Concordia University

Department of Mathematics and Statistics

Course	**Number**	**Section**	
MATH	208	All	
Examination	**Date**	**Time**	**Page**
Mid Term FEB. 2013.		1 Hour 30 minutes	2
Instructor	**Course Examiner**		**Marks**
D. Dryanov, L. Dube & U. Tiwari	D. Sen		60
Special Instructions:	**Answer ALL questions.**		

Formulae:

$$\mathbf{A = P(1+i)^n,\ A = Pe^{rt},\ FV = PMT\,\frac{(1+i)^n - 1}{i},\ PV = PMT\,\frac{1-(1+i)^{-n}}{i}}$$

[10] **Q 1.** A small company manufactures picnic tables. The weekly fixed cost is \$1,200 and the variable cost is \$45 per table. Find the total daily cost of producing x picnic tables. How many picnic tables can be produced for a total weekly cost of \$4,800?

[10] **Q 2.** Solve for x in the following equations:

(a) $4^{5x-x^2} = 4^{-6}$

(b) $16^{x-1} = 4^{1+x}$

(c) $3\log_b 2 + \frac{1}{2}\log_b 25 - \log_b 20 = \log_b x$

(d) $\log_a x + \log_a(x+1) = \log_a 6$

[10] **Q 3.** A person borrows \$3,600 and agrees to repay the loan in monthly installments over a period of 3 years. The agreement is to pay 1% of the unpaid balance each month for using the money and \$100 each month to reduce the loan. What is the total cost of the loan over the 3 years?

[10] **Q 4.** Parents have set up a sinking fund in order to have \$140,000 in 15 years for their children's college education. How much should be paid semiannually into an account paying 6.8% compounded semiannually?

[10] **Q 5.** American Express's online banking division offered a money market account with an annual percentage yield(APY) of 5.65%.

(a) If interest is compounded monthly, what is the equivalent annual nominal rate?

(b) If a company wishes to have $1,000,000 in this account after 8 years, what equal deposit should be made each month?

[10] **Q 6.** If you buy a television set for $800 and agree to pay for it in 18 equal monthly payments at 1.5% interest per month on the unpaid balance.

(a) How much are your payments?

(b) How much interest will you pay?

Good Luck!

Solutions to Math208 Midterm, May 2011

Course: 2012 Math208H/4; Instructor: Dr. V. Kalvin

February 11, 2013

[10] Q 1. The total daily cost of producing x picnic tables is $C(x) = \$45 \cdot x + \$1{,}200/7$.

The total weekly cost of producing x picnic tables is $C(x) = \$45 \cdot x + \$1{,}200$. If x tables can be produced for a total weekly cost of \$4,800, then

$$C(x) = \$4{,}800.$$

Therefore we need to solve the equation $\$45 \cdot x + \$1{,}200 = \$4{,}800$, which gives $x = 80$. Thus 80 picnic tables can be produced for a total weekly cost of \$4,800.

[10] Q 2. (a)

$$\begin{gathered} 4^{5x-x^2} = 4^{-6}, \\ 5x - x^2 = -6, \\ x^2 - 5x - 6 = 0, \\ (x+1)(x-6) = 0 \\ x = -1;\ x = 6. \end{gathered}$$

Answer: $x \in \{-1, 6\}$

(b)

$$\begin{gathered} 16^{x-1} = 4^{1+x}, \\ (4^2)^{x-1} = 4^{1+x}, \\ 4^{2x-2} = 4^{1+x}, \\ 2x - 2 = 1 + x, \\ x = 3. \end{gathered}$$

Answer: $x \in \{3\}$.

(c)

$$\begin{gathered} 3\log_b 2 + \frac{1}{2}\log_b 25 - \log_b 20 = \log_b x, \quad b > 0, \\ \log_b 2^3 + \log_b 25^{1/2} - \log_b 20 = \log_b x, \\ \log_b 8 + \log_b 5 - \log_b 20 = \log_b x, \\ \log_b(8 \cdot 5 : 20) = \log_b x, \end{gathered}$$

$$\log_b 2 = \log_b x,$$
$$x = 2.$$

Check:
$$3\log_b 2 + \frac{1}{2}\log_b 25 - \log_b 20 = \log_b 2.$$

As it was found before, the left hand side of the latter equality is equal to $\log_b 2$. Thus $x = 2$ satisfies the initial equation.

Answer: $x \in \{2\}$.

(d)

$$\log_a x + \log_a(x+1) = \log_a 6, \quad a > 0,$$
$$\log_a\big(x(x+1)\big) = \log_a 6,$$
$$x(x+1) = 6,$$
$$x^2 + x - 6 = 0,$$
$$(x+3)(x-2) = 0,$$
$$x = -3;\ x = 2.$$

Check: $x = -3$ does not satisfy the initial equation as $\log_a(-3)$ has no sense as well as $\log_a(-3+1)$.

