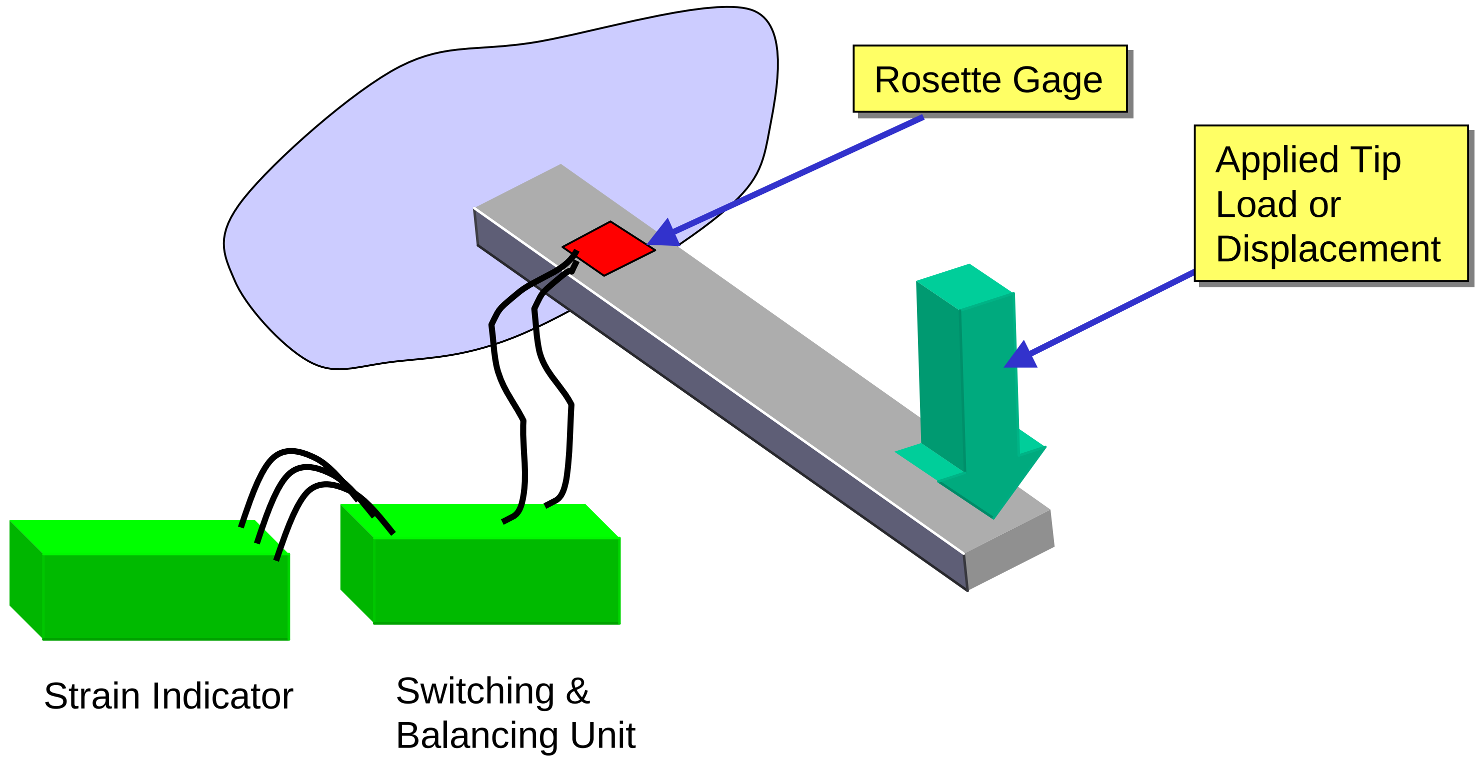
Beam Bending Lab

- Background
  - *beams are a very common structural element*
  - *the beam idealization works for many situations*
  - *beam theory has been presented in several structures classes*
- Objectives of the lab
  - *review and reiterate the essential features of simple beam theory*
  - *study a simple beam configuration in the laboratory*
  - *acquire strain, load and displacement data in the lab*
  - *compare the measured behavior with the predicted*

