1. The structural shape shown in Figure Q1 pivots at point A as it is gently lowered by slipping on the rope which is fixed at point B. The structure weighs 1500 N and its centre of gravity is as indicated in the figure. What is the pull tension of the rope T if there is to be impending motion of the structure downward? The coefficient of friction between the rope and the surface of the structure is 0.35.

(*All dimensions in m.*)

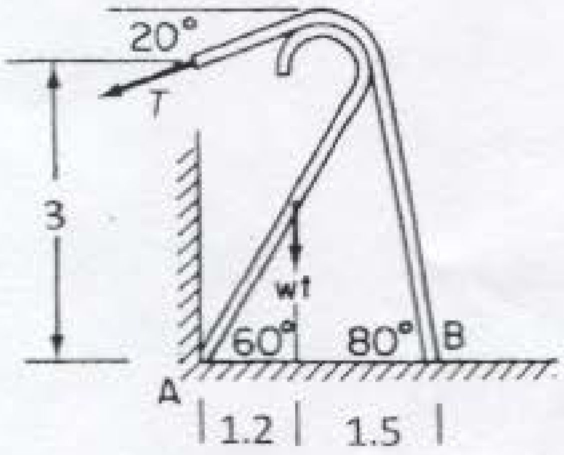


Figure Q1

$\overset{\frown}{A}$: $1500(1.2) + P(2.7 \sin 80°) = T(2.819)$

i.e. $2.819T - 2.659P = 1800$ (i)

Impending motion of structure downward $\Rightarrow$ $T > P$

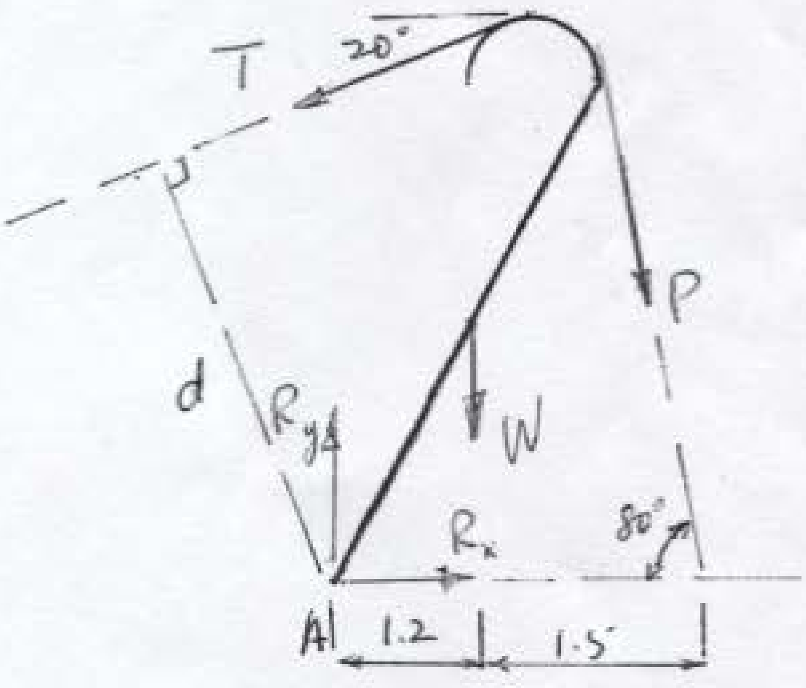
$\therefore$ $T/P = e^{\mu\theta} = e^{(0.35)(\frac{100}{180}\pi)}$

$\Rightarrow$ $T = 1.842P$ --- (ii)

(i)+(ii) : $2.534P = 1800$

$\rightarrow$ $P = 710.34N$

$\therefore$ $T = \underline{\underline{1308.45N}}$.



$W = 1500$ N

$d = 3 \cos 20° = 2.819$ m

2. Figure Q2 shows a steel tube A, fixed to a rigid wall; it has an internal diameter $D = 37.5$ mm and wall thickness $t = 1.25$ mm. The tube is coaxially connected to a steel shaft of diameter $d = 25$ mm via a polymer insert B that is perfectly bonded to each member as shown. The shaft is subjected to either an axial load P or a torque T at different stages of the service cycle, and it can displace through a circular opening in the wall. Answer the following questions with the help of free body diagrams.

