MA129 Mock Final

Name: _____

Time Allowed: 120 minutes
Total Value: 85 marks
Number of Pages: 9

Instructions:

Cheat Sheet: One 8.5" × 11" page of study notes (both sides) is allowed as a reference while completing the mock test. Please note, that the cheat sheet is permitted for the mock test only!!

Non-programmable, non-graphing calculators are permitted. No other aids allowed.

Check that your test paper has no missing, blank, or illegible pages. Note that test questions appear on ***both*** *sides of the paper.*

Answer in the spaces provided.

Show all your work. Insufficient justification will result in a loss of marks.

1. [2 *marks*] Solve the equation for x: $4^{3x+1} = 16^{2x-5}$

2. [5 *marks*] Solve the equation for x: $\log_2(x-4) = 3 - \log_2(x-2)$

3. [3 *marks*] Determine the domain of the function $f(x) = \dfrac{\ln\left(x^2 - 4\right)}{x + 5}$. Express your answer using interval notation.

4. [4 *marks*] Determine the equation of the line tangent to the curve $y = f(x) = \ln x + x^2$ at $x = 1$. Express your answer in the form $y = mx + b$.

5. [2 *marks*] Determine the derivative of the function: $g(x) = x^2 \log_4(3 - x)$

6. [4 *marks*] Determine y'' given: $y = 3^{2x+1} + \left(2x^3 - 1\right)^{1/3}$

7. [6 *marks*] Suppose you own an apartment building containing 100 units. If you charge \$400 per month for each unit, then all units can be rented out. For every \$20 increase in monthly rent, you will lose one customer. What monthly rent should you charge to maximize your revenue? Show that the corresponding revenue is an absolute maximum.

8. [3 *marks*] Given the following matrices,

$$A = \begin{bmatrix} 3 & -1 & 2 \\ 0 & 1 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & 1 \\ 0 & -1 \\ 1 & 0 \end{bmatrix},$$

determine $(A - 2B)\,C$.

9. [9 *marks*] Let $f(x) = \dfrac{x^2}{x-2}$. Then $f'(x) = \dfrac{x^2-4x}{(x-2)^2}$ and $f''(x) = \dfrac{8}{(x-2)^3}$.

(a) State the intervals on which f is increasing and those on which f is decreasing.

(b) Determine the coordinates of all relative maximum points and relative minimum points.

(c) State the intervals on which the graph of f is concave up and those on which it is concave down.

10. [2 *marks*] Evaluate the integral: $\displaystyle\int \frac{x^4 + 3x^2 - 4}{x^2}\, dx$

11. [2 *marks*] Evaluate the integral: $\displaystyle\int \left(\frac{1}{x} + \frac{x}{e} - x^e + e^x - 2^{e+1}\right)\, dx$

12. [4 *marks*] Evaluate the definite integral: $\displaystyle\int_0^1 x^2\sqrt{3x^3 + 1}\, dx$

13. [4 *marks*] Evaluate the integral: $\displaystyle\int \frac{4x}{x^2 + 1}\ln\left(x^2 + 1\right)\, dx$

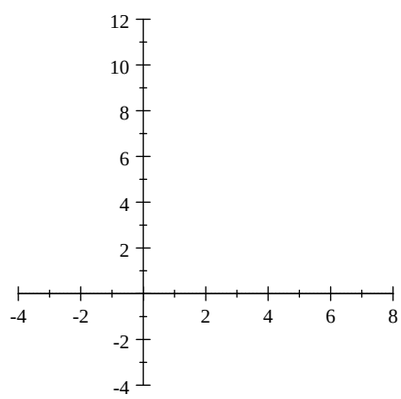
14. [4 *marks*] Simplify each of the following expressions:

(a) $5^{2\log_5 4} - \ln e^3$

(b) $\int_0^{10} f(x)\ dx$, given $\int_4^0 f(x)\ dx = 5$, $\int_2^7 f(x)\ dx = 12$, $\int_2^4 f(x)\ dx = 3$, $\int_{10}^7 f(x)\ dx = 1$.

15. [8 *marks*] Consider the parabola given by $y = x^2 - 7x + 10$.

(a) Find the vertex and x-intercepts of the parabola, and sketch its graph.



(b) Determine the total area between the parabola and the x-axis, from $x = 0$ to $x = 4$.

16. [6 *marks*] After a new product has been launched onto the market, its sales volume is given by

$$S = (AT + 450)\left(A + T^2\right)^{-1/2},$$

where T = time (in months) since the product was first introduced and A = amount (in hundreds of dollars) spent monthly on advertising.

(a) Verify that the partial derivative of sales volume with respect to time is:

$$\frac{\partial S}{\partial T} = \left(A^2 - 450T\right)\left(A + T^2\right)^{-3/2}.$$

(b) Suppose advertising is fixed at \$9000 per month [i.e., $A = 90$]. Determine when the monthly sales volume will begin to decrease.

17. [9 *marks*] Let $f(x,y) = 8x^3 + y^3 - 12xy - 15$.

(a) Determine all the critical points of $f(x,y)$.

(b) For each of the critical points found above, use the second derivative test to determine whether it corresponds to a relative maximum, a relative minimum or neither (saddle point).

18. [8 *marks*] A firm has an order for 10 000 units of its product and has two plants at which to manufacture these units. Let x be the number of units to be produced at *Plant A* and y denote the number to be manufactured at *Plant B*; that is, $x + y = 10\ 000$.

    It is known that the cost function is given by: $c(x, y) = 48x^3 + 3y^3 + 25\ 000$.

    Use the method of Lagrange multipliers to determine how many units should be produced at each plant to minimize this cost function. Then determine this minimum cost.

    [You can assume the cost is minimized at the single critical point obtained.]