Simple Beam Theory

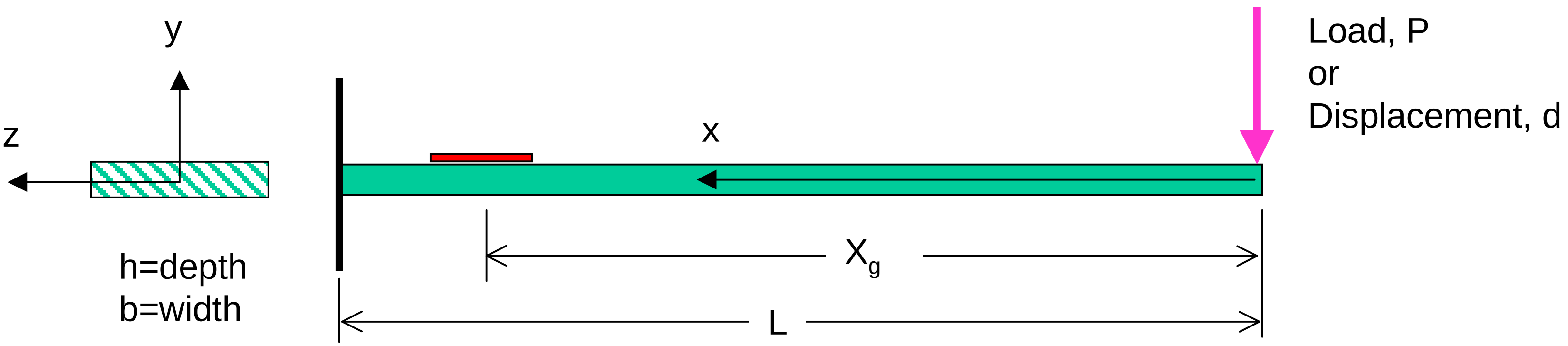
- Experiment:
  - *aluminum beam with simple rectangular cross section*
  - *cantilever support conditions*
  - *loading at the tip using either*
    - *applied tip load*
    - *applied tip deflection*
- Measurements:
  - *applied tip load or displacement*
  - *strains in all elements of rosette gage*
  - *strains in any other strain gages*
- Analysis
  - *determine strain state at rosette gage as function of applied load or applied tip displacements*
  - *estimate the flexural stiffness for loads applied at beam tip*
  - *determine if uniaxial stress state is a reasonable assumption*
  - *estimate values for E and* ν *from strain data*
  - *assess validity of Euler-Bernoulli assumptions*

The Experiment



- Setup: measure beam dimensions and distance from tip load point to center of rosette gage (and any other strain gages)
- Test A: apply tip loads and measure rosette strain and any other strain gage readings
- Test B: apply tip deflection and repeat above measurements

Analysis of the Beam



<u>Stresses</u>:

$$\sigma_x = -\frac{M_z y}{I_z}$$

where

$$M_z = -Px_g$$

$$I_z = \frac{bh^3}{12}$$

$$y = \pm h/2$$

The result is:

$$\sigma_x = \pm\frac{Px_g h}{2I_z} = \pm 6\frac{P}{A}\frac{x_g}{h} = \pm 6\frac{P}{A}\frac{x_g}{L}\frac{L}{h}$$

<u>Note</u>: we assume that $\sigma_y = \sigma_z = \tau_{yz} = \tau_{xz} = 0$ and that only τ_{xy} is nonzero (but $G = \infty$ so $\gamma_{xy} = 0$ also).