(A) Consider the situation when the torque $T = 35$ Nm is applied.

(i) What is the mean shear stress and its direction on the cylindrical surface of the polymer connector where it is bonded to the tube?

(ii) What is the mean shear stress on the cross-section of the steel tube at the station C between the rigid wall and the end of the polymer connector?

(B) Consider the situation when the axial load $P = 2$ kN is applied.

(i) What is the mean shear stress and its direction on the cylindrical surface of the polymer connector where it is bonded to the tube?

(ii) What is the mean direct stress on the cross-section of the steel tube at the station C?

(iii) If the shaft displaced 0.55 mm along its axis relative to the steel tube, what is the shear strain in the polymer, assuming that the steel shaft and tube are rigid compared to the polymer connector?

(iv) Assuming linear elastic behaviour, determine the approximate shear modulus of the polymer.

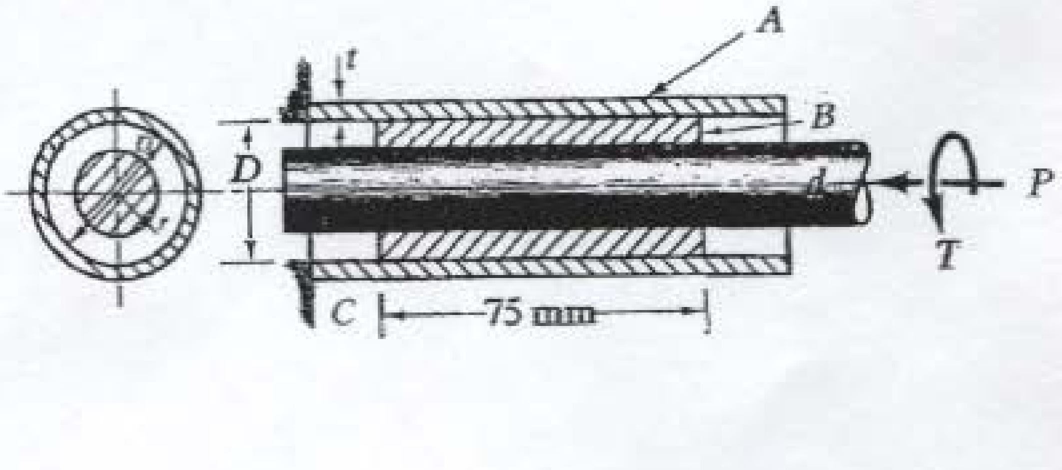
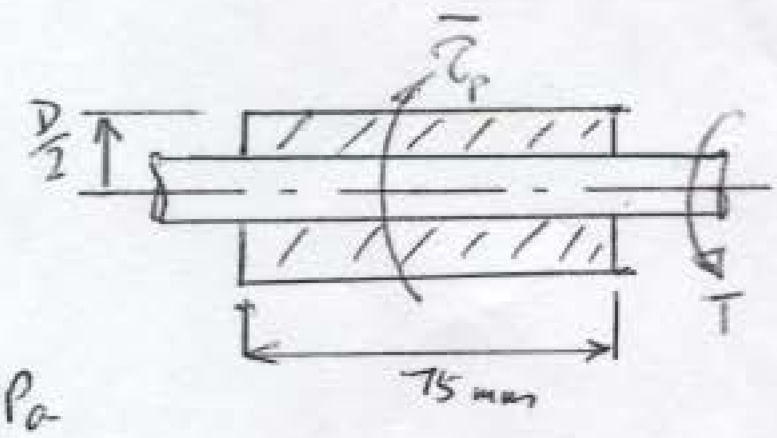


Figure Q2

(A) $T = 35$ Nm.