Next, $x = 2$ gives $\log_a 2 + \log_a 3 = \log_a 6$, which is true (since $\log_a 2 + \log_a 3 = \log_a(2 \cdot 3)$).

Answer: $x \in \{2\}$.

[10] Q 3. In this problem the periodic payments are unequal, so it is not about an ordinary annuity!!! Here every payment is a sum of \$100 (to reduce the balance) and the interest (which is the balance multiplied by the interest rate i = 1% per period). The balance before the first payment is \$3,600. Therefore the interest for the first month is I_1 = \$3,600 · i = \$3,600 · 0.01 = \$36 (and the first payment is \$100 + I_1 = \$136). The balance for the second month is B_2 = \$3,600 − \$100 and, in general, the balance for the n-th month is B_n = \$3,600 − $(n-1)$\$100, $n \leq 36$. Therefore the interest for the second month is $\mathrm{i} \cdot \mathrm{B}_2$ and, in general, the interest for the n-th month is

$$I_n = \mathrm{i} \cdot \mathrm{B_n} = 0.01 \cdot (\$3{,}600 - (\mathrm{n}-1)\$100) = \$37 - \mathrm{n};$$

and the n-th payment is \$100 + I_n, $n \leq 36$, so you can see that the payments are unequal. The total cost of the loan is the total interest plus \$3,600. The total interest is the sum

$$\sum_{n=1}^{36} I_n = I_1 + I_2 + \cdots + I_{35} + I_{36} = \$36 + \$35 + \cdots + \$2 + \$1.$$

Observe that this is the sum of first 36 elements of an arithmetic sequence with the common difference $d = -\$1$, $a_1 = \$36$, and $a_{36} = \$1$. Thus $\sum_{n=1}^{36} I_n = (a_1 + a_{36}) \cdot 36/2 = \$37 \cdot 18 = \$666$. The total cost of the loan is the total interest \$666 plus \$3,600, which is \$4,266.

[10] Q 4. It is about a sinking fund, so we should find PMT from the formula

$$\mathrm{FV} = \mathrm{PMT}\,\frac{(1+\mathrm{i})^{\mathrm{n}}-1}{\mathrm{i}}.$$

Thus we have

$$\mathrm{PMT} = \mathrm{FV}\,\frac{\mathrm{i}}{(1+\mathrm{i})^{\mathrm{n}}-1},$$

where FV = \$140,000, $\mathrm{i} = \frac{6.8\%}{2} = 0.034$, and

$$n = 2(\text{number of payments per year}) \cdot 15(\text{ years}) = 30.$$

Therefore

$$\mathrm{PMT} = \$140{,}000\frac{0.034}{(1+0.034)^{30}-1} \simeq \$2{,}756.92.$$

[10] Q 5. (a) As we know, $\mathrm{APY} = \left(1+\frac{r}{m}\right)^m - 1$, where APY = 0.0565, $m = 12$ (compounded monthly means 12 times per year), and r is the equivalent annual interest rate. Hence we need to solve

$$0.0565 = \left(1+\frac{r}{12}\right)^{12} - 1$$

for r. We have

$$1.0565 = \left(1+\frac{r}{12}\right)^{12},$$

$$(1.0565)^{1/12} = 1 + \frac{r}{12},$$

$$r = 12(1.0565)^{1/12} - 12 \simeq 0.055 = 5.5\%.$$

(b) It is a sinking fund problem. From the formula

$$\mathrm{FV} = \mathrm{PMT}\,\frac{(1+\mathrm{i})^{\mathrm{n}}-1}{\mathrm{i}}$$

we find that

$$\mathrm{PMT} = \mathrm{FV}\,\frac{\mathrm{i}}{(1+\mathrm{i})^{\mathrm{n}}-1}.$$

Here FV = \$1,000,000, $\mathrm{i} = \frac{r}{12} \simeq 0.0046$, and

$$n = 8(\text{years}) \cdot 12(\text{payments per year}) = 96.$$

Thus we have

$$\mathrm{PMT} = \$1{,}000{,}000\frac{0.0046}{(1+0.0046)^{96}-1} \simeq \$8{,}308.96.$$

[10] Q 6. (a) It is about an amortization. From

$$\mathrm{PV} = \mathrm{PMT}\,\frac{1-(1+\mathrm{i})^{-n}}{\mathrm{i}}$$

we find that

$$\mathrm{PMT} = \mathrm{PV}\,\frac{\mathrm{i}}{1-(1+\mathrm{i})^{-n}}.$$

Here PV = \$800, i = 1.5% = 0.015 per period (per month), and $n = 18$. Thus we get

$$\mathrm{PMT} = \$800\frac{0.015}{1-(1+0.015)^{-18}} \simeq \$51.05.$$

(b) The total interest paid is

$$\mathrm{PMT}\cdot 18 - \$800 \simeq \$51.05 \cdot 18 - \$800 = \$118.90.$$