Strain State in Beam

- Assumed uniaxial stress state but this does not imply similar strain state…
- Shear, τ_{xz}, is zero and so $\gamma_{xz}=0$.
- Rosette gage is located on top surface of beam and will see a 2D strain state: ε_x, ε_z, γ_{xz}

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu\sigma_z]$$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu\sigma_x]$$

$$\gamma_{xz} = \frac{1}{G}\tau_{xz} = \frac{2(1+\nu)}{E}\tau_{xz}$$

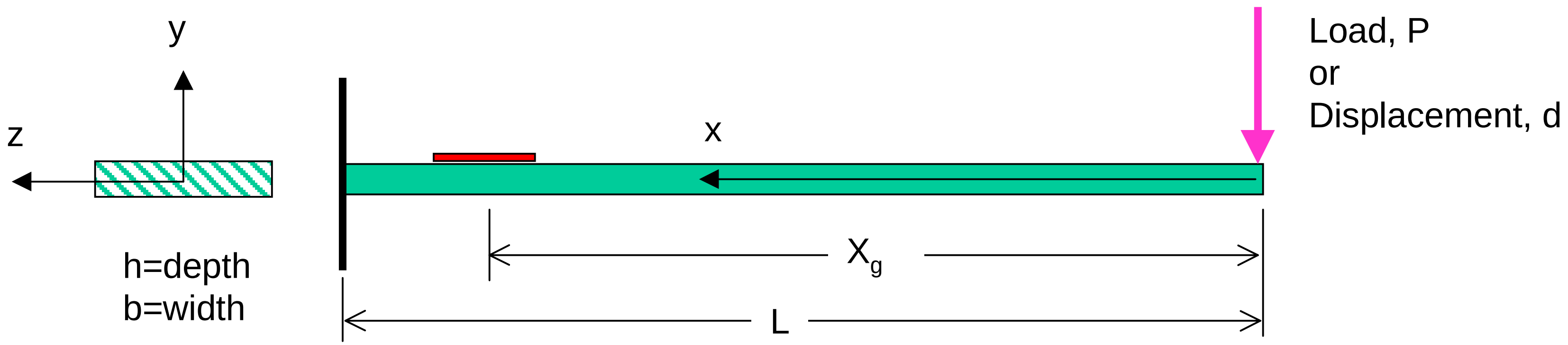
$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_z)$$

$$\sigma_z = \frac{E}{1-\nu^2}(\varepsilon_z + \nu\varepsilon_x)$$

$$\tau_{xz} = G\gamma_{xz} = \frac{E}{2(1+\nu)}\gamma_{xz}$$

- Can use these to compute either:
  - *expected strains at rosette from beam theory, or*
  - *2D stress state from measured strain state at rosette*

Analysis of Beam - cont'd



Lateral Deflection:

$$v(x) = \frac{1}{3}\frac{PL^3}{EI}\left[-\frac{1}{2}\left(\frac{x}{L}\right)^3 + \frac{3}{2}\left(\frac{x}{L}\right) - 1\right]$$

Tip Deflection

$$d = \frac{1}{3}\frac{PL^3}{EI_y} = 4\frac{PL^3}{Ebh^3} = 4\frac{PL}{EA}\left(\frac{L}{h}\right)^2$$

Notes on Rosette Gage Data Analysis

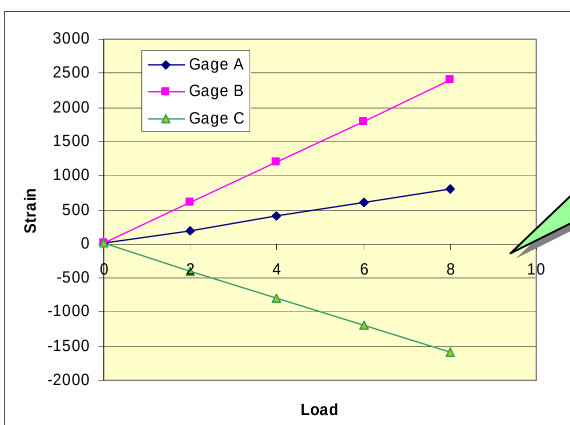
- You will have multiple readings from each gage in a rosette corresponding to different load values.
- How can you get a single “result” for each gage in the rosette?