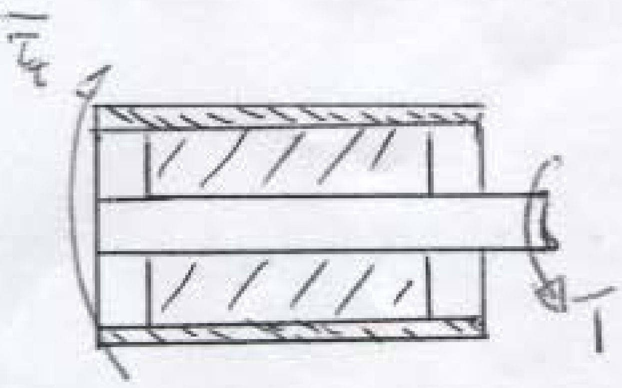
(i) $\bar{\tau}_p\left[(2\pi \frac{D}{2})(0.075)\right]\left(\frac{D}{2}\right) = 35$ Nm

$\therefore \bar{\tau}_p = \dfrac{2 \times 35}{0.0375^2 \times 0.075\,\pi}$ Pa $=$ $\underline{0.211 \text{ MPa}}$

(ii) Area of X-section of the tube:

$A = \pi D t = \pi(0.0375)(0.00125)$ m^2

i.e. $A = 1.4726 \times 10^{-4}$ m^2



$(\bar{\tau}_t A)\left(\frac{D}{2}\right) = T$

$\therefore \bar{\tau}_t (1.472 \times 10^{-4})\left(\frac{0.0375}{2}\right) = 35$ Nm

$\therefore \bar{\tau}_t = \underline{12.68 \text{ MPa}}$

(B) $P = 2$ kN

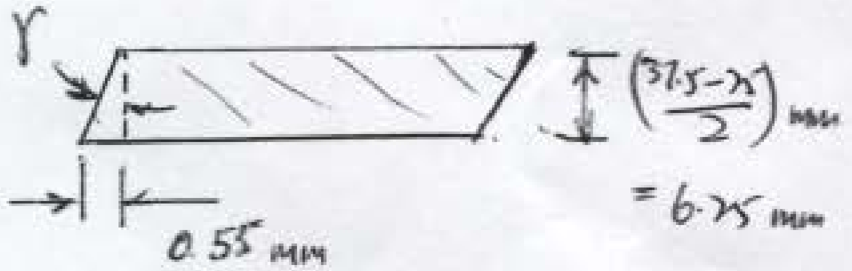
(i) $\bar{\tau}_p(\pi D \times 0.075) = 2000$ N.

$\therefore \bar{\tau}_p = \dfrac{2000}{\pi(0.0375)(0.075)}$ Pa $= \underline{0.226 \text{ MPa}}$

(ii) $\bar{\sigma}_t (A) = P \Rightarrow \bar{\sigma}_t = \dfrac{2000}{1.4726 \times 10^{-4}}$ Pa $= \underline{13.58 \text{ MPa}}$

(iii) $\tan\gamma = \dfrac{0.55}{6.25} = 0.088 \simeq \gamma$

$\left[\ \gamma = 5.03^{\circ} \times \dfrac{\pi}{180} = 0.088\ldots\ \right]$



$G = \dfrac{\tau}{\gamma} = \dfrac{0.226}{0.088} = \underline{2.57 \text{ MPa}}$.

3. Figure Q3 shows two concentric cylinders of length 0.75 m which are attached to thick, rigid end plates, used for transport of a pressurized, hazardous fluid in the inner cylinder. The outer cylinder, made of an aluminium alloy, is a safety feature which also provides some mechanical protection; it has an internal diameter of 300 mm and wall thickness of 1.5 mm. The inner cylinder is made of stainless steel; it has an inner diameter of 250 mm and wall thickness of 5 mm. The two cylinders are stress-free before the inner cylinder is filled with the fluid up to a pressure of 3.5 MPa.

Under the maximum pressure, calculate:

(a) the stresses in the walls of the cylinders; and

(b) the change in the internal radius of the inner cylinder.

Material properties:

	Young's modulus	*Poisson's ratio*
Stainless steel	207 GPa	0.3
Aluminium alloy	110 GPa	0.33

