUTION: We need to find the position and velocity of the helicopter at t=2s (moment when the bag is released) $y = 2.00t^3$: At $t = 2.00$ s , $y = 2.00(2.00)^3 = 16.0$ m and

$$v_y = \frac{dy}{dt} = 6.00t^2 = 24.0 \text{ m/s} \uparrow .$$

If the helicopter releases a small mailbag at this time, the equation of motion of the mailbag is

$$y_b = y_{bi} + v_i t - \frac{1}{2}gt^2 = 16.0 + 24.0t - \frac{1}{2}(9.80)t^2 .$$

Setting $y_b = 0$, $0 = 16.0 + 24.0t - 4.90t^2$.

Solving for t, (only positive values of t count), $\boxed{t = 5.49 \text{ s}}$.

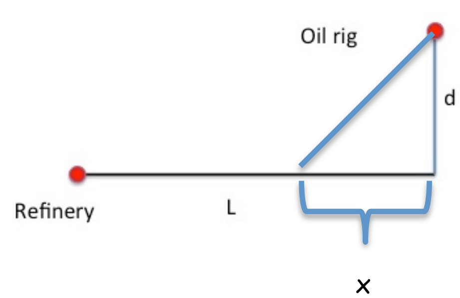
4 Two railroad tracks intersect at right angles at station O. At 10AM the train A, moving west with constant speed of 50 km/h, leaves the station O. One hour later train B, moving south with the constant speed of 60 km/h, passes through the station O. Find minimum distance between these trains.

$$D(t) = \sqrt{(v_A t)^2 + (60 - v_B t)^2} \Rightarrow \frac{dD}{dt} = 0 \Rightarrow \frac{2v_A^2 t + 2(60 - v_B t)(-v_B)}{2\sqrt{(v_A t)^2 + (60 - v_B t)^2}} = 0$$

$$\frac{2v_A^2 t + 2(60 - v_B t)(-v_B)}{2\sqrt{(v_A t)^2 + (60 - v_B t)^2}} = 0 \Rightarrow v_A^2 t + (60 - v_B t)(-v_B) = 0 \Rightarrow 2500t - 3600 + 3600t = 0$$

$$t = \frac{3600}{6100} = \frac{36}{61} h = 35 \min 24.6s \Rightarrow D = \sqrt{(v_A t)^2 + (60 - v_B t)^2} = 38.41 km$$

5 John who is member of NGO missed the meeting of his protest group at the refinery and now needs to get to the oil rig in the shortest time to join the demonstrators trying to disrupt the work of the petroleum company. John can run at 10km/h but can paddle only 3km/h.
a) How far from the Refinery should John enter the water
b) What is the minimum time it will take to get to the oil rig?
L=12km, d=4km



While on land John's speed is VL=10km/h. He will run at that speed the L-x distance.

It will take him time t_L given by $t_L = \frac{L-x}{v_L} = \frac{12-x}{10}$ hr

The distance he will need to swim/paddle with speed V_W is $\sqrt{d^2+x^2}$ it will take him $t_W = \frac{\sqrt{d^2+x^2}}{v_W} = \frac{\sqrt{4^2+x^2}}{3}$ hr

The total time to get from the R to OR is $t_L + t_W = \frac{L-x}{v_L} + \frac{\sqrt{d^2+x^2}}{v_W} = \frac{12-x}{10} + \frac{\sqrt{4^2+x^2}}{3}$

We need to find the entry point (coordinate x along the shore) such that this time is minimum.

$$\frac{d(t_L+t_W)}{dx} = -\frac{1}{v_L} + \frac{1}{v_W}\frac{2x}{2\sqrt{d^2+x^2}} = 0$$

$$-\frac{1}{10} + \frac{1}{3}\frac{2x}{2\sqrt{16+x^2}} = 0$$

$$\frac{1}{3}\frac{x}{\sqrt{16+x^2}} = \frac{1}{10} \Rightarrow 10x = 3\sqrt{16+x^2} \Rightarrow 100x^2 = 9(16+x^2) \Rightarrow 91x^2 = 144 \Rightarrow x = \sqrt{\frac{144}{91}} = 1.26(km)$$

So the point at which John needs to enter the water is 10.74km away from refinery.

It will take

$$t_L + t_W = \frac{L-x}{v_L} + \frac{\sqrt{d^2+x^2}}{v_W} = \frac{10.74}{10} + \frac{\sqrt{4^2 + 144/91}}{3} = 2.47hr$$