Load	Gage A	Gage B	Gage C	ε1	ε2	θ
0	6	5	9	8	8	-0.515
2	206	605	-391	1164	1164	-0.583
4	403	1207	-799	2333	2333	-0.583
6	605	1803	-1194	3486	3486	-0.583
8	808	2401	-1594	4646	4646	-0.582
4	406	1204	-794	2326	2326	[illegible]

Which princ. strain or angle do you use to compare to theory?

Any row contains consistent results for a given load.

Could add a row and compute averages of data for Load and Gages A-C...



Or do regression analysis of each column and use these results...

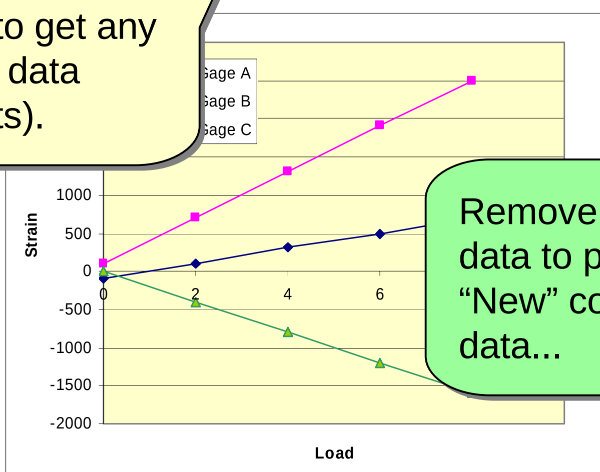
Notes on Rosette Gage Data Analysis - cont'd

- Best approach:
  - *do regression analysis for gage A-C data; intercept is the zero error for each gage so subract it out (new column)*
  - *now compute ε_1, ε_2, θ for each load*
  - *do regression analysis on ε_1, ε_2, θ columns; slopes provide the final answer (output per unit load) to compare with theory*

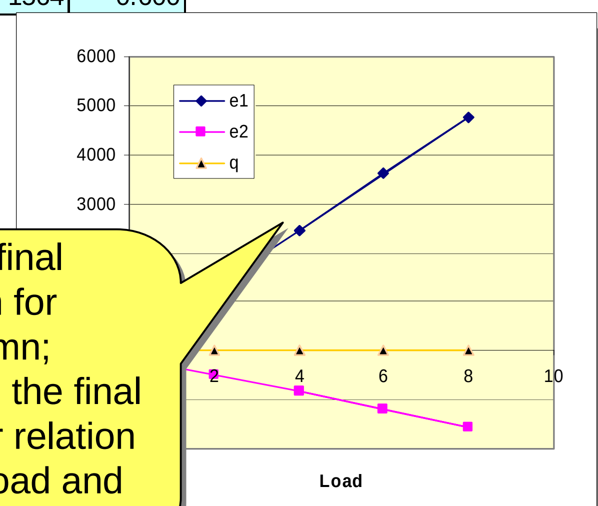
Load	Gage A	Gage B	Gage C	New A	New B	New C	ε1	ε2	θ
0	-99	104	7	1	4	7	162	-157	[illegible]
2	106	702	-395	206	602	-395	1287	-479	-0.6 [illegible]
4	307	1303	-793	407	1203	-793	2446	-836	-0.61 [illegible]
6	502	1903	-1195	602	1803	-1195	3607	-1202	-0.605
[illegible]	7?5	2507	-1598	805	2407	-1598	4776	-1564	-0.600

Compute principal strains and directions for each load.

Use regression analysis to get any offsets in data (intercepts).



Remove offsets in data to produce "New" corrected data...



NOW run final regression for each column; slopes are the final answer for relation between load and principal strains.