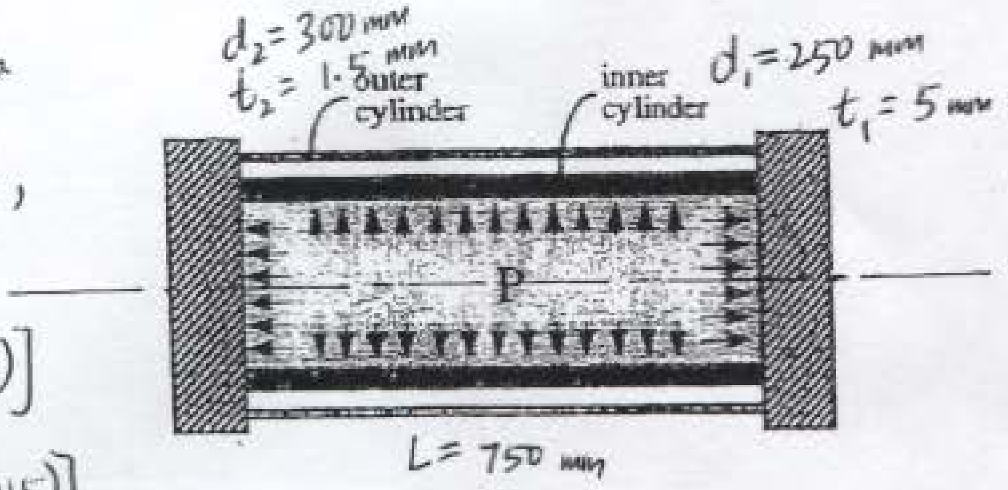


Figure Q3

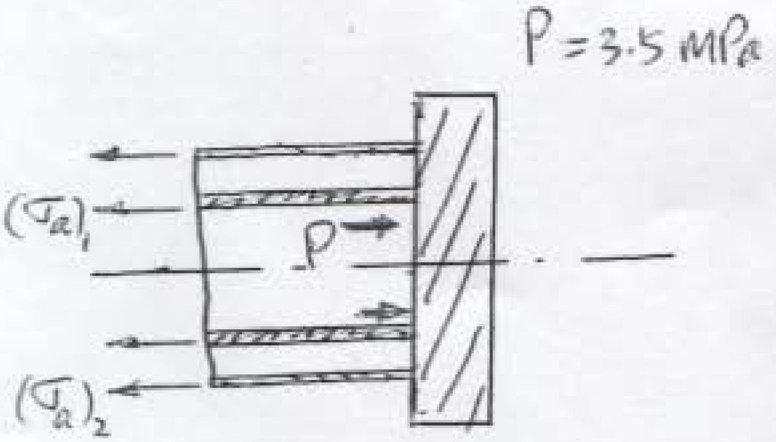
(a)

Eqm: $(\sigma_\theta)_1 = \frac{P(\frac{d_1}{2})}{t_1} = \frac{(3.5)(\frac{250}{2})}{5}$ MPa

i.e. $(\sigma_\theta)_1 = \underline{87.5 \text{ MPa}}$ (i)

Axial eqn: $P\left[\frac{\pi(0.250)^2}{4}\right] = (\sigma_a)_1\left[\pi(0.250)(0.005)\right] + (\sigma_a)_2\left[\pi(0.300)(0.0015)\right]$

$\Rightarrow (\sigma_a)_1 + 0.36(\sigma_a)_2 = 43.75$ (ii)



Compatibility: $(\varepsilon_a)_1 = (\varepsilon_a)_2$ (iii)

Stress/Strain: $(\varepsilon_a)_2 = \frac{1}{E_2}(\sigma_a)_2$ (iv)

$(\varepsilon_a)_1 = \frac{1}{E_1}\left[(\sigma_a)_1 - \nu_1(\sigma_\theta)_1\right]$ (v)

(iii) – (v) $\rightarrow \frac{E_1}{E_2}(\sigma_a)_2 = (\sigma_a)_1 - \nu_1(\sigma_\theta)_1$

i.e. $(\sigma_a)_1 - 1.882(\sigma_a)_2 = 0.3(87.5) = 26.25$ (vi)

(ii)+(vi) $\rightarrow 2.242(\sigma_a) = 70 \Rightarrow (\sigma_a)_2 = \underline{31.22 \text{ MPa}}$; $(\sigma_a)_1 = \underline{32.51 \text{ MPa}}$

(b) $(\varepsilon_\theta)_1 = \left(\frac{\delta r}{r}\right)_1 = \frac{1}{E_1}\left[(\sigma_\theta)_1 - \nu_1(\sigma_a)_1\right] = \frac{1}{207\times10^3}\left[87.5 - 0.3(32.51)\right] = 0.3756\times10^{-3}$

$\therefore (\delta r)_1 = 0.3756\times10^{-3} \times 125 \text{ mm} = \underline{0.047 \text{ mm}}$.